Problem 1

(a)
There are two safety constraints:
- on max fuel temperature and
- on critical heat flux

1. Max fuel temperature is constrained to 2200K
assume that it occures at the peak power location: $F_{xy} \times F_z = 1.4 \times 1.6 = 2.24$

Peak linear power:	q’	~ (2200 - 583) × 4 × 4π =81279 W/m
Core average linear power:	q'_{av}	= 81279 / 2.24 = 36285 W/m
Number of pins	N	= 3000MW / q’ / L = 20670 pins

2. Second constraint is the heat flux.
Assume that the maximum heat flux occurs in the same location as the peak power.

$q'' = q' / 2\pi R$, peak $q'' = q''_{DNB}/1.3 = 1300/1.3 = 1000$ kW/m^2

average $q''=1000/1.4/1.6 = 446$ kW/m^2
Number of pins = 3000 MW / q’’ / 2πRL = 53527 - DNB is more restrictive than T_{max}

in BWR, the limit is dryout, i.e. critical power per bundle:
Maximum power per pin in “hot” assembly = 6 MW / 100 = 0.06 MW
For a pin in average assembly = 0.06 MW / 1.6 = 0.0375 MW
Number of pins = 3000 MW / 0.0375 = 80000 pins - again more restrictive

(b)
It is a reasonable assumption since radial heat transfer rate is much higher than axial.
However, the critical heat flux reduces with axial height due to coolant heat up.
Therefore, MDNBR typically occurs in the upper half of the core, above the location of the peak flux.
Here, the DNB flux is given as a constant value.
Thus, assume DNB occurs at the location of peak heat flux.

(c)
PWR:
Find the core flow rate = $Q / C_p\, dT$ = $3000 \cdot 10^6 / 5941 / 30 = 16832$ kg/s
Find total flow area = $m / \rho / v = 16832 / 689.8 / 5.5 = 4.4366$ m^2
At 150 bar and 310 C
volume of fuel = N * π R2 * L = $53527\, \pi\, 0.005^2\, 4 = 16.816$ m^3
volume of coolant = A L = 4.4366 x 4 = 17.7464 m^3
Core power density = 3000 / (16.816 + 17.7464) = 86 MW/m^3

BWR:
Assume vapour density is negligible compared to the coolant.

The volume occupied by the coolant needs to be higher by 1/(1-void fraction).
Given that H/HM is the same, mass of the water per fuel pin (since the pin geometry is identical) is the same.
At 70 bar saturated, ρ = 760.65 kg/m^3
Mass of coolant in PWR core = 689.8 * 17.7464 = 12241.5 kg
Volume of liquid in BWR core = 12241.5 kg 80000/53527 / 760.65 kg/m3 = 24.0529 m^3
Volume of coolant = 24.0529 m^3 / 0.6 = 40.0881 m^3
Core power density = 3000 / (16.816 80000/53527+ 40.0881) = 46 MW/m^3

(d)
BWRs are cheaper to construct because of the fewer large components (no steam generators)
They however are more expensive to operate because of the radioactivity in the power plant.
Core physics and thus fuel cycle component of the cost is almost identical for both.

(e)
The power density is limited by the heat flux so increasing the surface area is beneficial.
Examples: internally and externally cooled fuel pins, larger number of thinner pins per assembly.
Power distribution can be flattened through fuel management, burnable poisons or enrichment variation.
DNB is also sensitive to coolant temperature and flow rate. So, reducing Tin and increasing m should allow higher power density.

Problem 2

(a)
H/HM ratio is proportional to coolant densities ratio and inversely proportional to volume of the coolant:

$((H/HM)_2/(H/HM)_1)= V_1/V_2$

Volume has linear dependence on temperature as given by
$\beta=1/V\ \partial V/\partial T \approx 1/V\ \Delta V/\Delta T \approx 1/V_1\ (V_2-V_1)/(T_2-T_1)$ or $(V_2/V_1)= (1+ \beta\Delta T)$
MTC = $\partial\rho/\partial T \approx (k_2-k_1)/(k_2\ k_1)\times 1/(T_2-T_1)$
Multiplication factor at H/HM = 5 at 80 C is given as 1.1
We need to pick a perturbed state temperature, find H/HM for it and get a new k from the graph.
Assume T_2 = 300 C $\Rightarrow (V_2/V_1)= (1+ \beta\Delta T) = (1+ 1.55*10^{-3}\times(300\text{-}50)) = 1.3875$
$\Rightarrow\ (H/HM)_2 = (H/HM)_1 \times V_2/V_1= (H/HM)_1 \times (1 + \beta\Delta T)^{-1} = 5 \times (1.3875)^{-1}\approx 3.6$
$\Rightarrow$ keff$_2$(3.6) $\approx$ 1.065 from the graph

