4M16 2025 crib v2

pmc55

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Question 1

1(a)

Deterministic effects are those in which the severity of the effect varies with the size of the dose and occur within a very short time of exposure.

For stochastic effects the statistical risk of an effect, usually some form of cancer, is proportional to the dose but the severity of the effect is not (as illustrated in the figure below).

The estimation is this increased risk is difficult because cancers have long and variable latent periods and radiation-induced cancers cannot be distinguished from those due to other causes. The fact that over 40% of the population of western countries die of cancer makes separate analysis of the nuclear risk almost impossible.

The only data available comes from the survivors of the atomic bomb attacks on Japan during World War 2 and from the records of workers exposed in known high radiation risk occupations, e.g. tin mines and the former luminous dial painters.

The latest ICRP recommendations are the result of a re-assessment of the Japanese data. All the data is at relatively high exposures so extrapolation to the dose levels resulting from nuclear energy programmes is uncertain.

This approach assumes that all radiation is potentially dangerous and that a low dose over a very long period has the same effect as a higher dose over a shorter period.

The other debatable stochastic effect is genetic damage, i.e. that radiation damages reproductive cells causing mutations of the genes and possible hereditary defects. There is no conclusive evidence to date that hereditary damage has ever occurred in humans as a result of ionising radiation.

Marks: 30%

1(b)

The effect of ionising radiation on matter is expressed in terms of energy deposited per unit mass, the absorbed dose (D). The SI unit of absorbed dose is the gray (Gy).

Because different types of radiation have different effects on living tissue a further factor is brought in. This called the radiation weighting factor (W_g) and gives rise to a measure of radiation dose called the equivalent dose (H_T). The SI unit is the sievert (Sv).

$$H_T = W_g D$$

The equivalent dose is a point source quantity and is further weighted by a tissue or organ weighting factor (W_T) to account for the fact that some parts of the body are more susceptible to radiation damage than others. The final result is an effective dose E (also measured in Sv) given by

$$E = \sum_{\text{organs}} W_T H_T$$

The gamma ray weighting factor, W_R, is 1.

Marks: 20%

1(c)

The decay constant of cobalt is

$$\lambda = \frac{\ln(2)}{T_{\frac{1}{2}}} = \frac{\ln(2)}{5.272 \times 365 \times 24 \times 3600} = 4.169 \times 10^{-9}\text{s}^{-1}$$

The specific activity of cobalt is

$$a = \frac{\lambda L}{M}$$

where L is Avogadro's constant and M is the molar mass of cobalt (both from the data book).

$$a = \frac{4.169 \times 10^{-9}[\text{s}^{-1}] \times 6.022 \times 10^{23}[\text{mol}^{-1}]}{0.05993[\text{kgmol}^{-1}]} = 4.189 \times 10^{16}\text{Bqkg}^{-1}$$

Marks: 15%

The formula in the data book can be used to calculate dose received by the worker:

$$D = \frac{1.6 \times 10^{-13} A \Sigma E_\gamma t}{4\pi\rho R^2}$$

In this case:

$$E_\gamma = 1.17 + 1.33 = 2.50\text{MeV}$$

$$R = 2\text{m}$$

$$t = 20 \times 60 = 1200\text{s}$$

Hence

$$D = 1.20\text{Gy}$$

So the whole-body ($W_T = 1$) effective dose is:

$$E = W_T W_R D = 1 \times 1 \times 1.20 = 1.20\text{Sv}$$

As the maximum permitted annual dose for employees is 20 mSv, this is unacceptable.

In order to get the effective dose down to 10 mSv, a decontamination factor of

$$F = \frac{1.20}{0.010} = 120$$

would be required before work could start.

Marks: 35%

Question 2

2(a)

The terms in the point kinetics equations are:

1. $\frac{\mathrm{d}P}{\mathrm{d}t}$ is the rate of change of power with time.
2. $\frac{\rho-\beta}{\Lambda}P$ is the (scaled to power) rate at which prompt neutrons are produced from fission.
3. λc is the (scaled to power) rate at which precursors decay and delayed neutrons are produced.
4. $\frac{\mathrm{d}c}{\mathrm{d}t}$ is the net rate of delayed neutron precursor production.
5. $\frac{\beta}{\Lambda}P$ is the (scaled to power) rate at which precursors are produced.

