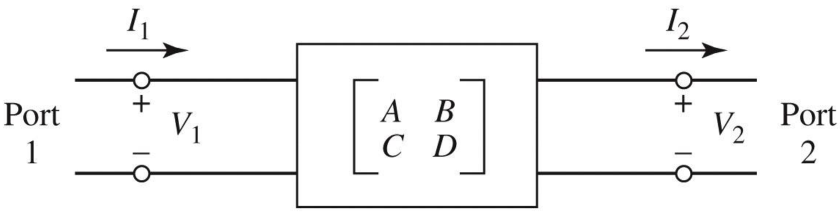
CRIB 4B24 2019

1. (a) When circuit dimensions become significant proportion of wavelength, lumped element models no longer apply. A full solution would require solving Maxwell's equations for the RF circuit, but this approach is computationally complex. Network analysis reduces sub-circuits to 'black-boxes' with ports where the relationship between the ports is known. Multiple sub-circuits can be connected by transmission lines and the behaviour of the larger system predicted accurately.

(b) (i)
ABCD parameters:



(a)

$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_2 \\ I_2 \end{bmatrix}$$

Useful because with I_1 and I_2 in the same sense, it is possible to combine cascaded blocks by simple matrix multiplication of the ABCD parameters.

(ii)
By inspection:
Shunt element:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 1/Z & 1 \end{bmatrix}$$

Series element:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$$

(iii)
For the transmission line:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 0 & 50j \\ \frac{j}{50} & 0 \end{bmatrix}$$

T-network using results above:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{50} & 1 \end{bmatrix} \begin{bmatrix} 1 & j30 \\ 0 & 1 \end{bmatrix}$$

For the load use another shunt element and consider the voltage across it as V2 (I2=0)
So overall network:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 & 50 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \frac{1}{50} & 1 \end{bmatrix} \begin{bmatrix} 1 & j30 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 50j \\ \frac{j}{50} & 0 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 1/100 & 1 \end{bmatrix}$$

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} -0.2 + 1j & 0 \\ 0.02 + 0.02j & 0 \end{bmatrix}$$

$$V_1 = AV_2 \quad (I_2 = 0)$$

$$V_2 = 4.287\angle -120°$$

(c) For a mismatch at port 3:

$$V_3^+ = \Gamma V_3^-$$

If port 2 is matched $V_2^+ = 0$

Condition for match is $\frac{V_1^-}{V_1^+} = 0$

$$V_1^- = S_{11}V_1^+ + S_{13}V_3^+$$

$$V_3^- = S_{31}V_1^+ + S_{33}V_3^+ = S_{31}V_1^+ + S_{33}\Gamma V_3^-$$

$$V_3^- = \frac{S_{31}V_1^+}{1 - \mathrm{S}_{33}\Gamma}$$

$$V_1^- = S_{11}V_1^+ + S_{13}\Gamma\frac{S_{31}V_1^+}{1 - \mathrm{S}_{33}\Gamma}$$

$$0 = S_{11} + S_{13}\Gamma\frac{S_{31}}{1 - \mathrm{S}_{33}\Gamma}$$

$$S_{11} = -S_{13}\frac{S_{31}}{1/\Gamma - \mathrm{S}_{33}}$$

$$\frac{1}{\Gamma} = -\frac{S_{13}S_{31}}{S_{11}} + S_{33}$$

$$\Gamma = \frac{S_{11}}{-S_{13}S_{31} + S_{33}S_{11}}$$

A popular question attempted by all candidates. Answers for the bookwork (a) and (b) (i) were mixed. The rest of part (b) was generally well answered. Part (c) clearly stretch the very best students to produce some good answers.

2. (a) (i) For a narrowband system, filters can be applied after any potential distortion inducing stages, so only the third order intermodulation products will matter which generate distortion products close to the carrier frequency which can't be easily filtered.
For a wideband system, there is potential for 2nd and 3rd harmonics to fall in the system bandwidth as well (and can't be filtered) so these must also be considered.
(ii) $v_i = V_0(\cos\omega_1 t + \cos\omega_2 t)$

$$v_o(t) = a_0 + a_1 V_0(\cos\omega_1 t + \cos\omega_2 t) + a_2 V_0^2(\cos\omega_1 t + \cos\omega_2 t)^2 + a_3 V_0^3(\cos\omega_1 t + \cos\omega_2 t)^3 + \cdots$$

$$= a_0 + a_1 V_0 \cos\omega_1 t + \frac{1}{2}a_2 V_0^2(1+\cos 2\omega_1 t) + \frac{1}{2}a_2 V_0^2(1+\cos 2\omega_2 t) + a_2 V_0^2\cos(\omega_1-\omega_2)t + a_2 V_0^2\cos(\omega_1+\omega_2)t + a_3 V_0^3\left(\frac{3}{4}\cos\omega_1 t + \frac{1}{4}\cos 3\omega_1 t\right) + a_3 V_0^3\left(\frac{3}{4}\cos\omega_2 t + \frac{1}{4}\cos 3\omega_2 t\right) + a_3 V_0^3\left[\frac{3}{2}\cos\omega_2 t + \frac{3}{4}\cos(2\omega_1-\omega_2)t + \frac{3}{4}\cos(2\omega_1+\omega_2)t\right] + a_3 V_0^3\left[\frac{3}{2}\cos\omega_1 t + \frac{3}{4}\cos(2\omega_2-\omega_1)t + \frac{3}{4}\cos(2\omega_2+\omega_1)t\right] + \cdots$$

