Version GTP/1

EGT3 / EGT2
ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 28 April 2023   2 to 3.40

Module 4M16

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4M16 Nuclear Power Engineering data sheet (8 pages)
Engineering Data Books

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 You have been asked to provide advice to the scriptwriters of a popular TV crime drama. It is believed that the former Russian spy Alexander Litvinenko, who died in November 2006, was "poisoned" by polonium-210. The writers have decided to use this method of poisoning as the basis for an episode in which a criminal kills their victims by 'spiking' their drinks with Po-210.

(a) Polonium-210 decays by alpha emission into stable lead-206 with a half-life of 138.4 days. Given that the atomic masses of Po-210 and Pb-206 are 209.98287 u and 205.97447 u respectively, show that the energy of the alpha particle released in the decay is 5.403 MeV, assuming it takes up all the energy released. [10%]

(b) Show that the power released by 1 g of Po-210 is 143.9 W. [15%]

(c) Rather than facing trial and imprisonment, the script envisages the criminal taking their own life by drinking a glass of champagne spiked with Po-210. Assuming the criminal weighs 60 kg, estimate the mass of Po-210 that must be ingested to give an initial whole-body equivalent dose rate of 10 Sv hr^{-1}. The radiation weighting factor for alpha particles is 20. State and justify any assumptions you make. [20%]

(d) The script also envisages a housekeeper who has sniffed an 'empty' vial that contained the Po-210 used to poison their employer being hospitalised a month after the sniffing incident. If 1 μg of Po-210 was inhaled when the vial was sniffed and the housekeeper weighs 60 kg, estimate the total whole-body equivalent dose received over 30 days and the effective dose, if it is assumed that all the energy from the decay of Po-210 is deposited in the lungs. The organ weighting factor for the lungs is 0.12. Assume that the housekeeper's lungs weigh 1 kg. Again, state and justify any assumptions you make. [30%]

(e) A radiation dose of 50 Sv will incapacitate a human almost immediately and it is unlikely that a human would survive an acute radiation dose of 10 Sv. In the light of this information and your calculations in (c) and (d) comment on the plausibility of these two elements of the script. [15%]

(f) The writers have also included a scene where the criminal threatens a police officer attempting to apprehend them by holding out a glass of champagne spiked with Po-210 and warning the officer to stay away for their own safety. Comment on the credibility of this scene. [10%]

2 An engineer is designing a cylindrical tank for the storage of liquids containing fissionable material in a fuel preparation plant. The tank is to be of diameter 0.9 m and will be surrounded by a non-reflecting neutron absorber. The fuel being processed is high assay low-enriched uranium (HALEU) containing U-235 at enrichment e of up to 0.20 (20 at%) and $(1 - e)$ U-238. The liquid contains 500 kg of uranium per m^3 of fluid.

(a) Using the nuclear data below, show that the macroscopic absorption cross-section of the solution is given by

$$\Sigma_a = 86.57e + 10.35 \ [\mathrm{m}^{-1}]$$

and find an equivalent expression for the macroscopic fission cross-section.
You can assume that the combined macroscopic absorption cross-section of everything in the solution other than the uranium is 10.0 m^{-1}. The molar mass of uranium can be taken to be 238 kg kmol^{-1} for all values of e.
Data: U-235: $\sigma_c = 107$ barns, $\sigma_f = 580$ barns; U-238: $\sigma_c = 2.75$ barns, $\sigma_f = 0$ barns. [15%]

(b) Given the average number of neutrons released in a U-235 fission reaction ν is 2.43, find an expression for η, the average number of neutrons released in fission per neutron absorbed in the solution, as a function of the enrichment e. [10%]

(c) The one-group, steady-state, source-free neutron diffusion equation for a cylindrical geometry, homogeneous system can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial \phi}{\partial r}\right) + \frac{\partial^2 \phi}{\partial z^2} + \frac{(\eta - 1)\Sigma_a}{D}\phi = 0$$

where all symbols have their usual meanings.
Show that the criticality condition for such a system is

$$\left(\frac{2.405}{R_0}\right)^2 + \left(\frac{\pi}{H_0}\right)^2 = \frac{(\eta - 1)\Sigma_a}{D}$$

where R_0 and H_0 are the extrapolated radius and height of the system. [40%]

