4A3 2019 nra27

Q1) (a) $\psi = \frac{\Delta h_0}{u^2}$ , $\frac{p_{03}}{p_{01}} = \left\{ \frac{T_{03}}{T_{01}} \right\}^{\frac{\eta_{pc}\gamma}{\gamma-1}}$

$$T_{03} = T_{01} + \frac{\Delta h_0}{c_p} = T_{01} + \frac{\psi u^2}{c_p} \Rightarrow \frac{p_{03}}{p_{01}} = \left[ 1 + \frac{\psi u^2}{c_p T_{01}} \right]^{\frac{\eta_{pc}\gamma}{\gamma-1}}$$

(b) High stage pressure ratio needs high $\eta_{pc}$, but this is a result of the other choices which can be made by the designer.

- high $\psi$: high stage loading requires lots of turning, which is limited by diffusion. Careful blade design can be used to keep the boundary layers healthy, but this is limited to a diffusion factor of ~0.6 in practice.

- high $U$: high blade speed gives higher pressure ratio per stage. Once the inlet relative Mach number rises above ~0.7 there will be supersonic flow on the early suction surface. The subsequent shock-boundary layer interactions downstream will likely separate the flow unless the passage is specifically designed to accomodate the shock.

This can be achieved with thin, very low camber blades, as the shocks themselves provide quite efficient compression.

(c) $\bar{u} = M_{b,0}\sqrt{\gamma R T_0}$ $\quad PR = \left[1 + \frac{\psi \bar{u}^2}{c_p T_{01}}\right]^{\frac{\eta_{pc}\gamma}{\gamma-1}} = \left[1 + \psi M_{b,0}^2(\gamma - 1)\right]^{\frac{\eta_{pc}\gamma}{\gamma-1}}$

Subsonic:

$M_{b,0} = 0.6$, $\eta_{pc} = 0.9$

$PR = 1.17$ $\quad n_{stage} = \log(1.45)/\log(1.17)$

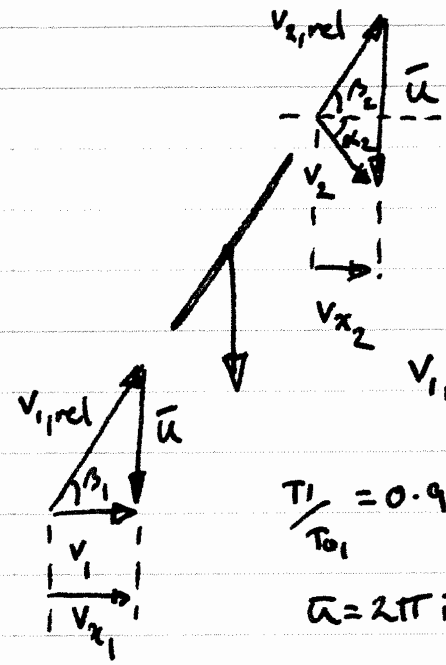
$= 2.4 \Rightarrow$ 3 stages

Transonic:

$M_{b,0} = 1.1$, $\eta_{pc} = 0.85$

$PR = 1.59 \Rightarrow$ 1 stage

d)




$\beta_1 = \beta_2$ $\quad N = 6000\,\text{rpm}$

$M_1 = 0.6$

$\bar{r} = 0.567\,\text{m}$

$T_{01} = 300\,\text{K}$

$V_{1,rel} = \sqrt{\bar{u}^2 + V_1^2} \Rightarrow M_{1,rel} = \sqrt{\left\{\frac{\bar{u}^2}{\gamma R T_1} + M_1^2\right\}}$

