1. (a)

The total length of all pins is limited by the maximum fuel temperature which is a function of linear power. Given the coolant temperature and power distribution are uniform and there is no temperature drop across the cladding and gap, the maximum fuel temperature is given by:

$$T_{CL} = T_{coolant} + \frac{q'}{4\pi k}$$

Therefore, linear power is:

$$q' = 4\pi k\ (T_{CL} - T_{coolant}) = 4\pi\ \times 3\ \times (1500 - 600) = 33929 \frac{W}{m}$$

The total length of all fuel pins:

$$L = \frac{Q}{q'} = \frac{100 \times 10^6}{33929} = 2947\ m$$

The core volume expressed through the pin pitch $p$, single pin length $l$ and total number of pins $N$:

$$V = N \times l \times p^2 = L \times p^2$$

$$V = 2947 \times (1.5 \times 10^{-2})^2 = 0.663075\ m^3$$

$$l \approx \sqrt[3]{V} = \sqrt[3]{0.663075} = 0.872\ m$$

$$N = \frac{L}{l} = \frac{2947}{0.872} \approx 3380\ pins$$

These many pins are needed if the fuel temperature is the most constraining factor.

Now, we determine the total heat transfer area required based on the MDNBR.

The nominal heat flux should be below CHF by a margin given by MDNBR.

$$q'' = \frac{q_{CHF}}{MDNBR} = \frac{1.5 \times 10^6}{1.7} = 882{,}353 \frac{W}{m^2}$$

Total pin surface area:

$$S = \frac{Q}{q''} = \frac{100 \times 10^6}{882353} = 113.333\ m^2$$

The pin radius is then:

$$S = L \times 2\pi R; \qquad R = \frac{S}{2\pi L} = \frac{113.333}{2\pi \times 2947} = 0.00612\ m = 0.612\ cm$$

This is smaller than half the lattice pitch and, therefore, should be geometrically possible.

(b)

Mass of uranium needed as a result of the discharge burnup limit:

$$M = \frac{Power \times time}{Burnup} = \frac{100MW \times 10\,y \times 365\frac{d}{y}}{50\frac{MWd}{kg}} = 7300\,kg$$

Now, let us check if there is enough volume in the fuel design (a) to contain the needed uranium mass in order to meet the burnup constraint.

Density of HM:

$$\rho_{HM} \approx \rho_{UO_2} \times \frac{M_{238}}{M_{238} + 2 \times M_O} = 10.4 \times \frac{238}{238 + 2 \times 16} = 9.167\frac{g}{cm^3} = 9167\frac{kg}{m^3}$$

Minimum volume of fuel needed to meet the burnup constraint:

$$V_{burnup} = \frac{M}{\rho_{HM}} = \frac{7300}{9167} = 0.796335\,m^3$$

Minimum volume of fuel needed to meet the thermal constraints (fuel temperature and MDNBR):

$$V_{th} = \pi L R^2 = \pi \times 2947 \times 0.00612^2 = 0.346763\,m^3 < V_{burnup}$$

The burnup constraint is more limiting, pushing the design to use more pins and/or larger diameter pins. This will result in additional thermal margin; either in the form of lower fuel temperature or lower heat flux or both.

(c)

Ways of increasing reactivity:

- Increase enrichment: non-proliferation limit of 20%, higher cost of uranium and enrichment
- Increase self-shielding (larger diameter pins): lower heat flux, change in H/HM may reduce or cancel out the effect, less flow area, thus higher pressure drop.
- Change H/HM towards optimum: potential for positive MTC.
- Change core shape to cylinder (or a sphere), reduce leakage.
- Add reflectors, reduce leakage, will reduce shielding requirements and radiation damage to surrounding structures.

The initial reactivity should be chosen such that the core would remain critical after incurring all of the following reactivity decrements:

- core heat up to full power operating conditions
- buildup of xenon and samarium
- buildup of other fission products and depletion of fissile nuclides over the core life

This reactivity needs to be managed by a combination of control rods, soluble boron and burnable poisons. If all these measures are not able to ensure subcriticality of the core at any operating conditions during the core life, the chosen initial reactivity is unfeasible and would need to be reduced, sacrificing the core life, or operational flexibility or efficiency (lower operating temperatures and power).

