EGT3 / EGT2
ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 2 May 2025 2 to 3.40

Module 4M16

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4M16 Nuclear Power Engineering data sheet (8 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

1 (a) Define the terms *deterministic* and *stochastic* as applied to the effects of ionising radiations on human health. Discuss some of the difficulties in assessing stochastic effects. [30%]

(b) Define the terms *absorbed dose*, *equivalent dose* and *effective dose* as applied to health physics. What SI units are used to measure these doses? What is the weighting factor for γ-rays? [20%]

(c) It is necessary to carry out some tube-sealing operations that involve 'hands-on' access in an old Pressurised Water Reactor (PWR) steam generator. The main source of activity is equivalent to 1 g of 'plated out' cobalt-60. The worker must work at a distance of 2 m from this source for a period of 20 minutes.

Cobalt-60 has a half-life of 5.272 years and in each decay releases two γ-rays of energies 1.17 MeV and 1.33 MeV respectively.

(i) Show that the activity of 1 g of cobalt-60 is 4.19×10^{13} Bq. [15%]

(ii) What would be the maximum whole-body effective dose likely to be received by the worker? Would this be acceptable? What decontamination factor will be required if the effective dose from these operations is not to exceed 10 mSv?
Take the density of human tissue to be 1000 kg m^{-3} and the absorption cross-section for γ-rays in human tissue to be 3 m^{-1}. [35%]

2 One common form of the point kinetics equations with a single delayed neutron precursor group is:

$$\frac{\mathrm{d}P}{dt} = \frac{\rho - \beta}{\Lambda} P + \lambda c$$

$$\frac{\mathrm{d}c}{\mathrm{d}t} = \frac{\beta}{\Lambda} P - \lambda c$$

where P is the reactor power, c is the normalised delayed neutron precursor density, and all other symbols have their usual meaning.

(a) Explain what each term in the point kinetics equations represents. [20%]

(b) An engineer desires to ramp a reactor's power such that:

$$P(t) = P_0 + at$$

where P_0 is the initial power when the core is critical and a is the power ramping rate. By solving the point kinetics equations subject to the above power ramp, show that the reactivity variation required is:

$$\rho(t) = \frac{\Lambda a}{P_0 + at} + \frac{\beta a}{\lambda} \frac{(1 - \exp(-\lambda t))}{(P_0 + at)}$$ [60%]

(c) Explain the importance of delayed neutrons to reactor operation. [20%]

3 During nuclear fuel handling and transportation, it is necessary to ensure the fuel is subcritical. To a first approximation, the critical mass of the fuel-containing regions of such a system can be evaluated using the one-group, steady-state, source-free neutron diffusion equation for a multiplying system:

$$\nabla^2\phi + B^2\phi = 0$$

where $B^2 = (\nu\Sigma_f - \Sigma_a)/D$ and all symbols have their usual meanings. Assuming diffusion theory remains valid, in a region containing no material, this equation reduces to Laplace's equation:

$$\nabla^2\phi = 0$$

Consider a cube of fissile material, with density ρ, width H, vacuum boundary conditions, and a negligible extrapolation distance.

(a) Show that the critical mass for such a system is given by:

$$M = \rho\left(\sqrt{3}\right)^3 \frac{\pi^3}{B^3}$$ [30%]

(b) Consider a 1D slab problem of width H. The inside of this slab is hollow with a width of αH about the centre of the slab, where $0 < \alpha < 1$. Show that the criticality condition of this 1D slab is:

$$B^2 = \frac{\pi^2}{H^2(1-\alpha)^2}$$

Note that at the boundaries of different material regions the flux, ϕ, and the current $J = -D\nabla\phi$ must be continuous. Hint: $\cos(\theta) = \sin\left(\frac{\pi}{2} - \theta\right)$ [35%]

(c) Hence, in 3D, consider two separated slabs with width H in the y and z dimensions. The total width of the x dimension is also H but with a centred gap between the two slabs with a width of αH. Show that the criticality condition is:

$$B^2 = \frac{\pi^2}{H^2(1-\alpha)^2} + \frac{2\pi^2}{H^2}$$

and hence that the critical mass is:

$$M = \rho\frac{\pi^3}{B^3}\frac{\left(1 + 2(1-\alpha)^2\right)^{\frac{3}{2}}}{(1-\alpha)^2}$$ [30%]