$\Rightarrow$ MTC = $\partial\rho/\partial T\approx (k_2-k_1)/(k_2\ k_1)\times 1/(T_2-T_1) = (1.065 - 1.1)/(1.065*1.1)\times(1/(300 - 50)) = - 0.0001195$ K^{-1}

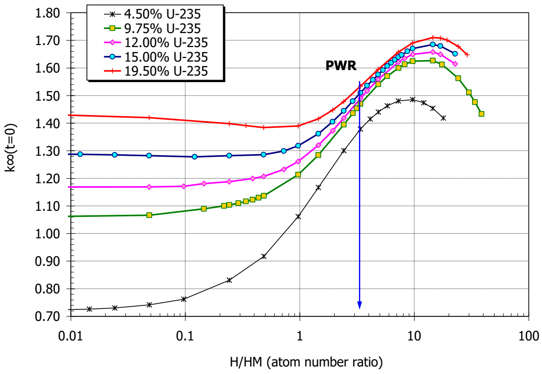
(b)

From the previous section,

If T_2 =300 C $\Rightarrow$ $keff_2(3.6) \approx 1.065$

reactivity change = $(k_2\text{-}k_1)/(k_2\ k_1)$= MTC $\times$ ΔT = -0.0298762

(c)



$k_{inf} \approx (\nu\ \Sigma_f(U235))/(\Sigma_a(U235) + \Sigma_a(U238) + \Sigma_a(H_2O))$

Increase up to H/HM~10 is due to better moderation, and therefore
- increase in σ_f(U235)
- reduction in resonance absorption of U238

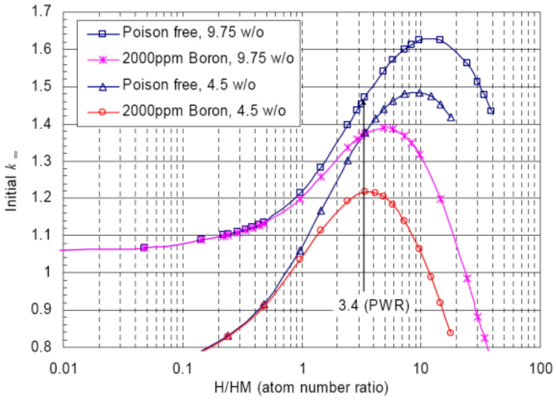
For H/HM above 10, the effect of moderation saturates, while parasitic absorption in H does not and leads to a decrease in k

For high enrichment and low H/HM, an increase in k-eff is due to higher neutron yield per fission as spectrum becomes harder.

(d).(i)

For lower enrichment, the curve is shifted down and slightly to the left because:
- more absorption in U238 ($N_{U238} \uparrow \Rightarrow \Sigma_a(U238)\ \ \uparrow$)
- less fission in U235 ($N_{U235} \downarrow \Rightarrow \Sigma_f(U235)\ \ \downarrow$)
- at high enrichment, larger fraction of neutrons can be absorbed in U235 during slowdown before reaching thermal energies. Therefore, slightly larger amount of moderator (higher H/HM) is required for the moderation effect to saturate.

Note that HM includes both U235 and U238 atoms. The number of H per U235 atom at the peak is roughly the same.