$T_1/T_{01} = 0.9328 \Rightarrow T_1 = 300 \times 0.9328 = 279.84\,\text{K}$

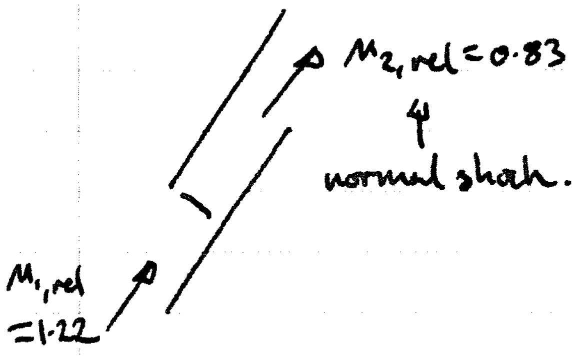
$\bar{u} = 2\pi \bar{r} \Omega = 2 \times \pi \times 0.567 \times 8000/60 = 356.26\,\text{m/s}$

(i) $\Rightarrow M_{1,rel} = \sqrt{\left\{\frac{356.26^2}{1.4 \times 287.1 \times 279.84} + 0.6^2\right\}} = 1.22$

ii) $V_{x_1} = V_1$ $\quad \frac{V_1}{\sqrt{c_p T_{01}}} = 0.3665 \Rightarrow V_1 = V_{x_1} = 0.3665\sqrt{1005 \times 300}$

$= 201.24\,\text{m/s}$

$\beta_2 = \beta_1 = -\tan^{-1}\left\{\frac{\bar{u}}{V_1}\right\} = -\cos^{-1}\left\{\frac{M_1}{M_{1,rel}}\right\} = -60.54^\circ$




$$T_{01,rel} = T_1\left\{1 + \frac{\gamma-1}{2} M_{1,rel}^2\right\} = 363.14\text{K}$$

$$\Rightarrow T_{02,rel} = 363.14\text{K}$$

$$T_2 = T_{02,rel} \Big/ \left\{1 + \frac{\gamma-1}{2} M_{2,rel}^2\right\} = 319.17\text{K}$$

$$V_{2,rel} = M_{2,rel}\sqrt{\gamma R T_2}$$

$$= 0.83\sqrt{1.4 \times 287.1 \times 319.17}$$

$$= 297.3 \text{ m/s}$$

$$V_{x_2} = V_{2,rel}\cos\beta_2 = 297.3\cos(60.54)$$

$$= 146.2 \text{ m/s}$$

$$\phi_1 = \frac{V_{x_1}}{\bar{u}} = \frac{201.24}{356.26} = 0.565$$

$$\phi_2 = \frac{V_{x_2}}{\bar{u}} = \frac{146.2}{356.26} = 0.4104$$

$$\bar{u} = V_{x_2}\tan(-\beta_2) + V_{x_2}\tan\alpha_2 \Rightarrow \frac{1}{\phi_2} = \tan(-\beta_2) + \tan\alpha_2$$

$$\alpha_2 = \tan^{-1}\left\{\frac{1}{\phi_2} - \tan(-\beta_2)\right\}$$

$$= \tan^{-1}\left\{\frac{1}{0.4104} - \tan(60.54)\right\}$$

$$= 33.68^\circ$$

$$\psi = \frac{\Delta h_0}{\bar{u}^2} = \frac{\cancel{\bar{u}}\Delta V_\theta}{\bar{u}^{\cancel{2}}} = \frac{V_{\theta_2} - \cancelto{0}{V_{\theta_1}}}{\bar{u}} = V_{x_2}\tan\alpha_2$$

$$= 146.2 \times \tan(33.68^\circ)$$

$$= 0.274$$

(iii) $$Y_p = \frac{p_{01,rel} - p_{02,rel}}{p_{01,rel} - p_1} = \frac{1 - p_{02,rel}/p_{01,rel}}{1 - p_1/p_{01,rel}}$$