Marks: 20%

2(b)

The power variation is inserted into the point kinetics equations, giving:

$$a = \frac{\rho(t) - \beta}{\Lambda}\left(P_0 + at\right) + \lambda c$$

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \frac{\beta}{\Lambda}\left(P_0 + at\right) - \lambda c$$

One must obtain an expression for c. This can be done by solving the second differential equation, which can be re-written as:

$$\frac{\mathrm{d}c}{\mathrm{d}t} + \lambda c = \frac{\beta}{\Lambda}P_0 + \frac{\beta a}{\Lambda}t$$

Among other ways, this can be solved by obtaining the homogeneous solution, followed by the particular integral. The homogeneous solution is:

$$c_{\mathrm{H}} = Ae^{-\lambda t}$$

The particular integral comes from guessing a solution of the form

$$c_{\mathrm{PI}} = Dt + E$$

Inserting this into the precursor equation gives:

$$D + \lambda Dt + \lambda E = \frac{\beta}{\Lambda} P_0 + \frac{\beta a}{\Lambda} t$$

which can be split into

$$D + \lambda E = \frac{\beta}{\Lambda} P_0$$

$$\lambda D = \frac{\beta a}{\Lambda}$$

Hence:

$$D = \frac{\beta a}{\lambda \Lambda}$$

and

$$E = \frac{\beta}{\lambda \Lambda} \left(P_0 - \frac{a}{\lambda} \right)$$

Hence the precursor evolution follows:

$$c(t) = Ae^{-\lambda t} + \frac{\beta}{\lambda \Lambda} \left(P_0 - \frac{a}{\lambda} + at \right)$$

The unknown coefficient can be found from the initial conditions (the reactor is critical). From the point kinetics equations when $\rho = 0$, this gives:

$$c(0) = \frac{\beta P_0}{\lambda \Lambda}$$

Hence, one can solve for A when $t = 0$:

$$A = \frac{\beta a}{\lambda^2 \Lambda}$$

so

$$c(t) = \frac{\beta a}{\lambda^2 \Lambda} e^{-\lambda t} + \frac{\beta}{\lambda \Lambda} \left(P_0 - \frac{a}{\lambda} + at \right)$$

This expression for the precursor density can then be inserted into the first equation and some rearrangement gives:

$$\rho(t) = \frac{\Lambda a}{P_0 + at} + \frac{\beta a}{\lambda} \frac{\left(1 - e^{-\lambda t}\right)}{(P_0 + at)}$$

Marks: 60%

2(c)

Delayed neutrons add inertia to a reactor's transient response. For a step reactivity insertion, the governing exponential time constant for a reactor would be on the scale of microseconds for a thermal reactor, making mechanical control essentially impossible. Delayed neutrons result in this time constant being on the order of seconds. However, this is provided that the reactivity insertion is smaller than β, otherwise the time constant begins to approach the prompt constant once again. The presence of delayed neutrons also limits how quickly a reactor may be shutdown, i.e., a reactors power (due to direct fission) will not instantaneously go to zero even for an extremely large negative reactivity insertion.

Marks: 20%

Question 3

3(a)

Students must show that $M = \rho\sqrt{3}^3 \frac{\pi^3}{B^3}$.

This follows from the buckling relationship for a cube. The diffusion equation in this case is:

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} + \frac{\partial^2\phi}{\partial z^2} + B^2\phi = 0$$

One applies separation of variable (given the separable boundary conditions) by asserting:

$$\phi(x, y, z) = X(x)Y(y)Z(z)$$

This gives:

$$X''(x)/X(x) + Y''(y)/Y(y) + Z''(z)/Z(z) + B^2 = 0$$

One can recognise each term as a constant to give:

$$B^2 = \alpha^2 + \beta^2 + \gamma^2 = 3\alpha^2$$

where the second equality follows due to the system being identical in each dimension. Solving one of the resulting ODEs gives:

$$X(x) = C\cos(\alpha x) + D\sin(\alpha x)$$

Symmetry about $x = 0$ allows one to set $D = 0$. The boundary condition gives:

$$0 = \cos(\alpha H/2)$$

implying that:

$$\alpha = \pi/H \tag{1}$$

Hence one has

$$B^2 = 3\pi^2/H^2$$

or

$$H = \sqrt{3}\pi/B$$

The critical mass is the volume of the cube, times the density. The volume is simply:

$$V = H^3 = \sqrt{3}^3 \frac{\pi^3}{B^3}$$

and multiplying by density gives the result.

Marks: 30%

3(b)

Considering a 1D slab gives the two diffusion equations:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} + B^2\phi = 0$$

in the material and:

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} = 0$$

in the cavity. The solution of the first equation gives:

$$\phi(x) = C\cos(Bx) + D\sin(Bx)$$

while the second gives, simply

$$\phi(x) = Ex + F$$

Symmetry about $x = 0$ can be used to simplify both equations to:

$$\phi(x) = C\cos Bx$$

$$\phi(x) = F$$

Boundary and continuity equations can then be applied.

$$\phi(H/2) = 0 = C\cos\left(BH/2\right)$$

$$\phi(\alpha H/2) = F = C\cos\left(B\alpha H/2\right)$$

$$\left.\frac{\mathrm{d}\phi}{\mathrm{d}x}\right|_{x=\alpha H/2} = 0 = -CB\sin\left(B\alpha H/2\right)$$

The first and third equations can be equated, giving:

$$\cos\left(BH/2\right) = \sin\left(B\alpha H/2\right)$$

Here the provided identity can be applied to obtain everything in terms of sines:

$$\sin\left(\frac{\pi}{2} - \frac{BH}{2}\right) = \sin\left(\frac{B\alpha H}{2}\right)$$