2. (a)

From CUED Thermofluids Databook p.20, latent heat of water evaporation at 1 bar:

$$h_{fg} = 2257.4\ \frac{kJ}{kg}$$

Amount of energy released during 72 hours of decay assuming conservatively infinite core operation prior to shutdown:

$$P(t) \approx P_0\ \times 0.066\, t^{-0.2}$$

$$E = \int_0^{t_s} P(t)dt = P_0\ \times 0.066 \int_0^{t_s} t^{-0.2}\, dt = P_0\ \times 0.066 \times 1.25 \times t_s^{0.8}$$

$$E = 200 \times 10^6 \times 0.066 \times 1.25 \times (72 \times 3600)^{0.8} = 3.535 \times 10^{11}\, J = 3.535 \times 10^8\ kJ$$

Mass of water needed:

$$m = \frac{E}{h_{fg}} = \frac{3.535 \times 10^8\ kJ}{2257.4\ \frac{kJ}{kg}} = 156596\ kg$$

(b)

The gravity pressure loss should compensate all other losses. Pressure loss in the HX and the core are given. Acceleration and shock losses are negligible. Therefore:

$$(\rho_{cold} - \rho_{hot})gZ = \Delta p_{core} + \Delta p_{HX} + \Delta p_{f_{pipes}}$$

The minimum possible flow rate to avoid boiling is determined by the heat balance. Noting that the primary coolant inlet temperature is approximately equal to the secondary water temperature in the tank and the saturation temperature at 100 bar is 311 °C (CUED TF Databook p.22) and the primary water $\overline{c_p} =\ 5200\, J/kg/K$

$$Q = \dot{m}c_p(T_H - T_C); \quad \dot{m} = \frac{Q}{\overline{c_p}(T_H - T_C)} = \frac{10 \times 10^6}{5200\ (311 - 100)} = 9.11\frac{kg}{s}$$

Neglecting the change in properties of water going up and down the loop for the purpose of friction loss calculation in the vertical pipes, using the friction factor correlation for smooth pipes (from the NE Databook, p.14) and saturated water density at 100 bar ($\rho = 1/0.001452 = 688.71\ kg\ m^{-3}$):

$$f\ = 0.184(\mathrm{Re})^{-0.2} = 0.184\left(\frac{\rho v D}{\mu}\right)^{-0.2}$$

$$v\ = \frac{\dot{m}}{\rho A} = \frac{4\dot{m}}{\rho \pi D^2}$$

Noting that $\rho v\ = \frac{\dot{m}}{A}$

$$f\ = 0.184\left(\frac{\dot{m}D}{A\mu}\right)^{-0.2} = 0.184\left(\frac{4\dot{m}}{\pi D\mu}\right)^{-0.2}$$

$$\Delta p_{f_{pipe_{hot}}} \approx \frac{\rho v^2}{2} f \frac{Z}{D} = \frac{1}{2\rho}\left(\frac{4\dot{m}}{\pi D^2}\right)^2\ 0.184\left(\frac{4\dot{m}}{\pi D\mu}\right)^{-0.2} \frac{Z}{D}$$

$$\Delta p_{f_{pipe_{hot}}} \approx \frac{1}{2 \times 688.71}\left(\frac{4 \times 9.11}{\pi \times 0.08^2}\right)^2 \; 0.184\left(\frac{4 \times 9.11}{\pi \times 0.08 \times 0.8 \times 10^{-4}}\right)^{-0.2} \frac{Z}{0.08} = 307.3\, Z$$

Approximating the cold leg water density by saturated density at 100 °C ($\rho = 1/0.001043 = 958.77\ kg\ m^{-3}$; this introduces only 0.3% error)

$$\Delta p_{f_{pipe_{cold}}} \approx \frac{1}{2 \times 958.77}\left(\frac{4 \times 9.11}{\pi \times 0.08^2}\right)^2 \; 0.184\left(\frac{4 \times 9.11}{\pi \times 0.08 \times 2.8 \times 10^{-4}}\right)^{-0.2} \frac{Z}{0.08} = 283.6\, Z$$

