

Q1

- (a) The decay reaction is
- $^{210}\text{Po} \rightarrow ^{206}\text{Pb} + ^4\text{He}$

The atomic mass of He-4 can be found on p2 of the 4M16 Data Sheet

$$\Delta = 209.98287 - 205.97447 - 4.00260 = 5.8 \times 10^{-3} \text{ u}$$

1 u  $\equiv$  931.5 MeV (4M16 Data Sheet p1)

$$\therefore \text{energy of } \alpha \text{ particle} = 5.8 \times 10^{-3} \times 931.5 = 5.403 \text{ MeV} \quad [10\%]$$

- (b) The number of Po-210 atoms in 1 g

$$N = \frac{m}{M} L = \frac{10^{-3}}{0.210} \times 6.022 \times 10^{23} = 2.868 \times 10^{21}$$

$$\lambda = \frac{\ln 2}{T_{1/2}} = \frac{0.693}{138.4 \times 24 \times 3600} = 5.797 \times 10^{-8} \text{ s}^{-1}$$

So, the activity of 1 g of Po-210 is

$$A = \lambda N = 5.797 \times 10^{-8} \times 2.868 \times 10^{21} = 1.663 \times 10^{14} \text{ Bq}$$

and the power

$$P = Ae$$

where  $e$  is the energy released per decay

$$\therefore P = 1.663 \times 10^{14} \times 5.403 [\text{MeV}] \times 1.602 \times 10^{-13} [\text{J/MeV}] = 143.9 \text{ W} \quad [15\%]$$

- (c) Assume that all the energy of the emitted alphas is deposited in tissue. There may be some self-shielding effects but these are hard to estimate; neglecting them will result in an over-estimation of the dose rate.

From (b), the ingestion of 1 g of Po-210 leads to an energy deposition rate of  $\dot{E} = 143.9 \text{ W}$ .

In a 60 kg human, this gives rise to a whole body absorbed dose rate

$$\dot{D} = \frac{\dot{E}}{m} = \frac{143.9}{60} = 2.398 \text{ Gy s}^{-1}$$

and an equivalent dose rate of

$$\dot{H}_T = W_R \dot{D} = 20 \times 2.398 = 47.96 \text{ Sv s}^{-1}$$

or

$$\dot{H}_T = 47.96 \times 3600 = 172.66 \times 10^3 \text{ Sv hr}^{-1}$$

If the ingestion of 1 g of Po-210 leads to this equivalent dose rate, the mass needed for the equivalent dose rate specified in the question is

$$m_{\text{Po}} = \frac{10}{172.66 \times 10^3} = 5.792 \times 10^{-5} \text{ g}$$

[20%]

- (d) 30 days is a significant fraction of the half-life of Po-210, so assuming constant activity over the period of irradiation would be a poor approximation. As a conservative (dose over-estimating) approximation assume that all the Po-210 inhaled stays in the body.

If 1  $\mu\text{g}$  is inhaled the initial activity will be  $10^{-6}$  times that calculated in (b), i.e.  $A_0 = 1.663 \times 10^8 \text{ Bq}$ .

Activity varies as  $A_0 \exp(-\lambda t)$ , so the total number of decays in 30 days ( $2.592 \times 10^6$  s)

$$n = \int_0^T A_0 \exp(-\lambda t) dt = \frac{A_0}{\lambda} [1 - \exp(-\lambda T)]$$

$$n = \frac{1.663 \times 10^8}{5.797 \times 10^{-8}} [1 - \exp(-5.797 \times 10^{-8} \times 2.592 \times 10^6)] = 4.002 \times 10^{14}$$

Assuming all the energy from the decays is deposited within the body (again, a conservative assumption)

$$E = 4.002 \times 10^{14} \times 5.403 [\text{MeV}] \times 1.602 \times 10^{-13} [\text{J/MeV}] = 346.2 \text{ J}$$

$$\therefore \text{ Absorbed dose } D = \frac{E}{m} = \frac{346.2}{60} = 5.770 \text{ Gy}$$

$$\therefore \text{ Equivalent dose } H_T = W_R D = 20 \times 5.77 = 115.4 \text{ Sv}$$

