Version GTP/4

EGT3 / EGT2
ENGINEERING TRIPOS PART IIB
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Friday 26 April 2024 2 to 3.40

Module 4M16

NUCLEAR POWER ENGINEERING

*Answer not more than **three** questions.*

All questions carry the same number of marks.

*The **approximate** percentage of marks allocated to each part of a question is indicated in the right margin.*

*Write your candidate number **not** your name on the cover sheet.*

STATIONERY REQUIREMENTS
Single-sided script paper

SPECIAL REQUIREMENTS TO BE SUPPLIED FOR THIS EXAM
CUED approved calculator allowed
Attachment: 4M16 Nuclear Power Engineering data sheet (8 pages)
Engineering Data Book

10 minutes reading time is allowed for this paper at the start of the exam.

You may not start to read the questions printed on the subsequent pages of this question paper until instructed to do so.

You may not remove any stationery from the Examination Room.

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1 Xenon-133 is a radionuclide used in nuclear medicine for the imaging of the lungs and of blood flow, particularly in the brain. It is a member of the decay chain of antimony-133 ($^{133}_{51}$Sb), a common uranium-235 fission product. $^{133}_{51}$Sb decays into metastable tellurium-133m with a half-life of 2.51 minutes. The majority of $^{133\mathrm{m}}_{52}$Te decays into iodine-133 with a half-life of 55.4 minutes. $^{133}_{53}$I decays into $^{133}_{54}$Xe with a half-life of 20.8 days. $^{133}_{54}$Xe decays into stable caesium-133 with a half-life of 5.24 days.

(a) The atomic masses of $^{133}_{53}$I, $^{133}_{54}$Xe and $^{133}_{55}$Cs are 132.90780 u, 132.90591 u and 132.90545 u, respectively. Stating any assumptions you make, estimate the maximum energy in MeV of the β-particles emitted in the decays between these isotopes. [10%]

(b) Given how much shorter the first two half-lives in the $^{133}_{51}$Sb decay chain are than the third, for modelling purposes $^{133}_{53}$I can be considered the primary fission product. Lumped models for the variation with time of the $^{133}_{53}$I population I and the $^{133}_{54}$Xe population X can then be established as follows:

$$\frac{dI}{dt} = \gamma_{\mathrm{Sb}}\Sigma_{\mathrm{f}}\phi - \lambda_{\mathrm{I}} I$$

$$\frac{dX}{dt} = \lambda_{\mathrm{I}} I - \lambda_{\mathrm{X}} X$$

where γ_{Sb} is the fission yield of $^{133}_{51}$Sb and other symbols have their usual meanings.

(i) Explain these two equations. What assumptions have been made here about the direct fission yields and neutron capture cross-sections of $^{133}_{53}$I and $^{133}_{54}$Xe? [15%]

(ii) Find expressions for the equilibrium $^{133}_{53}$I and $^{133}_{54}$Xe populations I_0 and X_0 in a reactor that has been operating at constant flux ϕ_0 for a prolonged period. [10%]

(c) In a nuclear accident in which fission products are released to the environment, the equations governing this decay chain are those given in (b) but with $\phi = 0$. In such accidents the release of radioactive iodine is a particular concern because iodine accumulates in the thyroid. If, at $t = 0$, $I = I_0$ and $X = 0$, show that the subsequent time variation of the $^{133}_{54}$Xe population is given by

$$X = \frac{\lambda_{\mathrm{I}} I_0}{\lambda_{\mathrm{X}} - \lambda_{\mathrm{I}}}\left[\mathrm{e}^{-\lambda_{\mathrm{I}} t} - \mathrm{e}^{-\lambda_{\mathrm{X}} t}\right]$$ [40%]

(d) Show that in this scenario the activity of $^{133}_{54}$Xe reaches a maximum after about two weeks, and comment on the significance of this result in respect of the dose received by someone ingesting $^{133}_{53}$I following a nuclear accident. [25%]

2 The one-group, steady-state, source-free neutron diffusion equation for a cylindrical geometry, homogeneous, *multiplying* system can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{\partial^2\phi}{\partial z^2}+B^2\phi=0$$

where $B^2=(\eta-1)\Sigma_a/D$ and all symbols have their usual meanings.

The solution of this equation for a critical reactor of radius R and height H is of the form

$$\phi(r,z)=\phi_0 J_0(\alpha r)\cos(\beta z)$$

where J_0 is an ordinary zero-order Bessel function, ϕ_0 is the flux at the centre of the reactor and $\alpha^2+\beta^2=B^2$.

(a) If extrapolation distances can be neglected, how are the values of α and β related to the values of H and R? [15%]

(b) Find the *axial form factor* for such a reactor. Extrapolation distances can again be neglected. [10%]

(c) The equivalent diffusion equation for a source-free cylindrical geometry, homogeneous, *non-multiplying* medium can be written as

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\phi}{\partial r}\right)+\frac{\partial^2\phi}{\partial z^2}-\frac{1}{M^2}\phi=0$$

where M is the neutron migration length in the medium.