(d) Without performing further calculations, how else could the system be modified to increase the critical mass? [5%]

4 (a) Why is it necessary to enrich the ^{235}U content of the fuel for most designs of civil reactors? Give typical values of the enrichment required for these reactors. [20%]

(b) Describe in detail the process most commonly used today for the enrichment of civil nuclear fuel. Additionally, briefly describe one historic method and one method under development. [20%]

(c) Define the term Separative Work and say why it is so important. What are the main criteria in fixing the tails concentration? [20%]

(d) A 1600 MW_e reactor requires fuel enriched to 5%. The reactor has an overall efficiency of 37% and operates at full load for 85% of the year. The burn-up is 45 GWd/te. Estimate the annual fuel requirement. Assume the ^{235}U content of natural uranium is 0.7% and the UOC has a concentration of 95% U. Estimate how much UOC will be required if the tails concentration is 0.3% ^{235}U. [20%]

(e) How much separative work will be needed? You may ignore losses at all stages of the fuel cycle excluding tails. [20%]

END OF PAPER

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MODULE 4M16

NUCLEAR POWER ENGINEERING

DATA SHEET

General Data

Speed of light in vacuum	c	$299.792458 \times 10^{6}\ \mathrm{m\,s^{-1}}$
Magnetic permeability in vacuum	μ_0	$4\pi \times 10^{-7}\ \mathrm{H\,m^{-1}}$
Planck constant	h	$6.62606957 \times 10^{-34}\ \mathrm{J\,s}$
Boltzmann constant	k	$1.380662 \times 10^{-23}\ \mathrm{J\,K^{-1}}$
Elementary charge	e	$1.6021892 \times 10^{-19}\ \mathrm{C}$

Definitions

Unified atomic mass constant	u	$1.6605655 \times 10^{-27}\ \mathrm{kg}$ (931.5016 MeV)
Electron volt	eV	$1.6021892 \times 10^{-19}\ \mathrm{J}$
Curie	Ci	$3.7 \times 10^{10}\ \mathrm{Bq}$
Barn	barn	$10^{-28}\ \mathrm{m^2}$

Atomic Masses and Naturally Occurring Isotopic Abundances (%)

	electron	0.00055 u	90.80%	$^{20}_{10}Ne$	19.99244 u
	neutron	1.00867 u	0.26%	$^{21}_{10}Ne$	20.99385 u
99.985%	$^{1}_{1}H$	1.00783 u	8.94%	$^{22}_{10}Ne$	21.99138 u
0.015%	$^{2}_{1}H$	2.01410 u	10.1%	$^{25}_{12}Mg$	24.98584 u
0%	$^{3}_{1}H$	3.01605 u	11.1%	$^{26}_{12}Mg$	25.98259 u
0.0001%	$^{3}_{2}He$	3.01603 u	0%	$^{32}_{15}P$	31.97391 u
99.9999%	$^{4}_{2}He$	4.00260 u	96.0%	$^{32}_{16}S$	31.97207 u
7.5%	$^{6}_{3}Li$	6.01513 u	0%	$^{60}_{27}Co$	59.93381 u
92.5%	$^{7}_{3}Li$	7.01601 u	26.2%	$^{60}_{28}Ni$	59.93078 u
0%	$^{8}_{4}Be$	8.00531 u	0%	$^{87}_{35}Br$	86.92196 u
100%	$^{9}_{4}Be$	9.01219 u	0%	$^{86}_{36}Kr$	85.91062 u
18.7%	$^{10}_{5}B$	10.01294 u	17.5%	$^{87}_{36}Kr$	86.91337 u
0%	$^{11}_{6}C$	11.01143 u	12.3%	$^{113}_{48}Cd$	112.90461 u
98.89%	$^{12}_{6}C$	12.00000 u		$^{226}_{88}Ra$	226.02536 u
1.11%	$^{13}_{6}C$	13.00335 u		$^{230}_{90}Th$	230.03308 u
0%	$^{13}_{7}N$	13.00574 u	0.72%	$^{235}_{92}U$	235.04393 u
99.63%	$^{14}_{7}N$	14.00307 u	0%	$^{236}_{92}U$	236.04573 u
0%	$^{14}_{8}O$	14.00860 u	99.28%	$^{238}_{92}U$	238.05076 u
99.76%	$^{16}_{8}O$	15.99491 u	0%	$^{239}_{92}U$	239.05432 u
0.04%	$^{17}_{8}O$	16.99913 u		$^{239}_{93}Np$	239.05294 u
0.20%	$^{18}_{8}O$	17.99916 u		$^{239}_{94}Pu$	239.05216 u
				$^{240}_{94}Pu$	240.05397 u