If all the energy from the decays is deposited in the lungs

$$\text{ Absorbed dose } D = \frac{E}{m} = \frac{346.2}{1} = 346.2 \text{ Gy}$$

$$\therefore \text{ Equivalent dose } H_T = W_R D = 20 \times 346.2 = 6924 \text{ Sv}$$

$$\therefore \text{ Effective dose } E = W_T H_T = 0.12 \times 6924 = 830.9 \text{ Sv} \quad [30\%]$$

- (e) The mass of Po-210 calculated in (c) would lead to a dose of 50 Sv in five hours. It is certainly credible that 58  $\mu\text{g}$  of Po-210 could be ingested in this manner, and thus the criminal could administer a potentially lethal amount of Po-210 to a victim or themselves in this way.

The calculation in (d) indicates that the inhalation of 0.1  $\mu\text{g}$  of Po-210 could lead to an effective dose of more than 10 Sv over the course of 30 days. The dose would be very localised rather than spread over the 60 kg of the whole organism, but an inhaled sample would pass close to several major organs and the lungs are among the most vulnerable to radiation exposure.

If one assumes that the total amount of Po-210 in the vial was 100  $\mu\text{g}$  (i.e. comfortably more than the 58  $\mu\text{g}$  needed according to the calculation in (c)), then 0.1  $\mu\text{g}$  represents 0.1% of this. A typical breath draws in  $\sim 1$  g of air, so 0.1  $\mu\text{g}$  would be a tiny fraction of a single breath, but whether such an amount could be inhaled in a couple of sniffs of an ostensibly empty vial does stretch credibility somewhat. [15%]

- (f) Being an  $\alpha$  emitter, Po-210 is only harmful if inhaled or ingested. The glass and the champagne itself will block almost all of the  $\alpha$  particles being emitted; there will be some  $\alpha$  particles emitted upwards from the surface of champagne. For the criminal's threat to deter the police officer, the officer will need to know (or think) that the champagne contains a radionuclide. If they don't know what the nuclide is and what radiation it emits, then the threat might work. If the officer knows the nuclide is Po-210 and its properties as an  $\alpha$  emitter, then they may understand that it will only be harmful if the criminal throws the drink at them in such a way that they inhale or ingest it – simply turning away at the crucial moment should be sufficient to keep them safe. One can assume that the police officer knows something of the criminal's *modus operandi*, if they are attempting to apprehend them, but it seems unlikely they will know much about radiation health physics (in common with most of the general public). The scene is therefore not implausible – but quite amusing to viewers who do have health physics knowledge. [10%]

There's no single correct answer to (f) and any well-argued answer either way received due credit.

This question was inspired by Series 1 Episode 3 of *Crossing Lines*.

### Assessor's Comments:

*All candidates: 54 attempts, Average raw mark 12.5/20, Maximum 18, Minimum 3.*

A reasonably popular question attempted by 69% of candidates.

Candidates generally showed a good understanding of the underlying health physics and the calculations required.

Marks were lost unnecessarily by some candidates due to failures to explain calculations being attempted sufficiently clearly to enable the cause of incorrect numerical answers to be diagnosed, impacting on the assessor's ability to award partial credit.

Marks were also lost unnecessarily due to failures to state and justify assumptions.

Some answers to part (d) revealed a lack of understanding of the differences between equivalent and effective doses.

Several answers neglected the effect of the decay of Po-210 over 30 days in answering (d).

Some calculations in part (d) went astray by overlooking the mass of the lungs in calculating the effective dose.

Many answers failed to recognise that the amounts of Po-210 determined or specified in (c) and (d) were in effect arbitrary (to provide the basis for calculations) and assumed that the criminal or housekeeper received exactly those doses in answering (e).