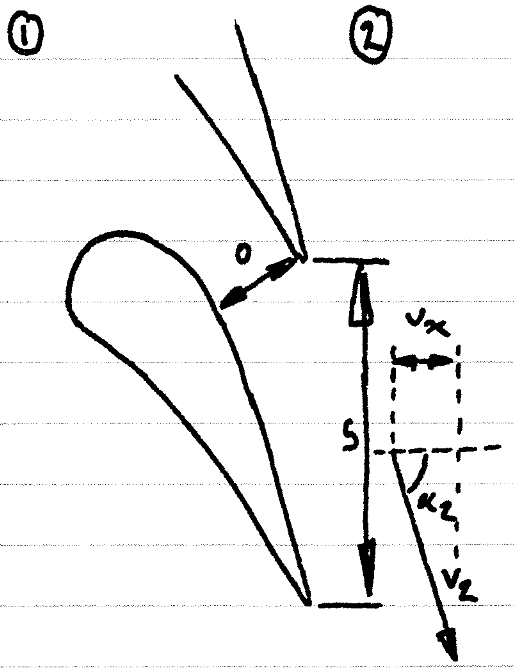
$p_{02,rel}$ is downstream of shock @ $M=0.83 \Rightarrow \frac{p_{02,rel}}{p_{01,rel}} = \frac{p_{0s}}{p_0} = 0.9907$

$p_{01,rel}$ is upstream of shock @ $M=1.22 \Rightarrow p_1/p_{01,rel} = 0.4017$

$$\Rightarrow Y_p = \frac{1-0.9907}{1-0.4017} = 0.0155$$ //

**Q1 Transonic compressor mean-line analysis:** 28/30 attempts, mean 63.7%, st. dev. 15.2%
Part (a) was an easy lead-in designed to prepare the ground for part (b) and (c) and was answered correctly by all candidates. Part (b) required the candidates to spot that blade loading (i.e. angles) *and* blade speed can be used by the designer to achieve high stage pressure ratio. This was less well answered, with candidates confusing the things that the designer can chose (i.e. blade angles and speed) as opposed to boundary conditions, that are generally out of the control of the designer, or the *result* of design choices like polytropic efficiency. Many candidates confused the *choice* of blade speed at design and a *change* in speed during operation of an actual machine. Many were also unduly worried about mechanical stresses. Very few spotted that thin, low-camber blades, as analysed in part (d) were the way forward. Part (c) used the expression derived in part (a) and was well answered, the only problems were numerical slips. Part (d) involved the mean-line analysis of a transonic compressor stage. Subsection (d)(i) was very well answered by almost all candidates. Subsection (d)(ii) was a fairly standard velocity triangle calculation. This was quite poorly answered, but in almost all cases the errors were simply due to poor diagrams and numerical slips. Subsection (d)(iii) was well answered by most candidates. The most common error for those who knew how to calculate the stagnation pressure loss coefficient was to use the inlet absolute, rather than the relative frame conditions calculated/given in subsection (d)(i).

Q 2)




$p_{01} = 8$ bar

$T_{01} = 450$ K

$N = 8000$ rpm

$o/s = 0.31$

$Y_p = 0.03$

$\eta_{is} = 0.85$

$\bar{r} = 0.3$ m

- h const. from throat to inlet.
- $V_x$ constant.
- all loss downstream of vane throat.

| | $\dot{m}\sqrt{c_pT_0}/Ap_0$ | $V/\sqrt{c_pT_0}$ | $p/p_0$ |
|---|---|---|---|
| $M^* = 1.0$ | 1.2810 | – | – |
| $M_2 = 0.95$ | 1.2783 | 0.5530 | 0.5595 |

