Q1

(a) The decay reaction is $$^{238}_{94}\mathrm{Pu} \rightarrow {}^{238-4}_{94-2}\mathrm{X} + {}^{4}_{2}\mathrm{He}$$

$$\therefore \quad A = 234 \quad \text{and} \quad Z = 92$$

So, the daughter product is $^{234}_{92}\mathrm{U}$. [5%]

(b) For the reaction $^{9}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \mathrm{n}$

$$\Delta u = 9.01219 + 4.00260 - 12.00000 - 1.00867 = 6.12 \times 10^{-3} \text{ u}$$

$$\therefore \quad \text{Energy released} = 6.12 \times 10^{-3} \times 931.5016 = 5.701 \text{ MeV}$$

Assuming the neutron receives all this energy plus the energy of the α, the maximum neutron energy is

$$5.701 + 5.593 = 11.294 \text{ MeV}$$ [10%]

(c)
$$\frac{dn}{dt} = -\nabla . \underline{j} + (\eta - 1)\Sigma_{\mathrm{a}}\phi + S$$

Steady-state: $$\therefore \quad \frac{dn}{dt} = 0$$

Fick's Law: $$\underline{j} = -D\nabla\phi$$

$$\therefore \quad 0 = -\nabla.(-D\nabla\phi) + (\eta - 1)\Sigma_{\mathrm{a}}\phi + S$$

As the system is uniform, it can be assumed that D is constant and the reactor is homogeneous

$$\therefore \quad 0 = D\nabla^2\phi + (\eta - 1)\Sigma_{\mathrm{a}}\phi + S$$

$$\therefore \quad \nabla^2\phi + \frac{(\eta - 1)\Sigma_{\mathrm{a}}}{D}\phi = -\frac{S}{D}$$

For a spherical geometry uniform reactor, the Laplacian $\nabla^2 = \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d}{dr}\right)$

$$\therefore \quad \frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) + \frac{(\eta - 1)\Sigma_{\mathrm{a}}}{D}\phi = -\frac{S}{D}$$ [20%]

(d)

(i) The complementary function solves

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) + \frac{(\eta - 1)\Sigma_{\mathrm{a}}}{D}\phi = 0$$

Now $$\frac{1}{r^2}\frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = \frac{1}{r^2}\left[r^2 \frac{d^2\phi}{dr^2} + 2r\frac{d\phi}{dr}\right] = \frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr}$$

$$\therefore \quad \frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} + \frac{(\eta - 1)\Sigma_{\mathrm{a}}}{D}\phi = 0$$

If $\psi = \phi r$, then $$\frac{d\psi}{dr} = \phi + r\frac{d\phi}{dr}$$

$$\therefore \quad \frac{d^2\psi}{dr^2} = \frac{d\phi}{dr} + \frac{d\phi}{dr} + r\frac{d^2\phi}{dr^2} = r\frac{d^2\phi}{dr^2} + 2\frac{d\phi}{dr}$$

$$\therefore \quad \frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \frac{1}{r}\frac{d^2\psi}{dr^2}$$

$$\therefore \quad \frac{1}{r}\frac{d^2\psi}{dr^2} + \frac{(\eta-1)\Sigma_a}{D}\frac{\psi}{r} = 0$$

Let $B^2 = \frac{(\eta-1)\Sigma_a}{D}$ $\quad\therefore \quad \frac{d^2\psi}{dr^2} + B^2\psi = 0$

This is an SHM equation, so the general solution is

$$\psi = A\sin(Br) + C\cos(Br)$$

$$\therefore \quad \phi_{cf} = \frac{A}{r}\sin(Br) + \frac{C}{r}\cos(Br)$$ [25%]

(ii) Considering $$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right) + \frac{(\eta-1)\Sigma_a}{D}\phi = -\frac{S}{D}$$

As the source S is uniform, the particular integral is, by inspection:

$$\phi_{pi} = -\frac{S}{(\eta-1)\Sigma_a}$$

Thus, the general solution is

$$\phi = \phi_{cf} + \phi_{pi} = \frac{A}{r}\sin(Br) + \frac{C}{r}\cos(Br) - \frac{S}{(\eta-1)\Sigma_a}$$

