EGT3 / EGT2
ENGINEERING TRIPOS PART IIB
ENGINEERING TRIPOS PART IIA

Friday 30 April 2021 1.30 to 3.10

Module 4M16

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet and at the top of each answer sheet.*

STATIONERY REQUIREMENTS
Write on single-sided paper.

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed.
Attachment: 4M16 Nuclear Power Engineering data sheet (8 pages).
You are allowed access to the electronic version of the Engineering Data Books.

10 minutes reading time is allowed for this paper at the start of the exam.

The time taken for scanning/uploading answers is 15 minutes.

Your script is to be uploaded as a single consolidated pdf containing all answers.

1 Start-up neutron sources for nuclear reactors are typically inserted in regularly spaced positions within the reactor core. One of the commonly employed start-up sources uses a mixture of plutonium-238 and beryllium-9.

(a) Plutonium-238 decays by α emission with a half-life of 87.7 years and has an atomic mass of 238.049553 u. Into which isotope does $^{238}\mathrm{Pu}$ decay? [5%]

(b) In a $^{238}\mathrm{Pu}$ - $^{9}\mathrm{Be}$ neutron source, α particles of energy 5.593 MeV from the decay of $^{238}\mathrm{Pu}$ interact with $^{9}\mathrm{Be}$ through the following reaction:

$$^{9}\mathrm{Be} + {}^{4}\mathrm{He} \rightarrow {}^{12}\mathrm{C} + \mathrm{n}$$

Estimate the maximum energy (in MeV) of the neutron released in this reaction. Relevant atomic masses can be found in the 4M16 data sheet. [10%]

(c) A start-up neutron source is uniformly distributed in the core of a spherical geometry reactor of radius *R*. Starting from the general form of the neutron diffusion equation given on page 6 of the 4M16 data sheet, derive the neutron diffusion equation for a steady-state uniform spherical system

$$\frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{d\phi}{dr}\right)+\frac{(\eta-1)\Sigma_\mathrm{a}}{D}\phi=-\frac{S}{D}$$

stating any assumptions made. [20%]

(d) (i) By using the substitution $\psi=\phi r$, or otherwise, show that the complementary function ϕ_cf of the differential equation in (c) for the case where $\eta>1$ is of the form

$$\phi_\mathrm{cf}=\frac{A}{r}\sin(Br)+\frac{C}{r}\cos(Br)$$

where $B^2=\dfrac{(\eta-1)\Sigma_\mathrm{a}}{D}$ and *A* and *C* are constants. [25%]

(ii) Hence find an expression for the flux distribution in the reactor for the case where $\eta>1$ with a boundary condition that $\phi=0$ at $r=R^{+}$, where $R^{+}>R$ is the extrapolated radius. [30%]

(iii) Explain what happens when BR^{+} reaches a value of π. [10%]

2 For axial coolant flow in a nuclear reactor with a 'chopped' cosine power distribution, Ginn's equation for non-dimensional temperature is

$$\theta = \sin\left(\frac{\pi x}{2L'}\right) + Q\cos\left(\frac{\pi x}{2L'}\right)$$

where L' is the flux half-length of the power distribution and Q is as defined on page 8 of the 4M16 data sheet.

(a) Explain qualitatively the origin of the sinusoidal and cosinusoidal terms in Ginn's equation. A detailed mathematical derivation is not required. [15%]

(b) Using the expression for θ in the 4M16 data sheet, show that, if L' is equal to the channel half-length L, the coolant temperature at mid-channel can be expressed as

$$T_{c1/2} = \frac{T_{\max} + \theta_{\max} T_{ci}}{1 + \theta_{\max}}$$

where $T_{\max}$ is a maximum temperature, $\theta_{\max}$ the corresponding maximum non-dimensional temperature and T_{ci} the coolant inlet temperature. [15%]

(c) A fuel channel of a nuclear reactor has a cylindrical shape with dimensions and properties given below. Coolant enters at $330\,°\mathrm{C}$. The following temperature constraints must all be satisfied:

The coolant exit temperature must not exceed $650\,°\mathrm{C}$
The cladding temperature must not exceed $900\,°\mathrm{C}$
The fuel temperature must not exceed $1500\,°\mathrm{C}$

The fuel channel experiences a full cosine neutron flux along its length, i.e. $L' = L$.