(a) (i) $$Y_p = \frac{p_{01} - p_{02}}{p_{01} - p_2} = \frac{p_{01}/p_{02} - 1}{p_{01}/p_{02} - p_2/p_{02}} \Rightarrow \frac{p_{01}}{p_{02}} = \frac{1 - Y_p\, p_2/p_{02}}{1 - Y_p} = \frac{1 - 0.03 \times 0.5595}{1 - 0.03}$$

$$= 1.0136$$

Stator, so $T_0^* = T_{02}$, all loss downstream of throat so $p_0^* = p_{01}$

$$\dot{m} = f\{M=1.0\}\frac{\cancel{\ell}\,o\; p_0^*}{\cancel{\sqrt{c_pT_0^*}}} = f\{M=0.95\}\frac{\cancel{\ell}\,s\cos\alpha_2\; p_{02}}{\cancel{\sqrt{c_pT_{02}}}}$$

$$\cos\alpha_2 = \frac{o}{s}\,\frac{p_{01}}{p_{02}}\,\frac{f\{M=1.0\}}{f\{M=0.95\}} = 0.31 \times 1.0136 \times \frac{1.2810}{1.2783}$$

$$= 0.3149$$

$$\Rightarrow \alpha_2 = 71.65^\circ$$
$$= 72^\circ$$ → N.B. use 2dp in following analysis.

$$U = \bar{r}\Omega = 0.3 \times 8000/60 \times 2\pi = 251.33 \text{ m/s}$$

$$V_2 = f\{M_2 = 0.95\}\sqrt{c_pT_{02}} = 0.5530\sqrt{1005 \times 450} = 371.89 \text{ m/s}$$

$$V_x = V_2\cos\alpha_2 = 371.89 \times 0.3149 = 117.1 \text{ m/s}$$

flow coefficent $\phi = V_x/u = 117.1/251.33 = 0.4659 = 0.47$

Error in flow coefficient if isentropic to $M_2 = 0.95$, $\frac{P_{01}}{P_{02s}} = 1$

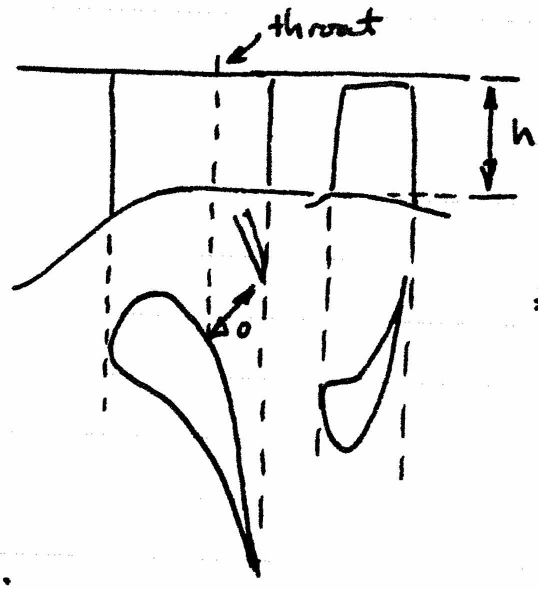
$$\Delta\phi = \frac{\phi - \phi_s}{\phi} = \frac{V_x/u - V_{xs}/u}{V_x/u} = 1 - \frac{V_{xs}}{V_x} = 1 - \frac{V_2\cos\alpha_{2s}}{V_2\cos\alpha_2}$$

$$= 1 - \frac{\cos\alpha_{2s}}{\cos\alpha_2}$$

From earlier $\cos\alpha_2 = \frac{o}{s}\left(\frac{P_{01}}{P_{02}}\right)\frac{f\{M=1.0\}}{f\{M=0.95\}}$

so $\frac{\cos\alpha_{2s}}{\cos\alpha_2} = \frac{1}{1.0136} \Rightarrow \Delta\phi = 1 - \frac{1}{1.0136} = 1.34\%$

(ii)




at throat $\frac{\dot{m}\sqrt{c_p T_0^*}}{h_0\, P_0^*} = f\{M=1.0\}$

$o = s(o/s) = 2\pi r\,(o/s)$

$$\Rightarrow h = \dot{m}\sqrt{c_p T_0} \Big/ \left(2\pi r\, \tfrac{o}{s}\, P_0^*\, f\{M=1.0\}\right)$$

$$h = 30 \times \sqrt{1005 \times 450} \Big/ \left(2\pi \times 0.3 \times 0.31 \times 8\times10^5 \times 1.2810\right)$$