On physical grounds ϕ must be finite at $r = 0$, so $C = 0$.

$$\therefore \quad \phi = \frac{A}{r}\sin(Br) - \frac{S}{(\eta-1)\Sigma_a}$$

With the boundary condition that $\phi = 0$ at $r = R^+$

$$\frac{A}{R^+}\sin(BR^+) - \frac{S}{(\eta-1)\Sigma_a} = 0 \quad \Rightarrow \quad A = \frac{SR^+}{(\eta-1)\Sigma_a\sin(BR^+)}$$

$$\therefore \quad \phi = \frac{1}{r}\frac{SR^+}{(\eta-1)\Sigma_a\sin(BR^+)}\sin(Br) - \frac{S}{(\eta-1)\Sigma_a}$$

$$\therefore \quad \phi = \frac{S}{(\eta-1)\Sigma_a}\left[\frac{R^+}{r}\frac{\sin(Br)}{\sin(BR^+)} - 1\right]$$ [30%]

(iii) When $BR^+ = \pi$, the flux becomes infinite because $\sin(\pi) = 0$. At this point the sphere becomes *critical* – it is able to sustain a steady-state flux distribution without an independent source. [10%]

Assessor's Comments:

All candidates: 87 attempts, Average raw mark 14.3/20, Maximum 19, Minimum 8.

A very popular question attempted by 96% of candidates and generally done quite well.
Answers to part (a) revealed a lack of familiarity with the content of the 4M16 data sheet among candidates.
The most common mistakes in part (b) were neglect of the energy of alpha and/or the mass of neutron in calculations.
Many candidates lost marks in part (c) as a result of not stating their assumptions.
A surprising number of candidates could not recognize the SHM equation. Others failed to seek the particular integral and tried to get the complementary function alone to fit the boundary conditions.
There was also confusion between the particular integral for the ϕ and ψ forms of the diffusion equation.
Several candidates incorrectly eliminated the sin term on grounds on symmetry.
Very few candidates correctly recognized the achievement of criticality when BR^+ reaches a value of π.

Q2

(a) The sinusoidal term reflects the variation in the coolant temperature along the channel. This depends on the total amount of heat transferred from the fuel and thus an integral of the cosinusoidal power distribution.

The cosinusoidal term reflects the temperature difference between the coolant and the location in question. This depends on the total thermal resistance and the local power density, which varies cosinusoidally. [15%]

(b) From the 4M16 data sheet $\theta = \dfrac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \sin\left(\dfrac{\pi L}{2L'}\right)$

As $L = L'$ $\quad \therefore \ \theta = \dfrac{T - T_{c1/2}}{T_{co} - T_{c1/2}} \ \Rightarrow \ \theta_{\max} = \dfrac{T_{\max} - T_{c1/2}}{T_{co} - T_{c1/2}}$

Noting that, by symmetry, $T_{co} - T_{c1/2} = T_{c1/2} - T_{ci}$

$$\therefore \ \theta_{\max}\left[T_{c1/2} - T_{ci}\right] = T_{\max} - T_{c1/2}$$

$$\therefore \ T_{c1/2} = \frac{T_{\max} + \theta_{\max} T_{ci}}{1 + \theta_{\max}}$$ [15%]

(c) Coolant exit temperature

$Q = 0 \ \Rightarrow \ \theta_{\max} = 1$

Coolant entry temperature $T_{ci} = 330\,°\mathrm{C}$

Coolant exit temperature $T_{co} = 650\,°\mathrm{C}$

$\therefore$ Coolant mid-channel temperature $T_{c1/2} = \frac{1}{2}\left[T_{ci} + T_{co}\right] = \frac{1}{2}\left[330 + 650\right] = 490\,°\mathrm{C}$ (5%)

Cladding temperature

The cladding temperature will be greatest at its interior surface.