Many answers to (e) failed to consider what a 'reasonable' amount of Po-210 that could be ingested was.

Few answers to part (e) gave any consideration to the viability of a fatal dose being inhaled by sniffing an 'empty' vial.

Few answers to part (f) gave any consideration to the health physics awareness of the arresting officer.

Q2

(a)

$$\sigma_a = \sigma_c + \sigma_f$$

$$\Sigma_a = \sum_i \Sigma_{ai}$$

$$\Sigma_{ai} = N_i \sigma_{ai} = \frac{f_i \rho_i N_A}{M_i} \sigma_{ai}$$

$f_i$  = isotopic abundance,  $\rho_i$  = mass per unit volume,  $M_i$  = molar mass,  $N_A$  = Avogadro's number

$$\begin{aligned} \therefore \Sigma_a &= \frac{e \times 500 \times 6.022 \times 10^{26}}{238} \times (107 + 580) \times 10^{-28} \\ &\quad + \frac{(1 - e) \times 500 \times 6.022 \times 10^{26}}{238} \times (2.75 + 0) \times 10^{-28} + 10.0 \end{aligned}$$

$$\therefore \Sigma_a = 86.914e + (1 - e) \times 0.3479 + 10.0 = 86.57e + 10.35$$

Only U-235 is fissionable

$$\therefore \Sigma_f = \frac{e \times 500 \times 6.022 \times 10^{26}}{238} \times 580 \times 10^{-28} = 73.38e \text{ m}^{-1}$$

[15%]

(b)

$$\eta = \frac{\nu \Sigma_f}{\Sigma_a} = \frac{2.43 \times 73.38e}{86.57e + 10.35} = \frac{178.31e}{86.57e + 10.35}$$

[10%]

(c) Let  $B^2 = (\eta - 1)\Sigma_a/D$  and assume that  $\phi(r, z) = F(r)Z(z)$ .

$$\begin{aligned} \therefore \frac{Z}{r} \frac{\partial}{\partial r} \left( r \frac{\partial F}{\partial r} \right) + F \frac{\partial^2 Z}{\partial z^2} + B^2 FZ &= 0 \\ \therefore \frac{1}{Fr} \frac{\partial}{\partial r} \left( r \frac{\partial F}{\partial r} \right) + \frac{1}{Z} \frac{\partial^2 Z}{\partial z^2} + B^2 &= 0 \end{aligned}$$

This implies that

$$\frac{1}{Z} \frac{d^2 Z}{dz^2} + \beta^2 = 0$$

and

$$\therefore \frac{1}{Fr} \frac{d}{dr} \left( r \frac{dF}{dr} \right) + \alpha^2 = 0$$

with

$$\alpha^2 + \beta^2 = B^2$$

The equation in  $z$  is an SHM equation, so the general solution is

$$Z(z) = A \sin(\beta z) + C \cos(\beta z)$$

If the origin of the coordinate system is at the centre of the cylinder, then by symmetry

$$A = 0$$

With the boundary condition that  $\phi = 0$  at  $z = \pm H_0/2$ , then

$$\beta \frac{H_0}{2} = \frac{\pi}{2} \implies \beta = \frac{\pi}{H_0}$$

The equation in  $r$  is a Bessel equation of zero order, so the general solution is

$$F(r) = P J_0(\alpha r) + Q Y_0(\alpha r)$$

As  $r \rightarrow 0$ ,  $Y_0(\alpha r) \rightarrow -\infty$ . This would give infinite flux at the centre of the system, which is physically impossible, so  $Q = 0$ .