$= 33.7\,\text{mm}$

[OR:

$$h = \dot{m}\sqrt{c_p T_{02}} \Big/ \left(s\cos\alpha_2\, P_{02}\, f\{M=0.95\}\right) = \dot{m}\sqrt{c_p T_{02}} \Big/ \left(2\pi r\cos\alpha_2\, P_{01} \Big/ \left(\tfrac{P_{01}}{P_{02}}\right) f\{M=0.95\}\right)$$

$= 33.7\,\text{mm}$ ]

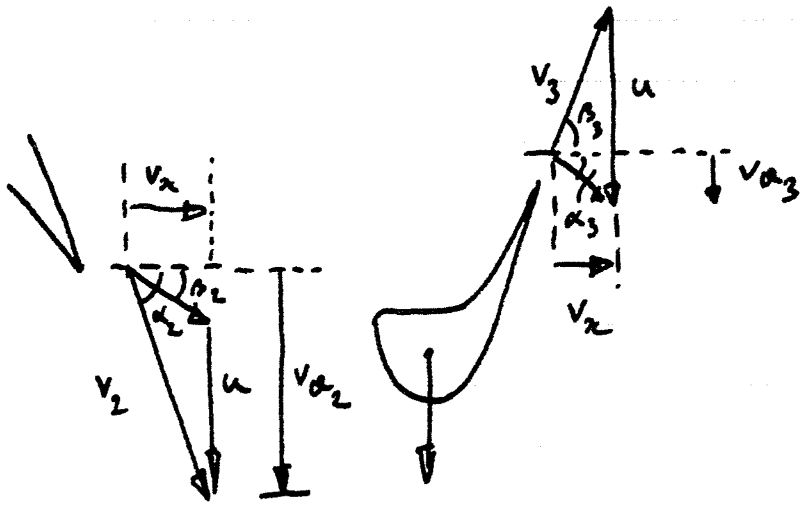
iii) Stage loading $\psi = \frac{\Delta h_0}{u^2} = \frac{u\Delta V_\theta}{u^2} \Rightarrow \Delta V_\theta = u\psi = 251.33 \times 1.2$
$= 301.39$ m/s

$V_{\theta_2} = V_2 \sin\alpha_2 = 352.98$ m/s $\qquad V_{\theta_3} = V_{\theta_2} - \Delta V_\theta = 51.38$ m/s

$$\alpha_3 = \tan^{-1}\left\{\frac{V_{\theta 3}}{V_x}\right\} = \tan^{-1}\left\{\frac{51.38}{117.1}\right\} = 23.69^\circ$$

$$-\beta_3 = \tan^{-1}\left\{\frac{u - V_{\theta 3}}{V_x}\right\} = \tan^{-1}\left\{\frac{251.33 - 51.38}{117.1}\right\} = 59.64^\circ$$

$$\beta_2 = \tan^{-1}\left\{\frac{V_{\theta_2} - u}{V_x}\right\} = \tan^{-1}\left\{\frac{352.98 - 251.33}{117.1}\right\} = 40.96^\circ$$




$$\text{Power} = \dot{m}\Delta h_0 = \dot{m}\psi u^2 = 30 \times 1.2 \times 251.33 = 2.274 \text{ MW}$$

$$\text{Power} = \cancel{\dot{m}} c_p T_{01}\left[1 - \frac{T_{03s}}{T_{01}}\right]\eta_{ts} = \frac{\cancel{\dot{m}}\psi u^2}{c_p T_0 \eta_{is}}$$