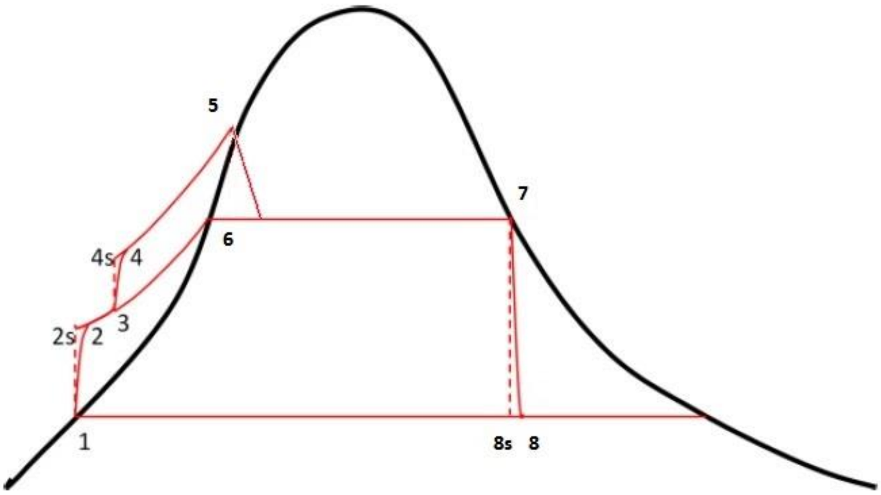
(d).(ii)

With an addition of Boron, the curve also shifts:

- down, due to absorption in Boron
- left, because the absorption in water effect is amplified and thus "overcomes" the moderation effect at lower values of H/HM
- The presence of Boron also makes the spectrum "harder" because it competes with U235 for thermal neutrons. This effect should shift the peak to the right but it is smaller in magnitude than the water absorption effect.
- at H/HM < 1, the spectrum becomes sufficiently hard so that Boron absorption becomes negligible. This is because Boron is primarily thermal absorber while almost transparent to fast neutrons. Therefore, at H/HM < 1, both curves almost overlap.

Problem 3

(a)



(b)

η = ((Net work)/(Heat added))=(w_T - w_F - w_R)/(h_b - h_4)

State 1: p=0.1 bar h_f= 191.8 h_g = 2583.9

State 7: p=86 bar h_f= 1345.5 h_g = 2749.4 s_f= 3.256 s_g = 5.705

State 8s: p=0.1 bar s_f= 0.649 s_g = 8.149

x = (s-s_f)/(s_g-s_f)=(5.705 – 0.649)/(8.149 – 0.649) = 0.674

h_{8s} = (h_g – h_f)x + h_f = 2392.1*0.674+191.8 = 1804.08

w_T = (2749.4 – 1804.08)*0.9 = 850.788 kJ/kg

$w_F \approx v_1 (p_2 - p_1) = 0.001 (8.6*10^6 - 1*10^5)$ = 8.500 kJ/kg

State 2: p=86 bar $h_2 = h_1 + w_F/\eta_p$ = 191.8 + 8.500/0.85 = 201.8

$w_R \approx v_3 (p_4 - p_3)/\eta_p = 0.001404 (14.0*10^6 - 8.6*10^6)/0.85$ = 8.9195 kJ/kg

Saturated water inside the flashing drum at 86 bar: h_6 = 1345.5

f - fraction of water flashed = 10%

State 3: p=86 bar $h_3=h_2 f + h_a (1 - f)$ = 201.8*0.1 + 1345.5*(1 – 0.1) = 1231.13

State 4: p=140 bar $h_4= h_3 + w_R$ = 1231.13 + 8.9195 = 1240.05 kJ/kg

State 5: p=140 bar T = 325 C h_5 = 1488.2 kJ/kg

Combining everything together with appropriate flow fractions:

$\eta = (f\, w_T - f\, w_F - w_R)/(h_5 - h_4)$ = (0.1* 850.788 - 0.1*8.5 – 8.9195)/(1488.2 – 1240.05) = 30.35 %

(c)

Net power generated = $f\, \dot{m} w_T - f\, \dot{m} w_F - \dot{m} w_R$ = 1000 MW

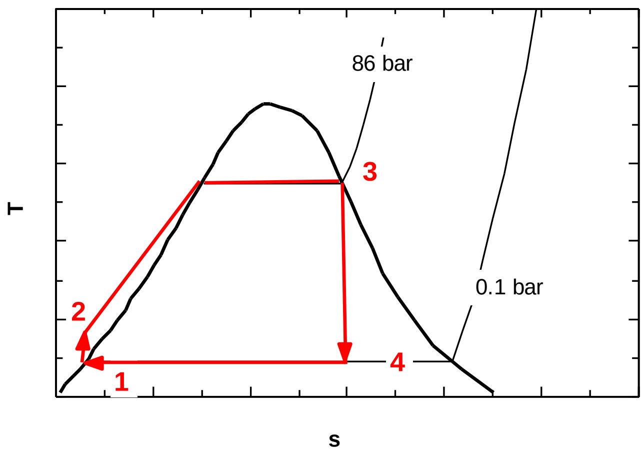
Flow through the heat exchanger inside RPV:

$\dot{m}$ = (10^6 kW)/ $f\, w_T - f\, w_F - w_R$ =(10^6 kW)/0.1* 850.788 - 0.1* 8.5 – 8.9195=13278.6 kg/s

Flow through the turbine:

$\dot{m} f$ = 13278.6 * 0.1 = 1327.86 kg/s

(d)



$\eta = (w_T - w_F)/(h_3 - h_2)$

State 3 and 4 are identical to States 5 and 6 in the previous cycle:

$w_T = 850.788$ kJ/kg

$h_3 = h_g = 2749.4$ kJ/kg

State 1 are also the same: p=0.1 bar $h_1 = h_f = 191.8$

$h_2 = h_1 + w_F/\eta_p = 191.8 + 8.500/0.85 = 201.8$

$\eta = (w_T - w_F)/(h_3 - h_2) = (850.788 - 8.500/0.85)/(2749.4 - 201.8) = 33.0\%$

The efficiency is higher because of the less pumping work.

In the flashing drum case, significantly higher pumping power as a proportion of the turbine work is required.

The efficiency can be improved if the flashing drum can be designed to flash a larger proportion of water into steam.

The reduction in the efficiency is somewhat mitigated by the fact that the average temperature of heat addition in the case of FD cycle is higher than in a standard PWR steam cycle without heat regeneration.

The difference in efficiencies become smaller if feed water heaters are employed.

(e)

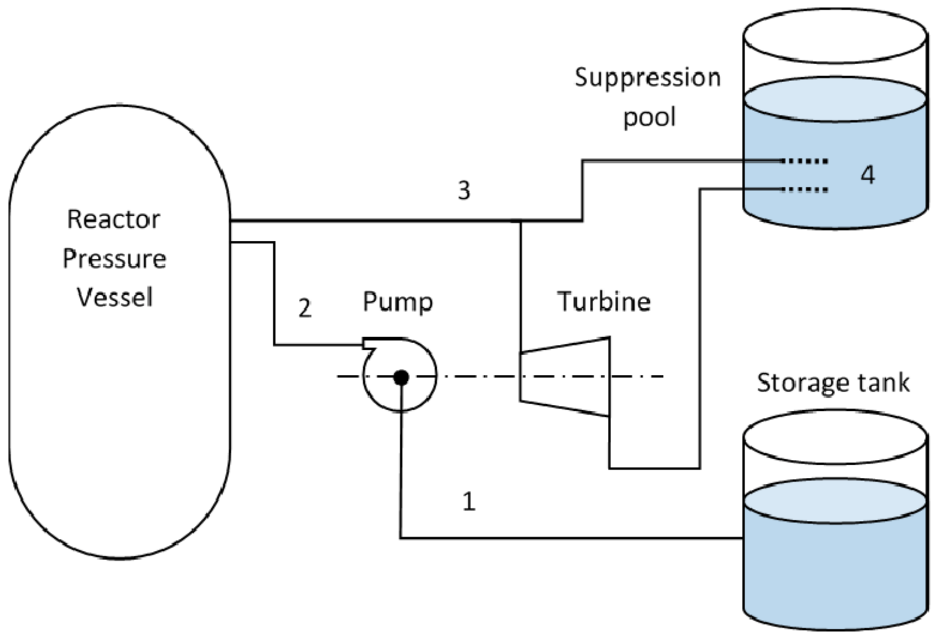
Integrating all primary system components into the RPV allows the elimination of the high pressure piping thus virtually eliminating the possibility of a large break LOCA.

Problem 4

(a)

- BWRs operate at lower pressure than PWRs making high pressure injection task easier.
- BWRs have many small penetrations in the reactor pressure vessel below the core level (e.g. recirculation pumps piping). If one of these is the sources of the leak, the core reflood cannot be done through water injection into the downcomer but by means of above core water sprays.
- There is no uncertainty in the location of the steam bubble in case of a small break LOCA.