The first zero of  $J_0(x)$  occurs at  $x = 2.405$ , so with a boundary condition that  $\phi = 0$  at  $r = R_0$

$$\alpha R_0 = 2.405 \implies \alpha = \frac{2.405}{R_0}$$

The requirement that  $\alpha^2 + \beta^2 = B^2$  thus means that

$$\left( \frac{2.405}{R_0} \right)^2 + \left( \frac{\pi}{H_0} \right)^2 = \frac{(\eta - 1)\Sigma_a}{D}$$

This is the *criticality condition*.

[40%]

(d) Using the results in parts (a) and (b)

$$\frac{(\eta - 1)\Sigma_a}{D} = \frac{1}{D} \left[ \frac{178.31e}{86.57e + 10.35} - 1 \right] [86.57e + 10.35]$$

$$\therefore \frac{(\eta - 1)\Sigma_a}{D} = \frac{1}{D} [178.31e - 86.57e - 10.35] = \frac{91.74e - 10.35}{D}$$

For the case where  $e = 0.15$  and  $D = 0.025$  m

$$\therefore \frac{(\eta - 1)\Sigma_a}{D} = \frac{91.74 \times 0.15 - 10.35}{0.025} = 136.44$$

If extrapolation distances can be neglected  $R = R_0$  and  $H = H_0$ , so the criticality condition becomes

$$\left(\frac{2.405}{R}\right)^2 + \left(\frac{\pi}{H}\right)^2 = \frac{(\eta - 1)\Sigma_a}{D} = 136.44$$

If  $R = 0.45$  m, the system becomes critical when

$$\left(\frac{2.405}{0.45}\right)^2 + \left(\frac{\pi}{H}\right)^2 = 136.44 \Rightarrow \left(\frac{\pi}{H}\right)^2 = 136.44 - 28.56 = 107.88 \Rightarrow H = 0.302 \text{ m}$$

[15%]

(e) With  $R = 0.45$  m, the criticality condition can be written as

$$\left(\frac{2.405}{0.45}\right)^2 + \left(\frac{\pi}{H}\right)^2 = \frac{91.74e - 10.35}{D}$$

As the height of solution in the tank increases, the term  $\left(\frac{\pi}{H}\right)^2$  in the criticality equation gets smaller. In the limit it becomes negligible and the limiting value of  $e$  can then be found from

$$\left(\frac{2.405}{0.45}\right)^2 = 28.56 = \frac{91.74e - 10.35}{0.025}$$

$$\therefore e = \frac{28.56 \times 0.025 + 10.35}{91.74} = 0.121$$

[20%]

### Assessor's Comments:

*All candidates: 77 attempts, Average raw mark 13.2/20, Maximum 19, Minimum 2.*

The most popular question attempted by all but one candidate and done well by many.

A common source of inaccuracy and/or error in answering (a) was a lack of attention to detail in calculating the required atom number densities.

Several candidates used an incorrect formula to calculate  $\eta$  in (b).

Many candidates lost marks in (c) by failing to justify steps in their derivations. There were also several cases of inconsistency between symmetry arguments and boundary conditions imposed.

A number of candidates calculated unphysical numerical results (values of  $\eta$  larger  $\nu$  or values of  $e$  larger than 1) and accepted these without comment.

Many candidates answered (e) by finding the value of  $e$  for which  $\eta$  is less than 1. Although this guarantees that criticality cannot be reached, it is a lower value of  $e$  than the value found if the system geometry is taken into account, as in the model answer.

Q3

(a) The *radial form factor* is defined as

$$F_R = \frac{\text{Power at the centre of the core}}{\text{The average power across the central plane at } x = 0}$$

Hence

$$F_R = \frac{P_0}{\frac{1}{\pi R^2} \int_0^R P_0 J_0\left(\frac{2.405r}{R}\right) 2\pi r dr} = \frac{R^2}{2 \int_0^R J_0\left(\frac{2.405r}{R}\right) r dr}$$