$$\Rightarrow \frac{T_{03s}}{T_{01}} = 1 - \frac{\psi u^2}{c_p T_0 \eta_{is}}$$

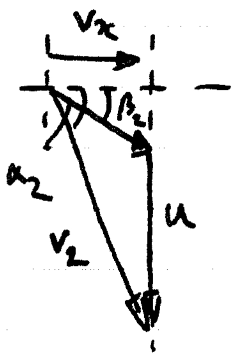
$$\frac{P_{03}}{P_{01}} = \left[1 - \frac{\psi u^2}{c_p T_0 \eta_{is}}\right]^{\frac{\gamma}{\gamma-1}} = \left[1 - \frac{1.2 \times 251.33^2}{1005 \times 450 \times 0.85}\right]^{\frac{1.4}{0.4}}$$

$$= 0.4636$$

(b)(i) new mass flow rate $\dot{m}'$, new temperature $T_0'$ $(T_{0*} = T_{01})$

throat still choked so $\dfrac{\dot{m}'\sqrt{c_p T_0'^*}}{A_0\, p_0^*} = \dfrac{\dot{m}\sqrt{c_p T_0^*}}{A_0\, p_0^*}$

$$\Rightarrow \dot{m}' = \dot{m}\sqrt{\frac{T_{01}}{T_{01}'}} = 30 \times \sqrt{\frac{450}{550}} = 27.14 \text{ kgs}^{-1}$$

(ii) 


keep velocity triangle the same shape $(T_{02} = T_{01})$

$$V_2 = f\{M = 0.95\}\sqrt{c_p T_{02}} \qquad \Rightarrow \qquad V_2' = V_2\sqrt{\frac{T_{02}'}{T_{02}}}$$
$$V_2' = f\{M = 0.95\}\sqrt{c_p T_{02}'}$$

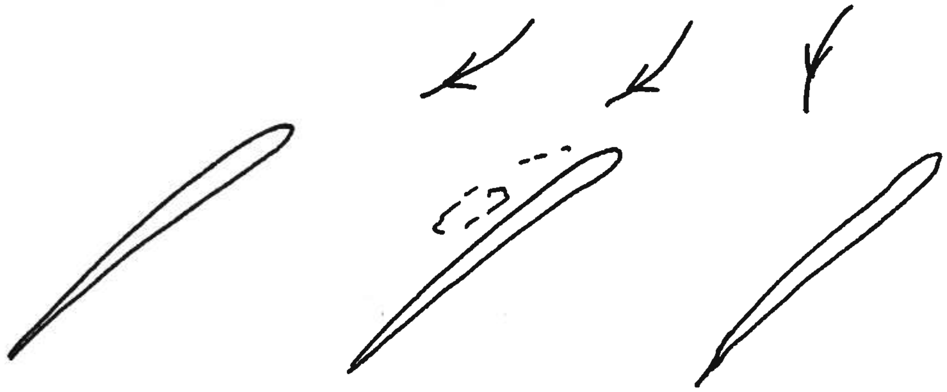
$$V_x = \dot{m}\Big/\rho_2 2\pi \bar{r} h$$
$$V_x' = \dot{m}'\Big/\rho_2' 2\pi \bar{r} h \qquad \Rightarrow \qquad \frac{V_x'}{V_x} = \frac{\dot{m}}{\dot{m}'}\frac{\rho_2'}{\rho_2} = \sqrt{\frac{T_{02}'}{T_{02}}}$$

so we just need to scale the blade speed by $\sqrt{\dfrac{T_{02}'}{T_{02}}}$

$$\Rightarrow \quad N_{new} = N\sqrt{\frac{T_{02}'}{T_{02}}} = 8000\sqrt{\frac{550}{450}} = 8844.3 \text{ rpm}$$

**Q2 Turbine stage:** 23/30 attempts, mean 59.7%, st. dev. 19.6%
Part (a)(i) was a standard compressible flow continuity question. Most candidates made a good attempt. Common mistakes included ignoring the stagnation pressure loss, or ignoring continuity and just using the geometric opening. The second part of this section required the candidates to assess the error in ignoring the stagnation pressure loss. This part was ignored by the majority of candidates, but done perfectly by all who attempted it. Part (a)(ii) was well answered, but many students didn't spot that the overall mean pitch of a machine is just the circumference. Part (a)(iii) was a fairly standard velocity triangle calculation. In order to find the exit conditions, they needed to the use the stage loading coefficient given in the preamble, which was spotted by the majority of candidates. There were many sign errors. Part (b) considered the process of recovering the inlet velocity triangles after a change in inlet temperature. There were far fewer good answers to this part.