(d) If extrapolation distances can be neglected, $e = 0.15$ and the value for the diffusion coefficient D of the system is 0.025 m, to what height can the tank be filled with the HALEU solution before it becomes critical? [15%]

(e) Assuming D remains unchanged, find the maximum value of e permissible if it must be guaranteed that criticality will not be reached whatever height the tank is filled to. [20%]

3 A nuclear reactor in the form of a bare (unreflected) cylinder of radius R and axial length $2L$ has regularly spaced fuel channels and is modelled in one-group diffusion theory. You can assume without proof that the power distribution in such a model is given by

$$P = P_0 \, J_0\left(\frac{2.405r}{R}\right)\cos\left(\frac{\pi x}{2L}\right)$$

where r is the radial coordinate, x is the axial coordinate, P_0 is the power density at the centre of the core, J_0 is an ordinary Bessel function of zero order, and extrapolation distances have been neglected.

(a) Show that the radial form factor for this reactor is 2.32. [20%]

(b) Carefully sketch on the same axes the expected forms of the variation of the coolant, cladding and fuel temperatures along a typical fuel channel of this reactor, identifying key features. [25%]

(c) It is proposed to reduce the coolant flow rates in all but the central channel so that the temperature rise in the coolant in each channel is the same as that in the central channel.

(i) Assuming no change in heat transfer coefficients, specific heat capacities or coolant inlet temperature, use the steady flow energy equation to find the required percentage reduction in coolant flow rate in a channel for which the power is equal to the mean channel power. [15%]

(ii) Why is it advantageous to reduce channel coolant flow rates in this way? [10%]

(d) The reactor power is limited by a hot spot within the fuel in the central channel. Explain why the hot-spot limitation will not apply to the mean channel after the coolant flow rate is reduced as in (c).
In considering this question, you may find the following form of Ginn's equation more helpful than the one given on the 4M16 data sheet:

$$\theta = 2\left(\frac{T - T_{ci}}{T_{co} - T_{ci}}\right) - 1 = \sin\left(\frac{\pi x}{2L}\right) + Q\cos\left(\frac{\pi x}{2L}\right)$$

where T_{ci} is the coolant inlet temperature and the other symbols are as defined on the data sheet. [15%]

(e) How, in practice, can form factors be improved? [15%]

4 In a 'lumped' model of the behaviour of a source-free nuclear reactor, the equations for the neutron population $n(t)$ and the delayed neutron precursor population $c(t)$ can be written as

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c$$
$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where all symbols have their usual meanings.

(a) What major simplifying assumptions underlie this model? [10%]

(b) A critical, source-free reactor has been operating in steady state for a prolonged period. It is then subject to a step change in reactivity. You can assume without proof that, for this model, the *in-hour equation* relating the inverse periods, p, to the reactivity, ρ, is

$$\rho = \Lambda p + \beta - \frac{\beta\lambda}{p + \lambda} = p\left[\Lambda + \frac{\beta}{p + \lambda}\right]$$

Sketch the relationship between the values of p that satisfy this equation and ρ, identifying any asymptotes, and discuss the implications for transients following step changes in reactivity. [30%]

(c) If $\beta = 0.0075$, $\lambda = 0.1\ \mathrm{s}^{-1}$ and $\Lambda = 0.5$ ms, estimate the ratio of precursors to neutrons in a source-free reactor at steady state. [10%]

(d) A source-free reactor with the parameter values in (c) has been operating in steady state with a neutron population n_0 for a prolonged period, and then is subject to a step increase in reactivity to $\rho = 0.0025$. In the *prompt jump approximation* the neutron population jumps instantaneously into equilibrium with the precursors. Using this approximation derive an expression for the subsequent variation of the neutron population with time, and hence show that the presence of delayed neutrons has a beneficial effect. [50%]