Substituting all the values into the first equation, we obtain the height difference:

$$(\rho_{cold} - \rho_{hot})gZ = \Delta p_{core} + \Delta p_{HX} + \Delta p_{f_{pipe_{hot}}} + \Delta p_{f_{pipe_{cold}}}$$

$$(958.77 - 688.71)\ \times 9.81\, Z = 3000 + 3000 + 307.3\, Z + 283.6\, Z$$

$$2649.3\, Z = 6000 + 590.9\, Z$$

$$Z \approx 2.91\, m$$

(c)

Examples of natural convection

- Decay heat removal through steam generators in the case of main pump failure. As long as feed water is supplied, the primary coolant will circulate between the core and the steam generators naturally. In new VVERs, steam generators can be emptied and air cooled.
- Water circulation between the core and IRWST in the AP1000
- The isolation condenser in the ESBWR
- Air flow between the shield building and containment vessel in the AP1000
- Passive reactor vessel cavity cooling for in-vessel molten core retention in the AP1000

3. (a)

The hydraulic diameter $\left(D = \frac{4A}{P}\right)$ did not change since both the wetted perimeter and flow area have doubled.

The coolant mass flux $(G = \rho\, v)$, however, is halved, since the flow area has doubled but the total mass flow rate remained the same. As a result Re number is also halved.

Pumping power is given by $W_p = \Delta p\, A\, v = \Delta p\, \frac{\dot{m}}{\rho}$, where $A$ is the flow area, $v$ – flow velocity and $\rho$ – coolant density.

Assume all coolant properties $\left(\rho, c_p, \mu\right)$ remain the same, because the core power, flow rate and, thus, the temperature rise across the core remain the same. We also assume that frictional pressure losses dominate. Therefore, $\Delta p_{core} \approx \Delta p_{friction}$ and $W_p \sim \Delta p_{friction}$

In the nominal case: $\Delta p_{friction1} = \frac{\rho v^2}{2}\, f\, \frac{L}{D} = \frac{\rho v^2}{2}\, 0.184(\mathrm{Re})^{-0.2}\, \frac{L}{D}$

In the modified case: $\Delta p_{friction2} = \frac{\rho\left(\frac{v}{2}\right)^2}{2}\, 0.184\left(\frac{\mathrm{Re}}{2}\right)^{-0.2}\, \frac{L}{2D}$

$$\frac{W_{p1}}{W_{p2}} = \frac{\Delta p_{friction1}}{\Delta p_{friction2}} = \frac{v^2}{v^2/4}\,\frac{v^{-0.2}}{(v/2)^{-0.2}}\,\frac{L}{L/2} = 6.96$$

(b)

Reducing the height while increasing core diameter will increase the core surface area, while preserving the volume. This will result in a net increase in neutron loss through leakage. This will make the core reactivity more sensitive to leakage.

$$k = \frac{Neutron\ production\ rate}{Absorption\ rate + Leakage\ rate}$$

Upon coolant heat-up and the corresponding reduction in coolant density, the neutron spectrum in the core will harden. Since most reaction cross-sections generally decrease with neutron energy, the core will become more neutron-transparent, increasing the neutron leakage as a result. A negative contribution to the coolant temperature coefficient like this is enhanced if the leakage contribution to the neutron balance is larger.

The obvious disadvantage of this strategy is the overall loss of reactivity due to higher leakage, which will result in a shorter fuel cycle length and will need to be compensated by higher enrichment.

Also, the reactor vessel diameter may need to be larger, leading to higher manufacturing and/or transportation costs. As evident from the results obtained in (a), these disadvantages will be at least partially compensated by substantially lower pumping power needs.

(c)

- In thermal reactors (e.g. LWRs), choosing an appropriate H/HM can bring MTC to the desired negative values range.
- Reducing coolant absorption (e.g. reducing reliance on soluble boron for reactivity control).
- Choosing appropriate burnable poisons which will increase absorption with spectrum hardening.
- Choosing reflector materials which will reflect neutrons with lower reactivity worth back into the core.
- Enhancing neutron leakage by introducing streaming channels or internal fertile blankets axially or radially.

4 (a)

The linear heat generation rate at the location of the maximum fuel temperature:

$$q' = q''' \times P^2 \times F = 100 \times 1.25^2 \times 2.5 = 390.6\ \frac{W}{cm} = 39.06\ \frac{kW}{m}$$

where $P$ is the lattice pitch and $F$ is the power peaking factor.