This equation is used to model the flux distribution in *axial reflectors*, placed at both ends of the cylindrical core considered above. The radial boundary condition for the reflectors is that $\phi=0$ at $r=R$.

(i) Show that the flux distribution in the reflectors is of the form

$$\phi(r,z)=[A\exp(\gamma z)+C\exp(-\gamma z)]J_0(\alpha r)$$

where α is the same as in (a) and A and C are constants, the values of which are determined by other boundary conditions. [30%]

(ii) What is the relationship between α, γ and M? [5%]

(d) Under certain conditions the resulting flux distribution within the reflector above the core ($z > 0$) is given by

$$\phi(r, z) = \phi_1 J_0(\alpha r)\exp(-\gamma z)$$

where ϕ_1 and γ are constants determined by the boundary conditions.
The core-reflector boundary conditions require that the flux ϕ and the neutron current $D\dfrac{\partial \phi}{\partial z}$ are continuous across the core-reflector interfaces at $z = \pm H/2$.

(i) Show that $\phi_1 = \phi_0 \dfrac{\cos(\beta H/2)}{\exp(-\gamma H/2)}$. [10%]

(ii) Find an expression for γ in terms of β, H and the diffusion coefficients in the core and reflector, D_c and D_r. [15%]

(e) If the use of axial reflectors allows the height of the active core to be reduced by 20%, estimate the new axial form factor, stating any assumptions you make. Comment briefly on the result. [15%]

3 (a) Describe and explain the in-core and out-of-core fuel management strategies typically used with CANDU reactors and Pressurised Water Reactors (PWRs). What is a loading pattern designer trying to achieve? How can they achieve these aims? [40%]

(b) A PWR has been designed for a three-batch fuel cycle with partial reloading after every 12 months of operation. The reactor provides base load and therefore always operates at full power. It is desired to extend the time between reloadings by switching over to a two-batch cycle without changing the enrichment of the fuel used or the reactor power.

(i) Assuming the *linear reactivity model* and the *partial reactivity model* can be applied, what will be the equilibrium cycle length in two-batch operation? What is the main disadvantage of two-batch operation compared to three-batch? [20%]

(ii) If the switch to two-batch operation is made from equilibrium three-batch operation, find the lengths of the first four cycles of two-batch operation. You can assume that during the transition from three-batch to two-batch operation some twice-burnt three-batch fuel will need to be discharged to make space for fresh two-batch fuel. [40%]

4 (a) Why is it necessary to enrich the U-235 content of fuel used in Pressurised Water Reactors (PWRs)? [10%]

(b) Discuss the historic and present-day benefits and disadvantages associated with the reprocessing of civil nuclear fuel. [20%]

(c) A large utility operating a number of PWRs requires 500 tonnes (as uranium metal) of fuel per year at an enrichment of 3.5% U-235. Taking the U-235 content of natural uranium to be 0.7% and assuming an enrichment plant tails of 0.3% U-235, calculate the amount of natural uranium and the number of separation work units (SWU) required per year. Losses in processing can be neglected. [15%]

(d) Estimate the savings per year in fresh natural uranium and separation work units if the fuel was reprocessed and the uranium recycled. Assume that the fuel leaves the reactor containing 96% uranium at an enrichment of 0.8% U-235. Take the total losses in the reprocessing plant to be 1%. [35%]

(e) If the cost of natural uranium is $100 per kg, the cost of a SWU is also $100 per kg, the cost of reprocessing is $3000 per kg (based on reactor fuel feed and including waste costs) and the cost of spent fuel disposal is $1000 per kg U (based on reactor fuel feed), is it worth reprocessing and recycling the uranium? [20%]

END OF PAPER

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Answers

Q1 (a) 1.761 MeV; 0.428 MeV

(b)(ii) $I_0 = \frac{\gamma_{\mathrm{Sb}}\Sigma_{\mathrm{f}}\phi_0}{\lambda_{\mathrm{I}}}$; $X_0 = \frac{\gamma_{\mathrm{Sb}}\Sigma_{\mathrm{f}}\phi_0}{\lambda_{\mathrm{X}}}$

Q2 (a) $\alpha = \frac{2.405}{R}$; $\beta = \frac{\pi}{H}$

(b) $\frac{\pi}{2}$

(c)(ii) $\alpha^2 - \gamma^2 = -\frac{1}{M^2}$

(d)(ii) $\gamma = \frac{D_c}{D_r}\beta\tan(\beta H/2)$

(e) 1.321

Q3 (b)(i) 16 months

(b)(ii) All four last 16 months

Q4 (c) 4000 tonnes; 2160.8 tonnes SWU

(d) 594 tonnes; 37.2 tonnes SWU

(e) Open cycle costs $\$1116.8 \times 10^6$, closed cycle costs $\$2053.0 \times 10^6$