Giving the required result:

$$B = \frac{\pi}{H(1-\alpha)}$$

Marks: 35%

3(c)

This requires combining the criticality conditions from the previous two problems. The separation variables for each dimension are known such that:

$$B^2 = \frac{\pi^2}{H^2(1-\alpha)^2} + 2\frac{\pi^2}{H^2}$$

as the x dimension problem has been replaced by the hollow slab problem. This can then be rearranged in terms of H:

$$H^2 = \frac{\pi^2}{B^2(1-\alpha)^2} + 2\frac{\pi^2}{B^2}$$

The volume of the fissile material is $V = H^3(1-\alpha)$. Inserting the expression for H gives the required result.

Marks: 30%

3(d)

The critical mass could be increased by including a neutron absorber in the hollow region. However, it could NOT be increased by adding absorber around the boundary of the problem – it is a vacuum boundary, it is already perfectly absorbing! Adding absorber would in fact decrease the critical mass (assuming it has a non-zero scattering cross section).

Marks: 5%

Question 4

4(a)

All fission reactors operate with a very sensitive neutron balance in which the number of neutrons generated per fission must equal those consumed by fission, capture and other losses. Nearly all civil nuclear power reactors use light water as a moderator and coolant and zircalloy cladding both of which have significant thermal neutron absorption cross sections which means that unlike the former Magnox graphite moderated CO2 cooled reactors with magnesium cladding there are insufficient neutrons to maintain a controlled chain reaction with natural uranium. Normal civil enrichment values are between 2.5% and 5%.

Marks: 20%

4(b)

Commercial enrichment processes use the difference in the size and mass of the U235 and the U238 atoms in the form of uranium hexa-fluoride (HEX). They use stagewise contacting processes in which the enrichment takes place in a number

of stages, each stage giving a small change in the concentration of the U235. In the earlier, now virtually obsolete diffusion process, the separation is affected by a semi porous membrane through which the slightly smaller U235 HEX atoms are more likely to pass. Because the separation per stage is very low the process requires a large number of stages, each with its own compressor and cooler. This process has the advantage of being relatively simple but very expensive in energy costs. The current process uses very high speed gas centrifuges in which the heavier U238 HEX is spun to the outside of the centrifuge whilst the slightly lighter U235 remains in the centre. This process requires fewer stages and hence less power and space but the construction of the centrifuges requires the use of very high grade materials to resist the enormous g forces. HEX has got to be used as it is the only uranium compound which is gaseous at relatively mild conditions (low temperatures).

Newer processes revolve round the use of specially tuned lasers which can excite one isotope without exciting the others. The excited isotope (or HEX molecule) can then be separated by an electromagnetic field. Despite much research and various claims from time to time such processes are still well into the future it is very difficult to compete with the centrifuge process. The original process envisaged the use of metallic uranium vapour but more recently the idea has been to use HEX instead.

Marks: 20%

4(c)

Separative work (SWU) is the unit in which enrichment is traded and is defined as:

$$S = E_w(2x_w - 1)\ln\left(\frac{x_w}{1-x_w}\right) + E_p(2x_p - 1)\ln\left(\frac{x_p}{1-x_p}\right) - E_f(2x_f - 1)\ln\left(\frac{x_f}{1-x_f}\right)$$

The approximation to:

$$S = E_w(-\ln x_w) + E_p(-\ln x_p) - E_f(-\ln x_f)$$

is generally valid for civil reactors where the enrichment is low. In the above, E_i is the total mass of feed, product, and waste.

As can be seen SWU increases as the enrichment required increases and the tails concentration decreases. Since the enrichment is dictated by the reactor physics and the feed concentration by nature the only variable is the tails concentration. The lower the tails the grater the SWU require but less feed will be needed so the optimum tails depend on the relationship between the uranium price and the cost of a SWU. When then uranium price is high it may be worth spending more on SWUs to reduce the feed requirements and the reverse is true if the SWU price is high.

Marks: 20%

4(d)

Thermal power is given by 1600/0.37 = 4324 MW(th)

Annual thermal output is given by 4324 x 0.85 x 365 = 1.341E6 MW days

Annual fuel requirement is given by 1.341E6/45000 = 29.81 te

Mass balance across enrichment plant

$$F = P + W$$

$$x_f F = x_p P + x_w W$$

$$F = 29.81 + W 0.007F = 0.05x29.81 + 0.003W$$

F = 350.3 te W = 320.49 te

As UOC is 95% U total UOC needed is 350.3/0.95 = 368.7 te

Marks: 20%

4(e)

$SWU = W(-\ln x_w) + P(-\ln x_p) - F(-\ln x_f) = 320.49(-\ln 0.003) + 29.81.(-\ln 0.05) - 350.3(-\ln 0.007)$

= 212.94 te SWU

Marks: 20%