END OF PAPER

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Data Sheet S/15

Revised December 2022

MODULE 4M16

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	$299.792458 \times 10^{6}\ \mathrm{m\,s^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Planck constant	h	$6.62606957 \times 10^{-34}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.380662 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Elementary charge	e	$1.6021892 \times 10^{-19}\ \mathrm{C}$

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}$ kg (931.5016 MeV)
Electron volt	eV	$1.6021892 \times 10^{-19}$ J
Curie	Ci	3.7×10^{10} Bq
Barn	barn	$10^{-28}\ \mathrm{m^2}$

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}$Ne	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}$Ne	20.99385 u
99.985%	$^{1}_{1}$H	1.00783 u	8.94%	$^{22}_{10}$Ne	21.99138 u
0.015%	$^{2}_{1}$H	2.01410 u	10.1%	$^{25}_{12}$Mg	24.98584 u
0%	$^{3}_{1}$H	3.01605 u	11.1%	$^{26}_{12}$Mg	25.98259 u
0.0001%	$^{3}_{2}$He	3.01603 u	0%	$^{32}_{15}$P	31.97391 u
99.9999%	$^{4}_{2}$He	4.00260 u	96.0%	$^{32}_{16}$S	31.97207 u
7.5%	$^{6}_{3}$Li	6.01513 u	0%	$^{60}_{27}$Co	59.93381 u
92.5%	$^{7}_{3}$Li	7.01601 u	26.2%	$^{60}_{28}$Ni	59.93078 u
0%	$^{8}_{4}$Be	8.00531 u	0%	$^{87}_{35}$Br	86.92196 u
100%	$^{9}_{4}$Be	9.01219 u	0%	$^{86}_{36}$Kr	85.91062 u
18.7%	$^{10}_{5}$B	10.01294 u	17.5%	$^{87}_{36}$Kr	86.91337 u
0%	$^{11}_{6}$C	11.01143 u	12.3%	$^{113}_{48}$Cd	112.90461 u
98.89%	$^{12}_{6}$C	12.00000 u		$^{226}_{88}$Ra	226.02536 u
1.11%	$^{13}_{6}$C	13.00335 u		$^{230}_{90}$Th	230.03308 u
0%	$^{13}_{7}$N	13.00574 u	0.72%	$^{235}_{92}$U	235.04393 u
99.63%	$^{14}_{7}$N	14.00307 u	0%	$^{236}_{92}$U	236.04573 u
0%	$^{14}_{8}$O	14.00860 u	99.28%	$^{238}_{92}$U	238.05076 u
99.76%	$^{16}_{8}$O	15.99491 u	0%	$^{239}_{92}$U	239.05432 u
0.04%	$^{17}_{8}$O	16.99913 u		$^{239}_{93}$Np	239.05294 u
0.20%	$^{18}_{8}$O	17.99916 u		$^{239}_{94}$Pu	239.05216 u
				$^{240}_{94}$Pu	240.05397 u

Simplified Disintegration Patterns

Isotope	$^{60}_{27}Co$	$^{90}_{38}Sr$	$^{90}_{39}Y$	$^{137}_{55}Cs$	$^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