So the voltage ratio will be:

$$\frac{\frac{3}{4}}{\frac{1}{4}} = 3$$

So power ratio is 9x

(b) (i) *Available gain* - $G_A = \frac{P_{avn}}{P_{avs}}$ ratio of power available from 2 port network to power available from source – assumes conjugate matching of source and load, depends on Z_S but not on Z_L
$S_{21} = 0.5$ (other ports matched so no other mechanism for power ending up at port 2

$$G_A = 0.25$$

(ii) $N_2 = G_A kTB + G_A N_{added}$
At thermal equilibrium $N_2 = kTB$

$$N_{add} = \frac{1-G_A}{G_A}kTB$$

$$T_e = \frac{N_{add}}{kB} = \frac{1-G_A}{G_A}T$$

$$F = 1 + \frac{T_e}{T_0} = 1 + \frac{1-G_A}{G_A}\frac{T}{T_0} = 1 + 3\frac{T}{T_0}$$

(an alternative solution is to recall that for a lossy passive circuit $F = 1 + \frac{(L-1)T}{T_0}$)

(iii) if port 3 is open circuit:
$\Gamma_{out3} = +1$

$$V_3^- = -V_3^+$$

From S parameter definitions:

$$V_2^- = S_{21}V_1^+ + S_{22}V_2^+ + S_{31}V_3^+$$
$$V_3^- = S_{31}V_1^+ + S_{32}V_2^+ + S_{33}V_3^+$$

$S_{22} = S_{33} = 0$ so those terms disappear.
If port 2 is match $V_2^+ = 0$, $S_{33} = 0$ so we get:

$$V_2^- = S_{21}V_1^+ + S_{31}S_{31}V_1^+$$

Available gain is increased to 0.5625 (9/16)

Apply equation above: F = $1 + \frac{7}{9}\frac{T}{T_0}$

The least popular question. Several missed the point of (a) (i) discussing noise rather than distortion. (b) polarized the responses with some near perfect and others making little attempt beyond the trivial part (i)

3 (a) (i)

Freespace path loss at 6.5e12m and 7GHz: 305dB

For 12W, 42dBi gain -> -222.2dBm (if no antenna gain at Rx)'

Note that the ground station antenna gain is going to influence both the noise (via background noise) and the signal level, so it appears on both sides of the SNR.

Antenna noise temperature:

$$T_b = \frac{1}{4\pi}\int_{\phi=0}^{2\pi}\int_{\theta=0}^{\pi} T_B(\theta,\phi)D(\theta,\ \phi)\sin\theta d\theta d\phi$$

$$T_b = \frac{1}{4\pi}\int_{\phi=0}^{\frac{\pi}{\sqrt{G}}}\int_{\theta=0}^{\frac{\pi}{\sqrt{G}}} 4\times G\sin\theta d\theta d\phi$$

$$T_b = \sqrt{G}\,(1-\cos\frac{\pi}{\sqrt{G}})$$

Signal at antenna output = -222.2+G dBm $= G\times 6.30\times 10^{-23}$

Noise at antenna output = kT_bB (linear)

SNR = 5dB = 3.162 (linear)

SNR = $\frac{P_r}{kT_bB}$

$$G\times 6.30\times 10^{-23} = kT_bB\times 3.162$$

$$G\times 1.995\times 10^{-23} = kB\sqrt{G}\,(1-\cos\frac{\pi}{\sqrt{G}})$$

$$\frac{1}{\sqrt{G}}(1-\cos\frac{\pi}{\sqrt{G}}) = 4.8\times 10^{-5}$$

By iteration:

$$G \approx 2200 = 33\text{dB}$$

(ii) A non ideal antenna will have a radiation resistance less than 1, this will result in thermal noise components related to the physical temperature of the antenna.

For a very high gain antenna pointed at a cool background temperature, components from sidelobes and back lobes pointing at high background temperatures are likely to dominant noise sources.

(iii) Given the very high losses, the requirement will be for the minimum BER for a given Eb/No, so BPSK/QPSK is likely to be the most suitable. Spectral efficiency and receiver complexity are not really concerns here.