Using the formula on page 8 of the 4M16 data sheet:

$$\frac{1}{U} = \frac{1}{h} + \frac{t_c}{\lambda_c} = \frac{1}{200} + \frac{10^{-3}}{16} = 0.0050625$$

$$\therefore \quad U = 197.53\,\mathrm{WK^{-1}m^{-2}}$$

Also, from page 8 of the 4M16 data sheet, $Q = \dfrac{\pi \dot{m} c_p}{UA}\dfrac{L}{L'}$ and $A = 4\pi r_o L$

Here $L = L' = 4$ m and $r_o = r_i + t_c = 7\times10^{-2} + 1\times10^{-3} = 7.1\times10^{-2}$ m

$$\therefore \quad Q = \frac{\pi \dot{m} c_p}{U 4\pi r_o L} = \frac{\dot{m} c_p}{U 4 r_o L} = \frac{0.5\times0.82\times10^3}{U\times4\times7.1\times10^{-2}\times4} = \frac{360.915}{U}$$

So, in this case, $$Q = \frac{360.915}{197.53} = 1.827$$

$$\therefore \quad \theta_{\max} = \sqrt{1+Q^2} = \sqrt{1+1.827^2} = 2.083$$

Using the result from (b), for the limiting cladding temperature,

$$T_{c1/2} = \frac{T_{\max} + \theta_{\max} T_{ci}}{1+\theta_{\max}} = \frac{900 + 2.083\times330}{1+2.083} = 514.9\,^\circ\mathrm{C} \qquad (30\%)$$

Fuel temperature

The fuel temperature will be greatest at its centre.

$$\therefore \quad \frac{1}{U} = \frac{1}{h} + \frac{t_c}{\lambda_c} + \frac{r_o}{h_b r_i} + \frac{r_o}{2\lambda_f} = \frac{1}{200} + \frac{10^{-3}}{16} + \frac{7.1\times10^{-2}}{2\times10^3\times7\times10^{-2}} + \frac{7.1\times10^{-2}}{2\times3} = 0.017403$$

$$\therefore \quad U = 57.46\,\mathrm{WK^{-1}m^{-2}}$$

$$\therefore \quad Q = \frac{360.915}{57.46} = 6.281$$

$$\therefore \quad \theta_{\max} = \sqrt{1+Q^2} = \sqrt{1+6.281^2} = 6.360$$

$$\therefore \quad T_{c1/2} = \frac{T_{\max} + \theta_{\max} T_{ci}}{1+\theta_{\max}} = \frac{1500 + 6.360\times330}{1+6.360} = 489.0\,^\circ\mathrm{C} \qquad (25\%)$$

Thus, the fuel temperature constraint is (just) the most limiting.

The channel power $P = \dot{m} c_p \left[T_{co} - T_{ci}\right] = 2\dot{m} c_p \left[T_{c1/2} - T_{ci}\right]$

$$\therefore \quad P = 2\times0.5\times0.82\times10^3\left[489-330\right] = 130380\ \mathrm{W} = 0.130\ \mathrm{MW} \qquad (10\%)\ \ [70\%]$$

Assessor's Comments:

All candidates: 66 attempts, Average raw mark 12.9/20, Maximum 20, Minimum 1.

A reasonably popular question attempted by 72.5% of candidates.

Part (a) was answered surprisingly poorly. Few candidates showed a convincing understanding of the origins of the form of Ginn's equation.

Unsuccessful efforts to answer part (b) usually failed to note and exploit the form of the coolant temperature distribution, meaning the temperature at mid-channel is just the average of inlet and outlet temperatures.

Answers to part (c) frequently went astray due to incorrect calculations of U: in some instances, the appropriate terms were excluded rather than included; in others, candidates did not show the values substituted in reaching wrong numerical answers, making it difficult to allocate partial credit.
Several candidates failed to appreciate that the maximum cladding temperature would occur on the inside surface of the cladding.
Other candidates tried to use the same values of U and Q for all three limiting cases.
Finally, a number of candidates failed to appreciate that the case with the lowest mid-channel temperature would be the limiting one.