Simplified Disintegration Patterns

Isotope	${}^{60}_{27}Co$	${}^{90}_{38}Sr$	${}^{90}_{39}Y$	${}^{137}_{55}Cs$	${}^{204}_{81}Tl$
Type of decay	β^-	β^-	β^-	β^-	β^-
Half life	5.3 yr	28 yr	64 h	30 yr	3.9 yr
Total energy	2.8 MeV	0.54 MeV	2.27 MeV	1.18 MeV	0.77 MeV
Maximum β energy	0.3 MeV (100%)	0.54 MeV (100%)	2.27 MeV (100%)	0.52 MeV (96%) 1.18 MeV (4%)	0.77 MeV (100%)
γ energies	1.17 MeV (100%) 1.33 MeV (100%)	None	None	0.66 MeV (96%)	None

Thermal Neutron Cross-sections (in barns)

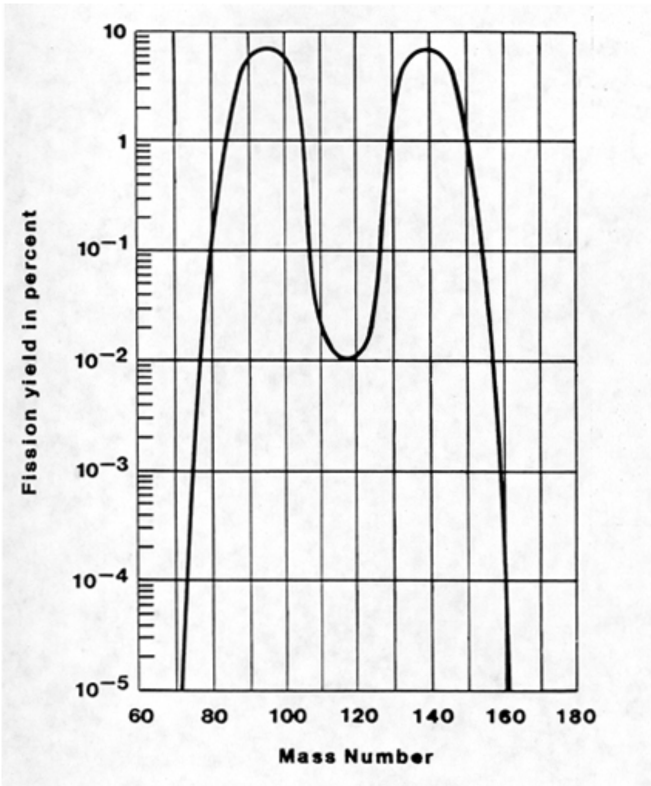
	"Nuclear" graphite	${}^{16}_{8}O$	${}^{113}_{48}Cd$	${}^{235}_{92}U$	${}^{238}_{92}U$	${}^{1}_{1}H$ unbound
Fission	0	0	0	580	0	0
Capture	4×10^{-3}	10^{-4}	27×10^{3}	107	2.75	0.332
Elastic scatter	4.7	4.2		10	8.3	38

Densities and Mean Atomic Weights

	"Nuclear" graphite	Aluminium Al	Cadmium Cd	Gold Au	Uranium U
Density / $kg\,m^{-3}$	1600	2700	8600	19000	18900
Atomic weight	12	27	112.4	196	238

Fission Product Yield

Nuclei with mass numbers from 72 to 158 have been identified, but the most probable split is unsymmetrical, into a nucleus with a mass number of about 138 and a second nucleus that has a mass number between about 95 and 99, depending on the target.