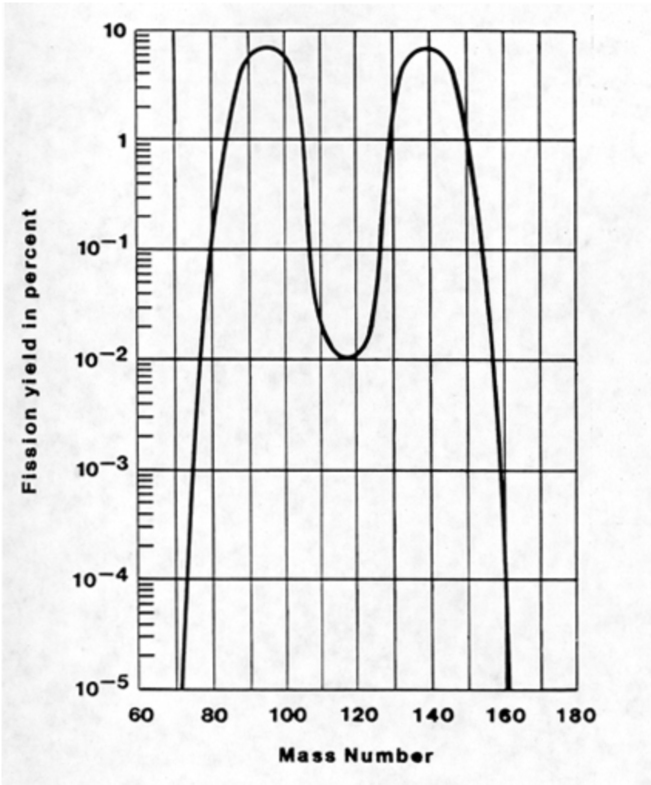
	"Nuclear" graphite	$^{16}_{8}O$	$^{113}_{48}Cd$	$^{235}_{92}U$	$^{238}_{92}U$	$^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90	→	Rb-90	→	Sr-90	→	Y-90	→	Zr-90
	33 s		2.7 min		28 yr		64 hr	

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131	→	Te-131	→	I-131	→	Xe-131
	23 min		24 min		8 days	

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135	→	Xe-135	→	Cs-135 (nearly stable)	Fission yield γ = 0.064
	6.7 hr		9.2 hr		

Xe-135 is a strong absorber of thermal neutrons, with σ_a = 3.5 Mbarn .

Pm-149	→	Sm-149	Fission yield γ = 0.014
	54 hr		

Sm-149 is a strong absorber of thermal neutrons, with σ_a = 53 kbarn .

Se-87	→	Br-87	→	either {Kr-86 + n} or {Kr-87	→	Rb-87}
	17 s		55 s		78 min	

This chain leads to a “delayed neutron”.

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	Total
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where n = neutron concentration

c = precursor concentration

β = delayed neutron precursor fraction $= \sum \beta_i$

λ = average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\overline{\mu})}$$

with $\overline{\mu} =$ the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Spherical geometry: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right) + R = 0$$

Solution is: $R(r) = A_1 J_0(r) + A_2 Y_0(r)$

$J_0(0) = 1$; $Y_0(0) = -\infty$;

The first zero of $J_0(r)$ is at $r = 2.405$.

$\int r J_0(\beta r)\, dr = \frac{r}{\beta} J_1(\beta r)$ where $J_1(0) = 0$ and $J_1(2.405) = 0.5183$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $\mathrm{g\,cm^{-3}}$	Σ_a $\mathrm{cm^{-1}}$	D cm	$L^2 = D/\Sigma_a$ $\mathrm{cm^2}$
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $\theta = \frac{T_M}{T_1} = \frac{2}{M+1}$

Enrichment of Isotopes

Value function: $v(x) = (2x-1)\ln\left(\frac{x}{1-x}\right) \approx -\ln(x)$ for small x

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $R_n \equiv \frac{L''_n}{P} = \frac{x_p - x'_{n+1}}{x'_{n+1} - x''_n}$

Stripping section reflux ratio: $R_n = \left[\frac{x_p - x_f}{x_f - x_w}\right]\left[\frac{x'_{n+1} - x_w}{x'_{n+1} - x''_n}\right]$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i | T = filling time

λ_j = decay constant of nuclide j | τ = decay hold-up time after filling

P = parent nuclide production rate

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L$ = surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity

4M16 Nuclear Power Engineering 2023

Answers

Q1 (c) 57.92 μg

(d) 115.4 Sv; 830.9 Sv

Q2 (a) $\Sigma_f = 73.38e$

(b) $\eta = \frac{178.31e}{86.57e+10.35}$

(d) 0.302 m

(e) 0.121

Q3 (c)(i) 56.9% reduction

Q4 (c) $\frac{c_0}{n_0} = 150$

(d) $n = \frac{\beta}{\beta-\rho} n_0 \exp\left(\frac{\rho\lambda}{\beta-\rho} t\right)$