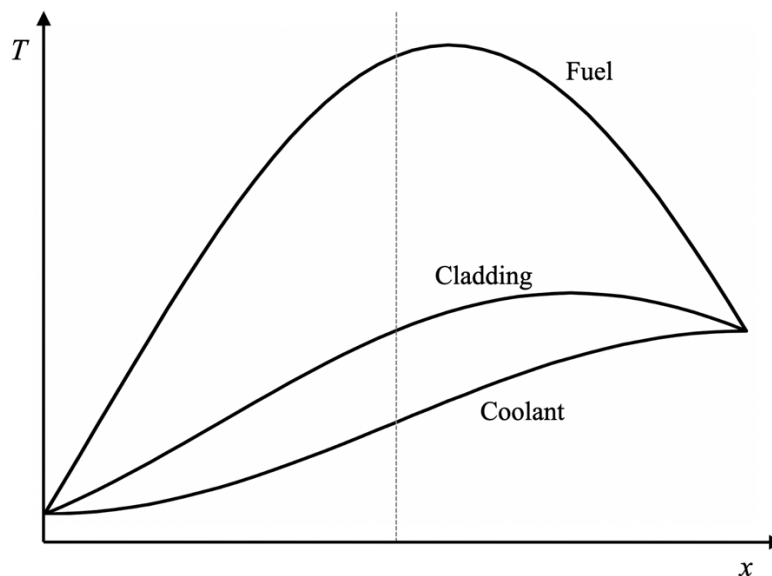
Now, using the information on 4M16 data sheet (p7):

$$\int_0^R J_0\left(\frac{2.405r}{R}\right) r dr = \frac{Rr}{2.405} J_1\left(\frac{2.405r}{R}\right) \Big|_0^R = \frac{R^2}{2.405} \times 0.5183 = 0.2155R^2$$

$$F_R = \frac{R^2}{2 \times 0.2155R^2} = 2.32$$

[20%]

(b)



Key features:

- $T_{coolant} = T_{cladding} = T_{fuel}$  at the ends as the power is zero there – not a ‘chopped’ cosine as extrapolation distances are negligible
- $T_{coolant}$  is symmetric about the channel centre
- $T_{coolant} < T_{cladding} < T_{fuel}$  except at the ends
- The differences  $(T_{cladding} - T_{coolant})$  and  $(T_{fuel} - T_{cladding})$  are cosines
- The maximum  $T_{cladding}$  occurs nearer the end of the channel than the maximum  $T_{fuel}$ ; both occur beyond half-way along the channel
- The maximum  $T_{coolant}$  occurs at the channel end (exit)
- $dT_{coolant}/dx = 0$  at the channel ends

[25%]

(c)

- (i) From the steady flow energy equation applied to a channel, the channel power  $\mathbf{P}$  is related to the coolant temperature rise by

$$\mathbf{P} = \dot{m}c_p(T_{co} - T_{ci})$$

The coolant inlet temperature  $T_{ci}$  will be the same for all channels. Thus, if  $T_{co}$  is also the same and  $c_p$  is constant, then  $\mathbf{P} \propto \dot{m}$ .

From part (a) the mean channel power  $\mathbf{P}_{ave} = \frac{1}{2.32}\mathbf{P}_{cen} = 0.431\mathbf{P}_{cen}$ , where  $\mathbf{P}_{cen}$  is the maximum channel power (the power of the central channel). Hence  $\dot{m}_{ave} = 0.431\dot{m}_{cen}$ , so a reduction of 56.9% in coolant flow rate is required. [15%]

- (ii) If channels are ‘gagged’ in this way, so that the coolant outlet temperature is the same for all channels, then this increases the average coolant outlet temperature and eliminates the entropy creation due to irreversible mixing of flows at different temperatures. Both of these effects will improve (increase) the efficiency of the thermodynamic (power production) cycle. [10%]
- (d) The temperature distributions can be determined using Ginn’s equation. In this case, as extrapolation distances are negligible:

$$\theta = 2 \left( \frac{T - T_{ci}}{T_{co} - T_{ci}} \right) - 1 = \sin \left( \frac{\pi x}{2L} \right) + Q \cos \left( \frac{\pi x}{2L} \right)$$

where  $Q = \pi \dot{m}c_p / UA$ .