Q3

(a) Many fission products are unstable. Some decay by neutron emission. Unlike the neutrons emitted promptly in fission, these neutrons are emitted some time after the fission reaction that produced the relevant fission product (at a time dependent on the decay constant of the fission product in question). These neutrons are in consequence known as delayed neutrons.

Delayed neutrons have a very significant, beneficial effect on reactor dynamics. They increase the average neutron lifetime and hence lengthen the dominant time constant governing the dynamic behaviour of the neutron population. [15%]

(b) The major simplifications of this model are that it assumes there is no spatial variation in behaviour, whereas, in practice, the reactor core is highly heterogeneous and the neutron population varies spatially, and it also assumes that there is only one type of precursor, whereas in reality there are a large number of them with widely varying production rates and half-lives. [10%]

(c) In steady-state operation $\frac{dc}{dt} = 0$

$$\therefore \quad \frac{\beta}{\Lambda} n_0 = \lambda c_0 \quad \Rightarrow \quad \frac{c_0}{n_0} = \frac{\beta}{\lambda \Lambda} = \frac{0.007}{0.1 \times 5 \times 10^{-4}} = 140$$ [10%]

(d)

$$\rho = p\left[\Lambda + \frac{\beta}{(p+\lambda)}\right] \quad \Rightarrow \quad \rho(p+\lambda) = p\left[\Lambda(p+\lambda) + \beta\right]$$

$$\therefore \quad \frac{\rho}{\Lambda}(p+\lambda) = p^2 + p\left[\lambda + \frac{\beta}{\Lambda}\right] \quad \Rightarrow \quad p^2 + p\left[\lambda + \frac{\beta - \rho}{\Lambda}\right] - \frac{\rho\lambda}{\Lambda} = 0$$

$$\therefore \quad p^2 + p\left[0.1 + \frac{0.007 - 0.001}{5 \times 10^{-4}}\right] - \frac{0.001 \times 0.1}{5 \times 10^{-4}} = 0$$

$$\therefore \quad p^2 + 12.1p - 0.2 = 0 \quad \Rightarrow \quad p = 0.0165 \text{ or } -12.1165 \text{ s}^{-1}$$

Hence the dominant time constant $T_+ = \frac{1}{p_+} = \frac{1}{0.0165} = 60.6 \text{ s}$ [15%]

(e) In the prompt jump approximation, the neutron population is assumed to stay in equilibrium with the precursor population. So, if $\frac{dn}{dt} = 0$ for a source-free reactor ($s = 0$)

$$\frac{\rho - \beta}{\Lambda} n + \lambda c = 0 \quad \Rightarrow \quad n = \frac{\lambda \Lambda}{\beta - \rho} c$$

Substituting for n $$\frac{dc}{dt} = \frac{\beta\lambda}{\beta-\rho}c - \lambda c = \frac{\rho\lambda}{\beta-\rho}c$$

$$\therefore \quad \frac{dc}{c} = \frac{\rho\lambda}{\beta-\rho}dt \quad \Rightarrow \quad \ln c = \frac{\rho\lambda}{\beta-\rho}t + \text{const} \quad \Rightarrow \quad c = A\exp\left(\frac{\rho\lambda}{\beta-\rho}t\right)$$

With a boundary condition that $c = c_0$ at $t = 0$, then

$$c = c_0 \exp\left(\frac{\rho\lambda}{\beta-\rho}t\right)$$ [20%]

(f)

(i) $$\frac{d\rho}{dp} = \left[\Lambda + \frac{\beta}{(p+\lambda)}\right] - p\frac{\beta}{(p+\lambda)^2}$$

$$\therefore \quad \frac{d\rho}{dp} = \Lambda + \frac{\beta}{(p+\lambda)}\left[1 - \frac{p}{(p+\lambda)}\right] = \Lambda + \frac{\beta\lambda}{(p+\lambda)^2}$$

$$\therefore \quad \left.\frac{d\rho}{dp}\right|_{p=0} = \Lambda + \frac{\beta}{\lambda} = 5\times10^{-4} + \frac{0.007}{0.1} = 0.0705$$

$$\therefore \quad p_+ \approx \frac{\rho}{0.0705} = \frac{0.001}{0.0705} = 0.0142\ \text{s}^{-1}$$

$$\therefore \quad T_+ = \frac{1}{p_+} = \frac{1}{0.0142} = 70.5\ \text{s}$$

which is a reasonable approximation of the result for the dominant time constant in (d), but not a conservative one as the time constant is overestimated.