The heat flux through the pin surface at this location:

$$q'' = \frac{q''' \times P^2 \times F}{2\pi R} = \frac{q'}{2\pi R} = \frac{390.6}{2\pi \times 0.48} = 129.5 \frac{W}{cm^2}$$

where $R$ is the pin radius.

Therefore, the temperature drop at the fuel-coolant interface can be obtained from:

$$q'' = h\Delta T \quad \text{or} \quad \Delta T = \frac{q''}{h} = \frac{129.5}{0.5} = 259\ ℃$$

The temperature drop in the fuel can be obtained by solving the heat conduction equation in cylindrical coordinates with constant thermal conductivity and heat source. The result can also be obtained by inspecting the maximum fuel temperature equation on p.13 of the Nuclear Data Book, noting that the term $\frac{q'}{4\pi k}$ corresponds to the temperature difference between the centre and the surface of the fuel.

Given that the coolant temperature is known, adding the temperature differences in the fuel and at the fuel-coolant interface, we can write:

$$T_{CL} = T_{coolant} + \frac{q''}{h} + \frac{q'}{4\pi k} = 300 + 259 + \frac{390.6}{4\pi \times 0.03} = 1595\ ℃$$

(b)

Assume that heat transfer to the coolant is much slower than the rate of heat generation increase during the transient. That is, all the extra energy generated within the fuel is nearly adiabatically deposited into the fuel, heating it up. This is a conservative assumption because some heat transfer to the coolant still occurs, leading to somewhat lower temperatures.

The core power distribution did not change, so the hot spot location remains the same. Also, assume that during the transient, power is deposited uniformly throughout the pellet.

The energy balance for the hot pellet analysed above:

$$\rho_{fuel} c_{p_{fuel}} \left(T_{fuel_{final}} - T_{fuel_{initial}}\right) = E = F \times \int_{0}^{200ms} \Delta q'''(t)dt$$

The final fuel temperature would be:

$$T_{fuel_{final}} = T_{fuel_{initial}} + \frac{F \times \int_0^{200ms} \Delta q'''(t)dt}{\rho_{fuel} c_{p_{fuel}}}$$

Let us calculate first the extra energy (in excess of what is being generated at steady state). The energy is the area under the curve – a triangle with a basis of 200ms and a height of:

$$\Delta q'''_{max} = (300 - 100) \times 2.5 = 500 \frac{W}{cm^3} \ of \ core$$

This is the average power density in the location of interest which must be converted into power density in the fuel:

$$\Delta q'''_{\max\_fuel} = 500 \left[\frac{W}{cm^3} \ of \ core\right] \times \frac{V_{cell}}{V_{fuel}} = 500 \times \frac{P^2}{\pi R^2} = 500 \times \frac{1.25^2}{\pi \times 0.48^2} =$$
$$= 1079 \left[\frac{W}{cm^3} \ of \ fuel\right]$$

Then, the peak fuel temperature after the transient:

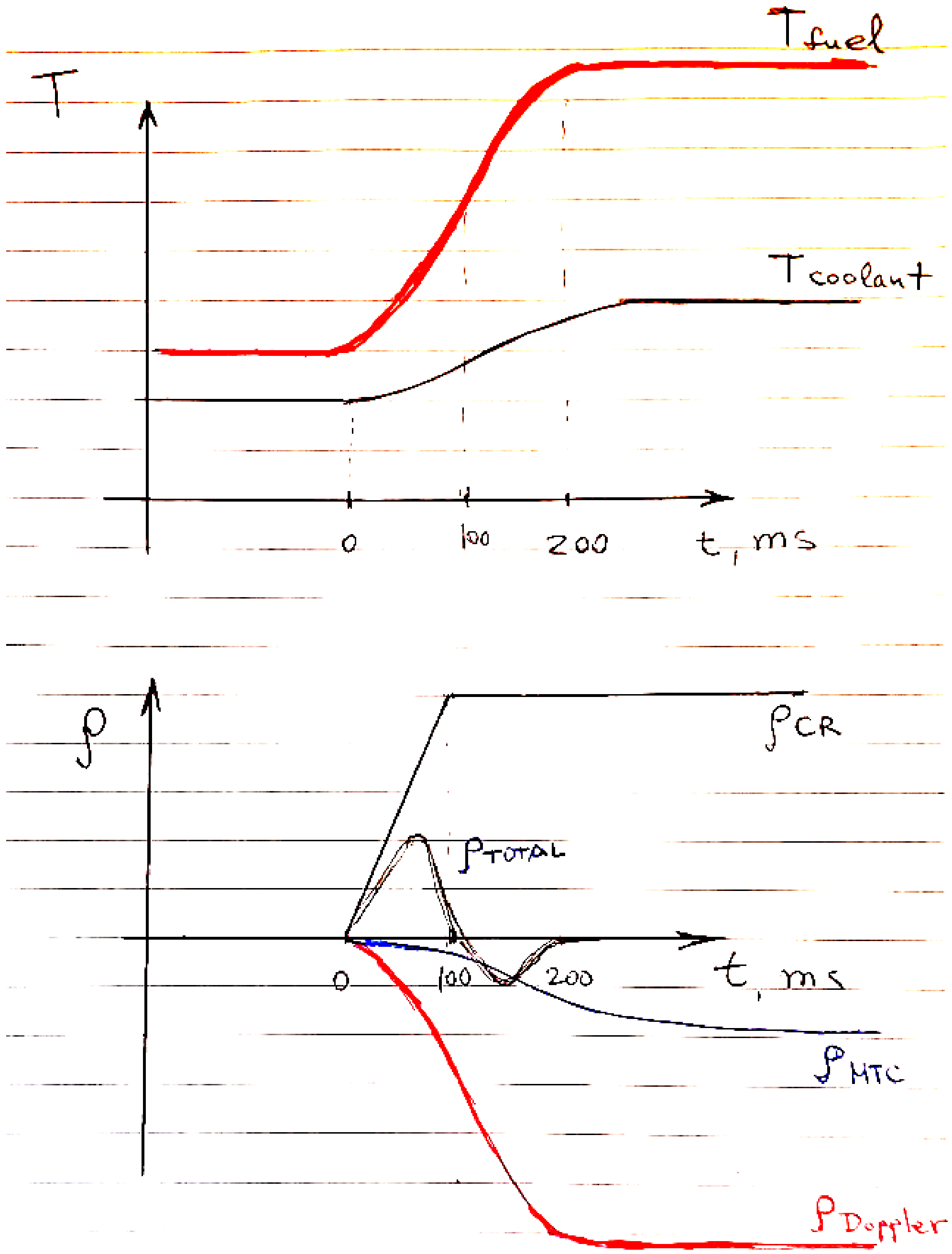
$$T_{fuel_{final}} = 1595 + \frac{\frac{1}{2} \times 1079 \times 10^6 \ W/m^3 \times 0.2s}{10000 \ kg/m^3 \ \times 350 \ J/kg/K} = 1626 \ ℃$$

A relatively modest increase in temperature with a resulting value well below the melting point.

(c)

Key features to note:

- The ejected control rod inserts reactivity rapidly and permanently – a nearly step function.
- This results in a power increase and a fuel temperature increase with a slight delay due to neutronic (finite neutron life-time and delayed neutrons) and thermal (fuel heat capacity) inertia.
- The rising fuel temperature invokes a negative reactivity contribution due to the Doppler Coefficient (DC).
- With a further delay due to the heat transfer time constant (finite heat diffusivity) to the coolant, the coolant temperature also begins to rise.
- The coolant temperature rise is smaller but MTC is typically higher. Therefore, negative MTC and DC contributions can be comparable.
- At some point, the Doppler and coolant temperature reactivity contributions fully compensate the external reactivity and the power increase is stopped.
- Further fuel and coolant temperature rises introduce negative net reactivity resulting in a drop in power.
- By the end of the transient, the fuel and coolant temperatures stabilise at new higher values. Negative Doppler and MTC reactivity contributions fully compensate the external reactivity. The core reactivity stabilises at zero and power stabilises at the initial level.



T
T fuel
T coolant
0
100
200
t, ms
ρ
ρ CR
ρ TOTAL
0
100
200
t, ms
ρ MTC
ρ Doppler