By finding the coolant temperature at mid-channel when each temperature constraint is active, identify which constraint is limiting, and, hence, calculate the maximum possible channel power. You can assume without proof that the maximum non-dimensional temperature is given by $\theta_{\max}^2 = 1 + Q^2$. [70%]

Data:	Fuel radius	r_i	7 cm
	Cladding thickness	t_c	1 mm
	Cladding thermal conductivity	λ_c	$16\,\mathrm{W K^{-1} m^{-1}}$
	Fuel thermal conductivity	λ_f	$3\,\mathrm{W K^{-1} m^{-1}}$
	Fuel-to-cladding heat transfer coefficient	h_b	$2\,\mathrm{kW K^{-1} m^{-2}}$
	Cladding-to-coolant heat transfer coefficient	h	$200\,\mathrm{W K^{-1} m^{-2}}$
	Coolant mass flow rate	$\dot{m}$	$0.5\,\mathrm{kg\,s^{-1}}$
	Coolant specific heat capacity	c_p	$0.82\,\mathrm{kJ\,kg^{-1} K^{-1}}$
	Channel length	$2L$	8 m

3 (a) Explain what is meant by the term *delayed neutrons*. Why are delayed neutrons so important to nuclear reactor dynamics? [15%]

(b) In a 'lumped' model of the behaviour of a nuclear reactor, the equations for the neutron population n and the delayed neutron precursor population c can be written as

$$\frac{dn}{dt} = \frac{\rho - \beta}{\Lambda} n + \lambda c + s$$
$$\frac{dc}{dt} = \frac{\beta}{\Lambda} n - \lambda c$$

where all symbols have their usual meanings.

What major simplifying assumptions underlie this model? [10%]

(c) Estimate the ratio of precursors to neutrons in steady-state operation in a reactor for which $\beta = 0.007$, $\lambda = 0.1\ \mathrm{s}^{-1}$ and $\Lambda = 0.5\ \mathrm{ms}$. [10%]

(d) A critical, source-free reactor has been operating in equilibrium for a prolonged period. It is then subject to a step increase in reactivity. For this model, the *in-hour equation* relating the inverse periods p of the subsequent kinetic behaviour of the neutron population to the reactivity ρ is

$$\rho = p\left[\Lambda + \frac{\beta}{(p+\lambda)}\right]$$

If $\rho = 0.001$, $\beta = 0.007$, $\lambda = 0.1\ \mathrm{s}^{-1}$ and $\Lambda = 0.5\ \mathrm{ms}$, show that the dominant time constant of the reactor's dynamic response is 60.6 s. [15%]

(e) Show that the *prompt jump approximation* predicts that the precursor population will vary as

$$c = c_0 \exp\left(\frac{\rho\lambda}{\beta - \rho} t\right)$$

where c_0 is the precursor population at the start of the transient (when $t = 0$). [20%]

(f) Compare the result for the dominant time constant obtained in (d) with:

(i) that predicted by approximating the relationship between ρ and p near the origin $(\rho = 0, p = 0)$ by a straight line of gradient $\left.\frac{d\rho}{dp}\right|_{p=0}$;

(ii) that predicted by the prompt jump approximation;

(iii) the dominant time constant if there were no delayed neutrons.

Comment on the significance of these comparisons. [30%]

4 (a) Define the term *separative work unit* (SWU) and explain why it is so important to the economics of nuclear fuel. What is the main criterion in determining the tails concentration? [25%]

(b) A 1200 MW(e) nuclear reactor requires fuel enriched to 4.0 wt% ^{235}U. The reactor has an overall efficiency of 30% and operates at full load for 90% of the year.

(i) Estimate the reactor's annual fuel requirement assuming a burnup of 40 MWd/kgU per annum. [10%]

(ii) Estimate how much uranium ore concentrate (UOC) will be required if the tails concentration is 0.3 wt% ^{235}U, the ^{235}U content of natural uranium is 0.7 wt% and the UOC has a U content of 95 wt%. Ignore losses at all stages of the fuel cycle except the tails. [15%]

(iii) How much separative work will be needed? Hence determine the cost of the reactor's annual fuel requirement if the cost of UOC is \$40/kg and separative work costs \$50/kg. [15%]

(iv) Optimization theory shows that the optimal tails concentration x_w is given by the solution of the equation

$$\frac{x_f}{x_w} = 1 + \frac{C_{\text{uoc}}}{x_{\text{u}} C_{\text{swu}}} + \ln\left(\frac{x_f}{x_w}\right)$$

where C_{uoc} is cost per unit mass of UOC, C_{swu} is the cost per unit mass of separative work, x_{u} is the U content (wt%) of the UOC and x_f is the ^{235}U content (wt%) of the feed.

By solving this equation, find the optimal tails concentration for the scenario described, and confirm that, for this value of x_w, the cost of the reactor's annual fuel requirement is indeed less than that calculated in (iii). [35%]

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Answers

Q1 (a) ^{234}U

(b) 11.294 MeV

(d)(ii) $\phi = \frac{S}{(\eta - 1)\Sigma_a}\left[\frac{R^+}{r}\frac{\sin(Br)}{\sin(BR^+)} - 1\right]$

Q2 (c) Coolant: 490.0 °C; Cladding: 514.9 °C; Fuel: 489.0 °C; 130 kW

Q3 (c) 140

(f)(i) 70.5 s

(f)(ii) 60.0 s

(f)(iii) 0.5 s

Q4 (b)(i) 32.85 te

(b)(ii) 319.85 te

(b)(iii) 172.37 teSWU; $\$21.413 \times 10^6$

(b)(iv) $x_w = 0.002405$; $\$21.056 \times 10^6$