(ii) In the prompt jump approximation, the neutron population is assumed to stay in equilibrium with the precursor population, so, from the result in (e), the dominant time constant will be

$$T_+ = \frac{\beta-\rho}{\rho\lambda} = \frac{0.007-0.001}{0.001\times0.1} = 60\ \text{s}$$

which is a very good (and slightly conservative) estimate.

(iii) With no delayed neutrons $$\frac{dn}{dt} = \frac{\rho}{\Lambda}n$$

So $$T_+ = \frac{\Lambda}{\rho} = \frac{5\times10^{-4}}{0.001} = 0.5\ \text{s}$$

Thus, delayed neutrons make the dominant time constant ×121 longer, making the reactor considerably more controllable. [30%]

Assessor's Comments:

All candidates: 88 attempts, Average raw mark 14.2/20, Maximum 20, Minimum 3.
The most popular question, attempted by 97% of candidates, and generally done quite well, although part (e) was skipped entirely by a surprising number of candidates.
Marks were often lost due to sloppy working, inadequately detailed comments and inadequately justified steps/approximations rather than lack of knowledge of how to approach delayed neutron kinetics analysis.

Some candidates incorrectly thought that a negative inverse period was impossible, rather than just not being associated with the dominant time constant.
Several candidates calculated the gradient in part (f)(i) correctly but did not find the associated time constant correctly.
In commenting on the significance of the time constant comparisons, many candidates failed to discuss the conservatism of the approximations.
There was also some confusion between the notion of supercriticality and the speed of response when the reactor is supercritical.

Q4

(a) Separative work is the unit in which enrichment is traded and is defined as

$$S = E_w\left(2x_w - 1\right)\ln\left(\frac{x_w}{1-x_w}\right) + E_p\left(2x_p - 1\right)\ln\left(\frac{x_p}{1-x_p}\right) - E_f\left(2x_f - 1\right)\ln\left(\frac{x_f}{1-x_f}\right)$$

where E_i and x_i are the total mass and concentration of the feed $(i = f)$, product $(i = p)$ and waste $(i = w)$, respectively.

The approximation to

$$S = E_w\left[-\ln(x_w)\right] + E_p\left[-\ln(x_p)\right] - E_f\left[-\ln(x_f)\right]$$

is generally valid for civil reactors where the enrichment is low.

S increases as the enrichment required (x_p) increases and the tails concentration (x_w) decreases. Since the enrichment is dictated by reactor physics and the feed concentration (x_f) by nature, the only variable is the tails concentration. The lower the tails concentration the greater the SWU required but less feed (E_f) will be needed, so the optimal tail concentration depends on the ratio of the uranium price and the cost of a SWU. When the uranium price is high it may be worth spending more on SWUs to reduce the feed requirements, and the reverse is true when the SWU price is high. [25%]

(b)

(i)
$$P_e = \eta P_{th} \quad \Rightarrow \quad P_{th} = \frac{P_e}{\eta} = \frac{1200}{0.3} = 4000 \text{ MW}$$

$\therefore$ Annual thermal output $E_{th} = 4000 \times 365 \times 0.9 = 1.314 \times 10^6$ MWd

$\therefore$ Annual fuel requirement $P = \frac{E_{th}}{B} = \frac{1.314 \times 10^6}{40} = 32.85 \times 10^3 \text{ kgU} = 32.85 \text{ te}$ [10%]