The primary fission products decay by β^- emission. Some important decay chains (with relevant half lives) from thermal-neutron fission of U-235 are:

Kr-90	$\rightarrow$	Rb-90	$\rightarrow$	Sr-90	$\rightarrow$	Y-90	$\rightarrow$	Zr-90
	33 s		2.7 min		28 yr		64 hr	

Sr-90 is a serious health hazard, because it is bone-seeking.

Sb-131	$\rightarrow$	Te-131	$\rightarrow$	I-131	$\rightarrow$	Xe-131
	23 min		24 min		8 days	

I-131 is a short-lived health hazard. It is thyroid-seeking.

I-135	$\rightarrow$	Xe-135	$\rightarrow$	Cs-135 (nearly stable)	Fission yield γ = 0.064
	6.7 hr		9.2 hr		

Xe-135 is a strong absorber of thermal neutrons, with σ_a = 3.5 Mbarn .

Pm-149	$\rightarrow$	Sm-149	Fission yield γ = 0.014
	54 hr		

Sm-149 is a strong absorber of thermal neutrons, with σ_a = 53 kbarn .

Se-87	$\rightarrow$	Br-87	$\rightarrow$	either {Kr-86 + n} or {Kr-87	$\rightarrow$	Rb-87}
	17 s		55 s		78 min	

This chain leads to a "delayed neutron".

Neutrons

Most neutrons are emitted within 10^{-13} s of fission, but some are only emitted when certain fission products, e.g. Br-87, decay.

The total yield of neutrons depends on the target and on the energy of the incident neutron. Some key values are:

Target nucleus	Fission induced by			
	Thermal neutron		Fast neutron	
	ν	η	ν	η
U-233	2.50	2.29	2.70	2.45
U-235	2.43	2.07	2.65	2.30
U-238	—	—	2.55	2.25
Pu-239	2.89	2.08	3.00	2.70

ν = number of neutrons emitted per fission

η = number of neutrons emitted per neutron absorbed

Delayed Neutrons

A reasonable approximation for thermal-neutron fission of U-235 is:

Precursor half life / s	55	22	5.6	2.1	0.45	0.15	
Mean life time of precursor ($1/\lambda_i$) / s	80	32	8.0	3.1	0.65	0.22	Total
Number of neutrons produced per 100 fission neutrons ($100\,\beta_i$)	0.03	0.18	0.22	0.23	0.07	0.02	0.75

Fission Energy

Kinetic energy of fission fragments	167 ± 5 MeV
Prompt γ-rays	6 ± 1 MeV
Kinetic energy of neutrons	5 MeV
Decay of fission products β	8 ± 1.5 MeV
γ	6 ± 1 MeV
Neutrinos (not recoverable)	12 ± 2.5 MeV
Total energy per fission	204 ± 7 MeV

Subtract neutrino energy and add neutron capture energy ⇒ ~200 MeV / fission

Nuclear Reactor Kinetics

Name	*Symbol*	*Concept*
Effective multiplication factor	k_{eff}	$\frac{\text{production}}{\text{removal}} = \frac{P}{R}$
Excess multiplication factor	k_{ex}	$\frac{P-R}{R} = k_{eff} - 1$
Reactivity	ρ	$\frac{P-R}{P} = \frac{k_{ex}}{k_{eff}}$
Lifetime	l	$\frac{1}{R}$
Reproduction time	Λ	$\frac{1}{P}$

Reactor Kinetics Equations

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$

$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where n = neutron concentration

c = precursor concentration

β = delayed neutron precursor fraction $= \sum \beta_i$

λ = average precursor decay constant

Neutron Diffusion Equation

$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_a \phi + S$$

where $\underline{j} = -D\nabla\phi$ (Fick's Law)

$$D = \frac{1}{3\Sigma_s(1-\overline{\mu})}$$

with $\overline{\mu}$ = the mean cosine of the angle of scattering

Laplacian ∇^2

Slab geometry: $\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

Cylindrical geometry: $\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$

Spherical geometry: $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2 \sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial}{\partial \theta}\right) + \frac{1}{r^2 \sin^2\theta}\frac{\partial^2}{\partial \psi^2}$

Bessel's Equation of 0th Order

$$\frac{1}{r}\frac{d}{dr}\left(r\frac{dR}{dr}\right)+R=0$$

Solution is: $R(r)=A_1J_0(r)+A_2Y_0(r)$

$J_0(0)=1;\ Y_0(0)=-\infty;$

The first zero of $J_0(r)$ is at $r=2.405$.