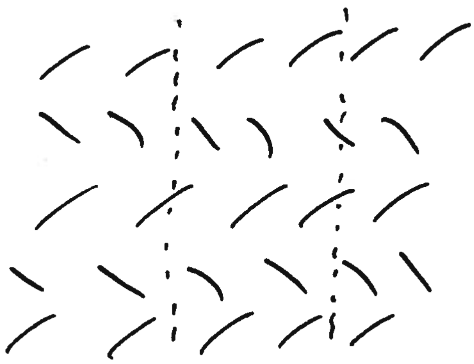
3(a)



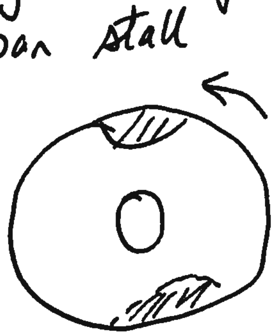
A local flow perturbation causes one blade passage to exhibit separation (or other rise in blockage). Flow is diverted around this blocked passage increasing incidence for blades to the left, which then separate, and reducing it for blades to the right which, if separated, then recover. The pattern grows to cover a number of passages – a cell – which propagates around annulus.

(b)

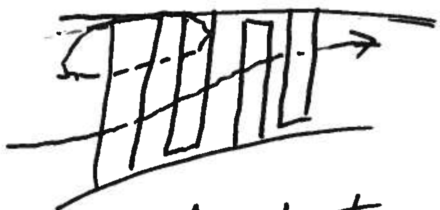
High hub-to-tip ratio likely to be full span stall




cell extends axially through machine



Low hub-to-tip ratio Likely to be part span



Confined to one (or a few) stages

(c) $$\eta_{is} = \frac{T_{02}^{is} - T_{01}}{T_{02} - T_{01}} = \frac{\left(\frac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_{02}}{T_{01}} - 1}$$

$$\Rightarrow \quad \frac{p_{02}}{p_{01}} = \left(1 + \eta_{is}\frac{T_{02} - T_{01}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$$

Also $$\frac{T_{02} - T_{01}}{T_{01}} = \frac{\Delta h_0}{c_p T_{01}} = \frac{\Delta h_0}{U^2}\frac{U^2}{c_p T_{01}}$$

and $$\frac{U^2}{c_p T_{01}} = \frac{U^2}{\gamma R T_{01}}\frac{\gamma R}{c_p} = (\gamma - 1) M_b^2$$

$M_b$ = blade Mach no based on stag sound speed

Hence $$\frac{p_{02}}{p_{01}} = \left[1 + \eta_{is}(\gamma-1) M_b^2 \frac{\Delta h_0}{U^2}\right]^{\frac{\gamma}{\gamma-1}}$$

Typically $\frac{\Delta h_0}{U^2} \leqslant .4 \quad M_b \leqslant .8 \qquad \eta_{is} \simeq 0.9$