If the coolant inlet and outlet temperatures are the same, the maximum temperature within a channel depends on the value of  $Q$ . If heat transfer coefficients and specific heat capacities are unchanged, then  $Q \propto \dot{m}$ . Thus, as  $\dot{m}$  is lower for the mean channel, the maximum fuel temperature will be lower than for the central channel and the hot-spot limitation will not apply. [15%]

- (e) Form factors can be improved by the use of:

- A reflector
- An ‘out-in’ loading pattern – fresh fuel at the periphery of the core and once/twice burnt fuel in the interior
- Burnable poisons to suppress the flux in assemblies that would otherwise be high power due to their location and reactivity
- Control rods to shape the flux

[15%]

### Assessor’s Comments:

*All candidates: 36 attempts, Average raw mark 10.7/20, Maximum 20, Minimum 2.*

The least popular question, attempted by only 46% of candidates.

The question’s unpopularity was possibly because it is more discursive than the other questions, possibly because it combined multiple 4M16 topics.

Part (a) was done well by candidates who could remember the definition of the radial form factor.

Many candidates lost marks in (b) by taking insufficient care in sketching the temperature distributions. The number of marks available and the use of “carefully” in the question were both clues as to the level of detail required for full credit.

Many candidates incorrectly believed that gagging coolant channels in the way proposed would eliminate hot spots (the hot spot, which is in the central channel, is unaffected), not appreciating

that the principal benefit gained is thermodynamic efficiency improvements in the reactor’s power generation cycle.

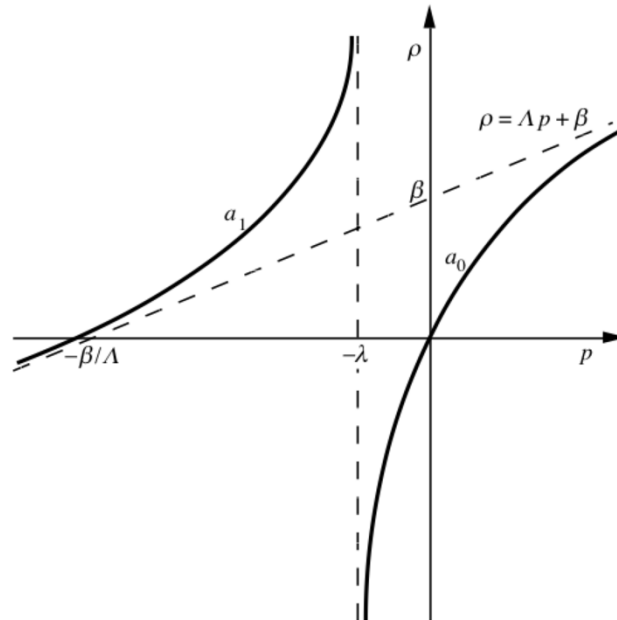
Many answers to (e) only mentioned the use of reflectors. For full credit candidates needed to mention three of the methods listed in the model answer.

Some candidates suggested making the reactor spherical to improve its form factors – hardly an answer describing what is done “in practice”!

Q4

- (a) The major simplifications of this model are that it assumes there is no spatial variation in behaviour, whereas in practice the reactor core is highly heterogeneous and the neutron population varies spatially, and it also assumes that there is only one type of delayed neutron precursor, whereas in reality there are a large number of them with widely varying production rates and half-lives. [10%]

(b)



This characteristic shows that for a negative value of  $\rho$ , there are two negative values of  $p$  that satisfy the equation. These correspond to negative decaying exponential terms in the solution for  $n$ . The more negative (given by the characteristic labelled  $a_1$ ) corresponds to a very short time constant; the less negative (given by the characteristic labelled  $a_0$ ) to a longer time constant, which ultimately dictates the system response.

