Version GTP/6

EGT3 / EGT2
ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

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Thursday 24 April 2014 2 to 3.30

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## Module 4M16

## NUCLEAR POWER ENGINEERING

*Answer not more than* ***three*** *questions.*

*All questions carry the same number of marks.*

*The* ***approximate*** *percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number* ***not*** *your name on the cover sheet.*

**STATIONERY REQUIREMENTS**
Single-sided script paper

**SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM**
CUED approved calculator allowed
Attachment: 4M16 data sheet (8 pages)
Engineering Data Book

**You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.**

1 (a) In one-group, steady-state diffusion theory, the equation for the neutron flux $\phi$ for a source-free, homogeneous reactor with spherical geometry is

$$\frac{1}{r^2}\frac{d}{dr}r^2\frac{d}{dr}\phi(r)+\frac{(\eta-1)\Sigma_a}{D}\phi(r)=0$$

where $\eta$ is the average number of neutrons released in fission per neutron absorbed, $\Sigma_a$ is the macroscopic absorption cross-section, and $D$ is the diffusion coefficient.

Show that the solution to this equation is

$$\phi(r)=\frac{A}{r}\sin(Br)$$

where $B^2=\dfrac{(\eta-1)\Sigma_a}{D}$ and $A$ is a constant. [30%]

(b) A graphite-moderated spherical reactor is fuelled with 3% enriched $UO_2$, which is uniformly distributed throughout the core and occupies 5% of its volume. Take the density of $UO_2$ to be $1.097\times10^4\ \mathrm{kg\,m^{-3}}$, its molar mass to be $270\ \mathrm{kg\,kmol^{-1}}$ and $\nu$, the average number of neutrons released in a $^{235}$U fission reaction, to be 2.43. Extrapolation distances and the absorption cross-sections of everything other than the uranium can be neglected. Data: $^{235}$U: $\sigma_c=107\ \mathrm{b}$, $\sigma_f=580\ \mathrm{b}$; $^{238}$U: $\sigma_c=2.75\ \mathrm{b}$, $\sigma_f=0\ \mathrm{b}$; $D=0.01\ \mathrm{m}$.

(i) Show that the value of $\eta$ for this fuel is 1.817. [20%]

(ii) Find the minimum volume reactor for criticality. [15%]

(c) In spherical geometry the *form factor*, $F$, is defined by

$$F=\frac{\text{maximum flux}}{\text{volume-average flux}}$$

Show that $F=\frac{1}{3}\pi^2$ for this minimum volume reactor. [25%]

(d) How can form factors be improved in practice? [10%]

2 The equations governing the behaviour of xenon-135 in a 'lumped' reactor model can be written as

$$\frac{dI}{dt} = \gamma_i \Sigma_f \phi - \lambda_i I$$

$$\frac{dX}{dt} = \gamma_x \Sigma_f \phi + \lambda_i I - \lambda_x X - \sigma X \phi$$

where all symbols have their usual meanings.

(a) Stating any assumptions made, show that the steady-state loss of reactivity $\rho_{\mathrm{Xe}0}$ due to xenon poisoning in a high power reactor approaches $-(\gamma_x + \gamma_i)/\nu$, where $\nu$ is the average number of neutrons released per fission. [20%]

(b) A reactor is operating in steady state with a neutron flux $\phi$ of $10^{18}$ m$^{-2}$s$^{-1}$. How does the steady-state loss of reactivity due to xenon poisoning compare with that predicted by the result in (a)?