(ii) Considering mass balance across the enrichment plant at 0.3% tails

Uranium $$F = P + W$$

^{235}U $$x_f F = x_p P + x_w W$$

$$\therefore \quad x_f(P + W) = x_p P + x_w W$$

$$\therefore \quad W(x_f - x_w) = P(x_p - x_f)$$

$$\therefore \quad W = P\frac{(x_p - x_f)}{(x_f - x_w)} = 32.85\frac{0.04 - 0.007}{0.007 - 0.003} = 271.01 \text{ te}$$

$$\therefore \quad F = P + W = 32.85 + 271.01 = 303.86 \text{ te}$$

As UOC is 95% U, the total mass of UOC needed

$$M_{\text{uoc}} = \frac{F}{0.95} = \frac{303.86}{0.95} = 319.85 \text{ te}$$ [15%]

(iii)
$$S = W\left[-\ln(x_w)\right] + P\left[-\ln(x_p)\right] - F\left[-\ln(x_f)\right]$$

$$\therefore \quad S = 271.01\left[-\ln(0.003)\right] + 32.85\left[-\ln(0.04)\right] - 303.86\left[-\ln(0.007)\right]$$

$$\therefore \quad S = 172.37 \text{ te SWU}$$

$$\text{Cost} = C_{\text{uoc}} M_{\text{uoc}} + C_{\text{swu}} S$$

$$\therefore \quad \text{Cost} = 40 \times 319.85 \times 10^3 + 50 \times 172.37 \times 10^3 = \$21.413 \times 10^6$$ [15%]

(iv) For the values given in the question $\dfrac{C_{\text{uoc}}}{x_{\text{u}} C_{\text{swu}}} = \dfrac{40}{0.95 \times 50} = 0.8421$

We therefore need to solve $z = 1.8421 + \ln(z)$ where $z = \dfrac{x_f}{x_w}$.

This transcendental equation can be solved by an iterative scheme

$$z_{n+1} = 1.8421 + \ln(z_n)$$

Using a first guess of $z_0 = 3$, this converges to

$$z = 2.9104$$

$$\therefore \quad x_w = \frac{x_f}{z} = \frac{0.007}{2.9104} = 0.002405$$

For this value of x_w

$$W = P\frac{(x_p - x_f)}{(x_f - x_w)} = 32.85\frac{0.04 - 0.007}{0.007 - 0.002405} = 235.92 \text{ te}$$

$$\therefore \quad F = P + W = 32.85 + 235.92 = 268.77 \text{ te}$$

$$\therefore \quad M_{\text{uoc}} = \frac{F}{0.95} = \frac{268.77}{0.95} = 282.92 \text{ te}$$

$$S = W\left[-\ln(x_w)\right] + P\left[-\ln(x_p)\right] - F\left[-\ln(x_f)\right]$$

$$\therefore \quad S = 235.92\left[-\ln(0.002405)\right] + 32.85\left[-\ln(0.04)\right] - 268.77\left[-\ln(0.007)\right]$$

$$\therefore \quad S = 194.79 \text{ te SWU}$$

$$\text{Cost} = C_{\text{uoc}} M_{\text{uoc}} + C_{\text{swu}} S$$

$$\therefore \quad \text{Cost} = 40 \times 282.92 \times 10^3 + 50 \times 194.79 \times 10^3 = \$21.056 \times 10^6$$

This is indeed less than the cost when $x_w = 0.003$, although only ~2% less. [35%]

Assessor's Comments:

All candidates: 31 attempts, Average raw mark 12.9/20, Maximum 19, Minimum 2.

Comfortably the least popular question, attempted by only 34% of candidates, but done very well by some.

Answers to part (a) were often lacking in detail (given the amount of credit available). Several answers incorrectly thought separative work was important principally in determining whether to reprocess or not, while failing to note that the relative cost of separative work compared to uranium feed is important in configuring any enrichment plant.

The calculations in parts (b)(i) to (iii) were generally done well with confusion over units being the main source of error.

The transcendental equation in part (b)(iv) confounded many. Those who recognized that an iterative solution was possible usually found the correct result, but did not always follow through in calculating the new feed and tails mass flows.