$\int rJ_0(\beta r)dr=\frac{r}{\beta}J_1(\beta r)$ where $J_1(0)=0$ and $J_1(2.405)=0.5183$.

Diffusion and Slowing Down Properties of Moderators

Moderator	Density $\mathrm{g\,cm^{-3}}$	Σ_a $\mathrm{cm^{-1}}$	D cm	$L^2=D/\Sigma_a$ $\mathrm{cm^2}$
Water	1.00	22×10^{-3}	0.17	$(2.76)^2$
Heavy Water	1.10	85×10^{-6}	0.85	$(100)^2$
Graphite	1.70	320×10^{-6}	0.94	$(54)^2$

In-core Fuel Management Equilibrium Cycle Length Ratio

For M-batch refueling: $\theta=\frac{T_M}{T_1}=\frac{2}{M+1}$

Enrichment of Isotopes

Value function: $v(x)=(2x-1)\ln\left(\frac{x}{1-x}\right)\approx-\ln(x)$ for small x

For any counter-current cascade at low enrichment:

Enrichment section reflux ratio: $R_n\equiv\frac{L''_n}{P}=\frac{x_p-x'_{n+1}}{x'_{n+1}-x''_n}$

Stripping section reflux ratio: $R_n=\left[\frac{x_p-x_f}{x_f-x_w}\right]\left[\frac{x'_{n+1}-x_w}{x'_{n+1}-x''_n}\right]$

Bateman's Equation

$$N_i = \lambda_1 \lambda_2 \ldots \lambda_{i-1} P \sum_{j=1}^{i} \frac{[1-\exp(-\lambda_j T)]\exp(-\lambda_j \tau)}{\lambda_j \prod_{\substack{k=1 \\ k \neq j}}^{i} (\lambda_k - \lambda_j)}$$

where N_i = number of atoms of nuclide i

λ_j = decay constant of nuclide j

P = parent nuclide production rate

T = filling time

τ = decay hold-up time after filling

Temperature Distribution

For axial coolant flow in a reactor with a chopped cosine power distribution, Ginn's equation for the non-dimensional temperature is:

$$\theta = \frac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\frac{\pi L}{2L'}\right) = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L = fuel half-length

L' = flux half-length

$T_{c1/2}$ = coolant temperature at mid-channel

T_{co} = coolant temperature at channel exit

$Q = \frac{\pi \dot{m} c_p}{UA} \frac{L}{L'}$

with $\dot{m}$ = coolant mass flow rate

c_p = coolant specific heat capacity (assumed constant)

$A = 4\pi r_o L$ = surface area of fuel element

and for radial fuel geometry:

$$\frac{1}{U} = \underbrace{\frac{1}{h}}_{\text{bulk coolant}} + \underbrace{\frac{1}{h_s}}_{\text{scale}} + \underbrace{\frac{t_c}{\lambda_c}}_{\text{thin clad}} + \underbrace{\frac{r_o}{h_b r_i}}_{\text{bond}} + \underbrace{\frac{r_o}{2\lambda_f}\left(1 - \frac{r^2}{r_i^2}\right)}_{\text{fuel pellet}}$$

with h = heat transfer coefficient to bulk coolant

h_s = heat transfer coefficient of any scale on fuel cladding

t_c = fuel cladding thickness (assumed thin)

λ_c = fuel cladding thermal conductivity

r_o = fuel cladding outer radius

r_i = fuel cladding inner radius = fuel pellet radius

h_b = heat transfer coefficient of bond between fuel pellet and cladding

λ_f = fuel pellet thermal conductivity