For a positive value of  $\rho$ , one negative and one positive value of  $p$  satisfy the equation. The negative one (given by the characteristic labelled  $a_1$ ) corresponds to a short time constant decaying exponential term. The positive one (given by the characteristic labelled  $a_0$ ) corresponds to a growing exponential term which dictates the system response. [30%]

- (c) In steady state  $\frac{dc}{dt} = 0$

$$\begin{aligned} \therefore \frac{\beta}{\Lambda} n_0 - \lambda c_0 &= 0 \\ \therefore \frac{c_0}{n_0} &= \frac{\beta}{\lambda \Lambda} = \frac{0.0075}{0.1 \times 0.5 \times 10^{-3}} = 150 \end{aligned}$$

[10%]



- (d) The prompt jump approximation (PJA) assumes that the neutron population remains in equilibrium with the precursor population, even when the latter is varying with time. Thus, for a source-free system:

$$\frac{dn}{dt} \approx 0 = \frac{\rho - \beta}{\Lambda} n + \lambda c \implies n = \frac{\Lambda \lambda}{\beta - \rho} c \quad (3.1)$$

Using this expression to substitute for  $n$  in the precursor equation gives:

$$\frac{dc}{dt} = \frac{\beta \lambda}{\beta - \rho} c - \lambda c = \frac{\rho \lambda}{\beta - \rho} c$$

By inspection, this has a solution:

$$c = c_0 \exp\left(\frac{\rho \lambda}{\beta - \rho} t\right) \quad (3.2)$$

if  $c = c_0$  at  $t = 0$ .

As the system was operating in steady state for a prolonged period,  $n = n_0$  and  $\rho = 0$  before the change in  $\rho$ . Then from part (c):

$$\beta n_0 = \Lambda \lambda c_0 \quad (3.3)$$

So, combining equations (3.1), (3.2) and (3.3), the variation in the neutron population after the change in  $\rho$  is given by:

$$n = \frac{\Lambda \lambda}{\beta - \rho} c = \frac{\Lambda \lambda}{\beta - \rho} c_0 \exp\left(\frac{\rho \lambda}{\beta - \rho} t\right) = \frac{\beta}{\beta - \rho} n_0 \exp\left(\frac{\rho \lambda}{\beta - \rho} t\right)$$

Thus, the time constant predicted by the PJA is

$$T_+ = \frac{\beta - \rho}{\rho \lambda} = \frac{0.0075 - 0.0025}{0.0025 \times 0.1} = 20 \text{ s}$$

In the absence of delayed neutron precursors, the time constant for the growth of the neutron population would be

$$T_+ = \frac{\Lambda}{\rho} = \frac{0.5 \times 10^{-3}}{0.0025} = 0.2 \text{ s}$$

Thus, the delayed neutrons helpfully increase the governing time constant by a factor of 100 in this case. [50%]

### Assessor's Comments:

*All candidates: 69 attempts, Average raw mark 12.0/20, Maximum 19, Minimum 2.*

A popular question, attempted by 88% of candidates and done well by quite a few of those. Answers to (a) sometimes confused the meanings of homogeneous (spatially invariant core properties) and 'lumped' (spatially invariant neutron and precursor populations).

Sketches of the relationship between  $p$  and  $\rho$  in (b) varied from the meticulous to the bizarre. As with Q3(b), a lack of care, despite the number of marks available, was evident in many answers. The quality of discussion in (b) was very variable – from the excellent to the non-existent.

Almost every candidate got (c) correct. Those who didn't contrived calculator errors.

While there were plenty of correct derivations of the prompt jump approximation (PJA) solution, plenty also went astray. Several candidates correctly found the solution for the variation of the precursor population but then overlooked the fact that the PJA assumes the neutron population stays in equilibrium with the precursors. Others tried, usually unsuccessfully to solve the original neutron kinetics equations without using the PJA at all.