Data: $\gamma_i = 0.061$; $\gamma_x = 0.003$; $\lambda_i = 2.874 \times 10^{-5}$ s$^{-1}$; $\lambda_x = 2.093 \times 10^{-5}$ s$^{-1}$; $\sigma = 2.75$ Mb; $\nu = 2.43$. [15%]

(c) After prolonged operation at this flux level, the reactor is shut down. If it is to be restarted after a 2-hour outage, how much reactivity is required to overcome the xenon poisoning? [50%]

(d) If this is all the reactivity available, will the reactor be able to continue operating once restarted? [15%]

3 (a) The reactivity $\rho$ of a Pressurized Water Reactor (PWR), which operates at constant power, varies linearly with time $t$ as

$$\rho = \rho_0 \left(1 - \frac{t}{T}\right)$$

where $\rho_0$ is the start-of-cycle reactivity and $T$ is the cycle length for one-batch operation. Derive an expression for the equilibrium cycle length $\mu$ in $M$-batch operation, and hence show that the discharge burnup $B$ accumulated by each batch in equilibrium $M$-batch operation is given by

$$B = \frac{2\alpha MT}{M+1}$$

where $\alpha$ is the amount of burnup accumulated in unit time of operation. [20%]

(b) Due to the number of maintenance operations to be undertaken during the refuelling outage, the length of the outage, $\Delta$, is independent of the number of fuel assemblies to be replaced. Show that the PWR's availability $A$ (the fraction of the time it spends at power) is given by

$$A = \frac{2T}{2T + (M+1)\Delta}$$

[10%]

(c) From an economic point of view it is desirable to maximise both the PWR's availability and the discharge burnup of the fuel. Why is it not possible to maximise availability and discharge burnup simultaneously? What are the maximum possible values of $A$ and $B$? [15%]

(d) In order to strike a suitable compromise between availability and discharge burnup, an engineer decides to maximise

$$f = \frac{3A}{A_{\max}} + \frac{B}{B_{\max}}$$

where $A_{\max}$ and $B_{\max}$ are the maximum possible values of $A$ and $B$. Show that

$$f = \frac{6(T+\Delta)}{2T + (M+1)\Delta} + \frac{M}{M+1}$$

and hence find the optimal batch refuelling scheme if $T = 135$ weeks and $\Delta = 4$ weeks. [30%]

(e) What is the total cycle length, including the refuelling outage, for this refuelling scheme? From a practical perspective is this cycle length optimal? If not, what changes would you recommend? [25%]

4 (a) Describe the basic steps in the reprocessing of spent nuclear fuel. Discuss the main waste streams arising and how they are handled. [30%]

(b) (i) What are the advantages and disadvantages of reprocessing spent nuclear fuel? [10%]

(ii) Why was reprocessing essential for first generation UK Magnox reactors? [5%]

(iii) Suggest ways of using the plutonium recovered by reprocessing spent fuel. [5%]

(c) A large utility operating a number of Pressurized Water Reactors (PWRs) requires 100 tonnes of fuel (as uranium metal) per year at an enrichment of 4% $^{235}\mathrm{U}$. Taking the $^{235}\mathrm{U}$ content of natural uranium to be 0.7% and assuming an enrichment plant tails of 0.3% $^{235}\mathrm{U}$, calculate the amount of natural uranium and the number of separation work units (SWUs) required annually. Processing losses other than enrichment tails can be neglected. [15%]

(d) Estimate the savings in fresh natural uranium and separation work units if the spent fuel from the PWRs was reprocessed and the uranium recycled. Assume that the fuel leaves the reactors containing 96% uranium at an enrichment of 0.8% $^{235}\mathrm{U}$. Take the losses in the reprocessing plant to be 1%. [25%]

(e) Assuming natural uranium costs \$70 per kg, 1 SWU(kg) costs \$100, reprocessing (including waste disposal) costs \$1000 per kg, and direct disposal of spent fuel costs \$400 per kg, is reprocessing economic in this case? Ignore any credit for the re-use of the plutonium. [10%]

**END OF PAPER**

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Answers

Q1 (b)(ii) $R = 0.206\ \text{m},\ V = 0.0366\ \text{m}^3$

Q2 (b) $-0.0245$ (exact), $-0.0263$ (in the limit)

(c) $0.0626$

(d) No

Q3 (c) $A_{\max} = \dfrac{T}{T + \mathit{\Delta}},\ B_{\max} \approx 2\alpha T$

(d) 4-batch operation is optimal

Q4 (c) 925 tonnes, 524.72 tonnes SWU

(d) 118.8 tonnes, 7.41 tonnes SWU

(e) No, reprocessing costs $48,545,100 more