(b) (i) Power at the backscatter device:

$$P_r = S_tA_e = \frac{S_t\lambda^2 G}{4\pi}$$

For short $\Gamma = -1$, for match $\Gamma = 0$

Power scattered $P_S = GP_r\Gamma$

$$\Delta\sigma = \frac{P_s}{S_t} = \frac{\lambda^2 G^2 |\Gamma_A - \Gamma_B|^2}{4\pi} = \frac{\lambda^2 G^2}{4\pi} = 0.004577$$

Power at reader/receiver (1W = 1000mW, -70dBm = 1e-7mW:

$$P_r = \frac{P_t G_{reader}^2 \lambda^2 \Delta\sigma}{(4\pi)^3 R^4} = \frac{1.004}{1984.4 \times R^4}$$

R = 8.47m

(ii) We have assumed matched polarisation of the antennas so no polarisation mismatch loss.

Assumed free space propagation, so no fading which in practice reduces reliable range, but may increase the absolute maximum range.

Ideal short at 2.5GHz will be difficult to achieve.

Many missed that the antenna gain would influence both the noise (via brightness temperature) and the signal level in part (a)(i) with no full solutions. Most managed sensible comments for (a)(ii) and (iii). In part (b) a common mistake was to not realise that the tag antenna gain appears squared since the signal is received, reflected then retransmitted.

4 (a)

For passive matching the S11 of the match must be less than 1.

Stable if $|\Gamma_{\text{in}}|$ and $|\Gamma_{out}| < 1$.

If $|\Gamma_{\text{in}}|$ or $|\Gamma_{out}| < 1$ for any source and load impedances then unconditionally stable. If only for some combinations then conditionally stable.

(b). For a passive device, minimum noise figure occurs when it is matched ($\Gamma_{in} = 0$). The noise figure of an amplifier is actually a function of the admittance presented to the device, so there is an optimum input impedance which leads to the lowest noise figure.

(c) (i)

$$\Delta = S_{11}S_{22} - S_{12}S_{21} = 0.377\angle - 109.2°$$

$$K = \frac{1 - |S_{11}|^2 - |S_{22}|^2 + |\Delta|^2}{2|S_{12}S_{21}|}$$

K = 2.66, $\Delta = 0.38$, device is unconditionally stable

(ii) Transistor gain: 4-> 6dB

Use conjugate match to get max G_L (won't effect noise since we can assume unilateral).

$$\Gamma_L = S_{22}^* = 0.5\angle 60°$$

$$G_L = \frac{1}{1 - |S_{22}|^2} = 1.33 = 1.25dB$$

For 8.5dB gain we need to have $G_s = 8.5 - 6 - 1.25 = 1.25dB = 1.33$

Now plot the gain circles corresponding to 1.25dB on the Smith chart:

$$G_{S_{max}} = \frac{1}{1 - |S_{11}|^2} = 1.56$$

So $g_s = \frac{1.33}{1.56} = 0.8525$

$$C_S = \frac{g_S S_{11}^*}{1 - (1 - g_S)|S_{11}|^2} = 0.54\angle 60°$$

$$R_S = \frac{\sqrt{1 - g_S}(1 - |S_{11}|^2)}{1 - (1 - g_S)|S_{11}|^2} = 0.26$$

Any point on this circle will give the correct gain, but we need min noise figure.

$$N = \frac{|\Gamma_S - \Gamma_{opt}|^2}{1 - |\Gamma_S|^2} = \frac{F - F_{min}}{4R_N/Z_0} \left|1 + \Gamma_{opt}\right|^2$$

To minimize F, we can see that N should also be minimised. Exact solution is difficult, but $\min|\Gamma_S - \Gamma_{opt}|$ will get close as a starting point, then iterate $\frac{|\Gamma_S - \Gamma_{opt}|^2}{1 - |\Gamma_S|^2}$ to find minimum

P1: $\Gamma_S = 0.58\angle 87° \; \frac{|\Gamma_S - \Gamma_{opt}|^2}{1 - |\Gamma_S|^2} = 0.0301$

P2: $\Gamma_S = 0.42\angle 90° \frac{|\Gamma_S - \Gamma_{opt}|^2}{1-|\Gamma_S|^2} = 0.058$

P3: $\Gamma_S = 0.54\angle 88° \frac{|\Gamma_S - \Gamma_{opt}|^2}{1-|\Gamma_S|^2} = 0.0297$

Take P3 as solution:

$$\frac{F - F_{min}}{4R_N/Z_0}\left|1 + \Gamma_{opt}\right|^2 = 0.0297$$

$$F - F_{min} = 0.02276$$

$$NF = 1.67dB$$

Another popular question. The bookwork part (a) was very disappointing with only a few good answers. Many referred to S parameters and K-Delta test which isn't what was asked for. Part (c)(ii) was the major part of the question and possibly should have been broken into stages for easier marking. Most understood the basic requirement, but many didn't realise that to minimise the noise figure the output match in this case needed maximum gain, so plotting multiple gain and noise figure circles could be avoided.

EGT3
ENGINEERING TRIPOS PART IIB

Candidate No.

Smith Chart to be detached and handed in with script if required