(b).(i)



(b).(ii)

Power to be removed 1 hour after shutdown following infinite prior operation:

$P = 0.0622\ P_0\ t^{-0.2} = 0.0622 * 2000\ (3600)^{-0.2} = 24.18$ MW

Work of the coolant injection pump (ideal): $w_p \approx v\ (p_2 - p_1) = 0.001\ (7*10^6 - 1*10^5) = 6.920$ kJ/kg

State 1	p= 1 bar	T = 25 C	h_1 =104.9 kJ/kg
State 2	p=70 bar		h_2 =104.9+6.9=111.8 kJ/kg

Injected water flow rate to remove the required power:

$\dot{m}_p = P/(h_3 - h_2) = (24.18*10^3)/(2772.6 - 111.8) = 9.1$ kg/s

State 3	p=70 bar	T=285.8 C	h_3 = 2772.6 kJ/kg	s = 5.815 kJ/kg/K

Power of the coolant inject. pump (real): $P_p = \dot{m}(h_2 - h_1)/\eta = (9.1\ (111.8 - 104.9))/0.80 = 78.38$ kW

Pressure at the turbine exhaust is atmospheric plus hydrostatic pressure of 5 m water in the suppression tank.

State 4 $\quad p = 1 + \rho g h \approx 1 + 0.5 = 1.5$ bar

Relevant data from tables:

State 4 1.5 bar $h_f = 467.1$ kJ/kg $h_g=2693.1$ kJ/kg
$s_f = 1.434$ kJ/kg/K $s_g=7.223$ kJ/kg/K

steam quality after expansion in turbine (ideal)

$x = (s - s_f)/(s_g - s_f)=(5.815 - 1.434)/(7.223 - 1.434)=0.757$

$h_4 = (h_g - h_f)x + h_f = (2693.1 - 467.1)*0.757 + 467.1 = 2152.18$ kJ/kg

$w_T = h_3 - h_4 = 2772.6 - 2152.18 = 620$ kJ/kg

Real turbine $w_T*\eta = 620 * 0.8 = 496$ kJ/kg;

real $h_4 = 2772.6 - 496 = 2276.6$ kJ/kg

Steam flow rate $\dot{m}_s= P_T/(h_3 - h_4)= P_P/(h_3 - h_4)= 78.38/496= 0.158$ kg/s

(b).(iii)

Total energy to be removed from the core between 1 and 25 hours:

$E_C =\int_{3600}^{3600*25} 0.0622 * 2000 * t^{-0.2}dt = 1.32047*10^6$ MJ

Mass of water needed to remove this energy:

$m = \int_{3600}^{3600*25} \dot{m}_p(t)dt = \int_{3600}^{3600*25} \frac{P(t)}{h_3-h_2}dt = (1.32047*10^9)/(2772.6 - 111.8)= 496.3*10^3$ kg

$V \approx 496.3$ m^3

(b).(iv)

Assuming perfect fluid mixing in the suppression pool and neglecting the water height change during steam discharge, heat balance for the pool at saturation:

$m_{steam\ @70bar}\, h_{steam\ @70bar} + m_{steam\ @1.5bar}\, h_{steam\ @1.5bar} + m_{water\ @25C}\, h_{water\ @25\ C} =$
$= (m_{steam\ @70bar} + m_{steam\ @1.5bar} + m_{water\ @25C})\, h_{sat\ water\ @1\ bar}$

Total energy needed to drive the pump/turbine:

$E_T = \int_{3600}^{3600*25} \frac{\dot{m}_p(t)*(h_2-h_1)}{\eta}\, dt = \int_{3600}^{3600*25} \frac{P(t)*(h_2-h_1)}{\eta*(h_3-h_2)}dt =$
$(1.32047*10^6*(111.8-104.9))/(0.8*(2772.6-111.8)) = 4280.31$ MJ

Mass of steam needed to drive the turbine/pump:

$m_s = E_T/(h_3-h_4) = (4280.31*10^3)/496= 8629.66$ kg

Mass of water in the suppression pool:

$m_{water\ @25C} = (m_{steam\ @70bar}(h_{steam\ @70bar} - h_{sat\ water\ @1\ bar}) + m_{steam\ @1.5bar}\,(h_{steam\ @1.5bar} - h_{sat\ water\ @1\ bar}))/(h_{sat\ water\ @1\ bar} - h_{water\ @25\ C}) =$
$= (496.3*10^3\,(2772.6 - 417.5) + 8629.66\,(2276.6 - 417.5))/(417.5 - 104.9) = 3.7904*10^6$ kg

$V \approx 3790$ m^3