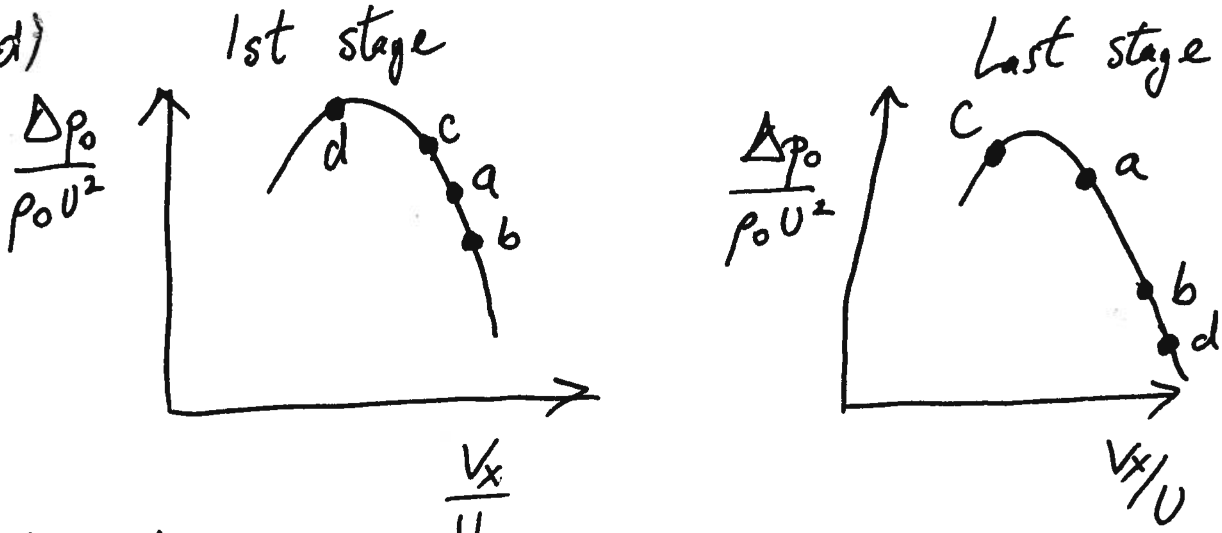
$$\therefore \quad \eta_{is}\frac{(\gamma-1) M_b^2 \Delta h_0}{U^2} \leqslant .4 \times .4 \times .8^2 \times .9 \simeq .09$$

Thus $$\frac{p_{02}}{p_{01}} \simeq 1 + \frac{\gamma}{\gamma-1}\eta_{is}(\gamma-1) M_b^2 \frac{\Delta h_0}{U^2}$$

or $$\frac{p_{02} - p_{01}}{\rho_{01} U^2} = \frac{p_{01}}{\rho_{01} U^2}\frac{\gamma}{\gamma-1}\eta_{is}\frac{U^2}{c_p T_{01}}\frac{\Delta h_0}{U^2}$$

$$= \frac{\gamma R}{(\gamma-1) c_p}\eta_{is}\frac{\Delta h_0}{U^2} \approx \eta_{is}\frac{\Delta h_0}{U^2}$$

(d)




<u>High speed</u>

As mass flow increases density rise across first stage is less than design ⇒ $V_x$ into second stage > $V_{x\,design}$ ⇒ less density rise. Process same through machine, so $V_x \uparrow$ point b for last stage near choke.

Opposite effect for point c — last stage sees much lower $V_x$

<u>Lower Speed</u>

$\Delta p$ from each stage <u>much</u> less than design ⇒ $V_x \uparrow$ through machine. Rear stage choking sets mass flow through machine. ∴ Point d at low $V_x/U$ for first stage, very high $\frac{V_x}{U}$ for last.

(e) 60% front stage stall — maybe part span

80% all stages together — probably full span

100% rear stage stall (usually leads to surge)

**Q3 High-speed multistage compressor stall:** 9/30 attempts, mean 73%, st. dev. 18.4%
This question was attempted by just under a third of the candidates. Those who attempted it gave very good solutions, with minimum guesswork. In part (a) almost all candidates gave a good diagram and explanation of the mechanisms of rotating stall. In part (b), there was some confusion over hub-to-tip ratios, but most candidates were able to describe the type of stall cells to be expected. The only common error/omission was that very few candidates included the extent of the different types of stall cell.