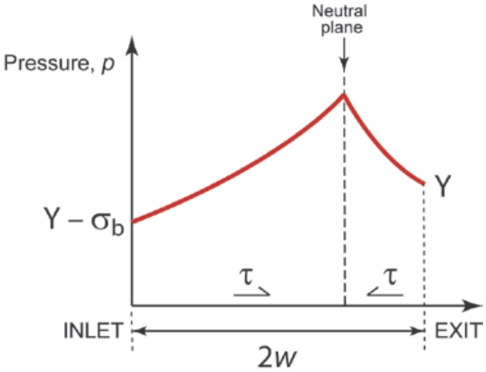
ENGINEERING TRIPOS Part IIA, MODULE 3C1 APRIL 2018
MANUFACTURING ENGINEERING TRIPOS Part IIA, MODULE 3P1

1. (a) Yield is controlled by dislocations, driven by shear stresses. Hydrostatic stress produces no shear stress in any orientation; deviatoric stress measures the deviation away from hydrostatic stress (and hence characterises shear). From Mohr's circle, the difference between any pair of principal stresses is twice the maximum shear stress on a plane at 45^o to both of the principal directions. Tresca considers only the largest of these three shear stresses; von Mises takes a balance of all three.

(b) (i) Exponential friction hill when Coulomb friction applies, with peak pressure at the neutral plane where the shear stress τ reverses (rolls moving faster than strip to left of this plane, and slower than the strip to the right).

Back tension decreases pressure at entry and whole curve to left of neutral plane, which therefore moves towards the exit.



(ii) The difference between axial tension (if any) and compressive pressure must be Y at inlet and exit. Hence $p - (-\sigma_b) = Y$ at inlet, and $p = Y$ at exit.
Assumptions: radius large compared to thickness and width of contact patch; friction constant and independent of slip velocity between roll and strip; assume p and σ_x are principal stresses (i.e. neglecting effect of friction on principal stresses).

(c) (i) For $x > 0$: $\frac{d\sigma_x}{dx} = -\left(\frac{4\mu}{h_1+h_2}\right)p$

Tresca yield criterion (compression positive): $p - \sigma_x = Y$, so $\frac{d\sigma_x}{dx} = \frac{dp}{dx}$

$$\frac{dp}{dx} = -\left(\frac{4\mu}{h_1+h_2}\right)p$$

Boundary condition: $x = (2w - A)$, $p = Y$

$$\int_{p(x)}^{Y}\frac{dp}{p} = -\left(\frac{4\mu}{h_1+h_2}\right)\int_{x}^{2w-A}dx \quad \text{so} \quad \ln\frac{p(x)}{Y} = \left(\frac{4\mu}{h_1+h_2}\right)(2w - A - x)$$

$$p(x) = \mathrm{Y}\exp\left[\left(\frac{4\mu}{h_1+h_2}\right)(2w - A - x)\right]$$

For $x < 0$: same method, with change of sign and boundary condition

$$\frac{dp}{dx} = \left(\frac{4\mu}{h_1+h_2}\right)p$$

Boundary condition: $x = -A$, $p = Y - \sigma_b$ (NB σ_b is negative)

$$\int_{p(x)}^{Y-\sigma_b}\frac{dp}{p} = \left(\frac{4\mu}{h_1+h_2}\right)\int_{x}^{-A}dx \quad \text{so} \quad \ln\frac{p(x)}{Y-\sigma_b} = \left(\frac{4\mu}{h_1+h_2}\right)(A + x)$$

$$p(x) = (\mathrm{Y} - \sigma_b)\ \exp\left[\left(\frac{4\mu}{h_1+h_2}\right)(A + x)\right]$$

(ii) Peak of friction hill at $x = 0$ when two pressures are equal:

$$\mathrm{Y}\exp\left[\left(\frac{4\mu}{h_1+h_2}\right)(2w - A)\right] = (\mathrm{Y} - \sigma_b)\ \exp\left[\left(\frac{4\mu}{h_1+h_2}\right)(A)\right]$$

$$\ln\frac{(\mathrm{Y}-\sigma_b)}{\mathrm{Y}} = \left(\frac{4\mu}{h_1+h_2}\right)[(2w - A) - A] = \left(\frac{8\mu}{h_1+h_2}\right)[w - A]$$

Hence: $A \;=\; w - \left(\frac{h_1+h_2}{8\mu}\right) \ln \frac{(\mathrm{Y}-\sigma_b)}{\mathrm{Y}}$

(iii) Maximum value of σ_b when peak reaches the exit, i.e. $A = 2w$, and the strip no longer passes between the rolls. This occurs when:

Hence: $\sigma_b \;=\; \mathrm{Y}\,(\exp - \left[\frac{8\mu w}{h_1+h_2}\right] \;-\; 1)$

(Q1 Analysis of metal rolling: low number of attempts, low average mark

Many candidates wasted time re-deriving the differential equations given in the question, and for assumptions simply restated those given. Common numerical errors were to apply incorrect boundary conditions from more standard problems (e.g. ± w from the symmetric case). Few integrated dp/dx to find p(x) by putting p and x as corresponding limits of integration, but many were not able to use a constant of integration instead.)

2. (a) Permanent mould: cavity machined in metal mould (or formed round a pattern in a ceramic mould) and mould used repeatedly – e.g. die casting.

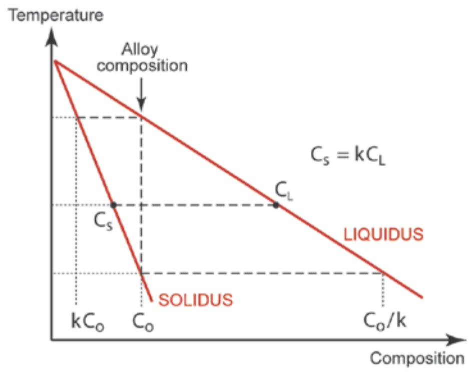
Permanent pattern: pattern (often wooden) used to shape cavity in a mould made from a material that can be broken up after casting; pattern removed prior to casting and re-used, e.g. sand casting.

Large parts: permanent pattern (cheaper to make pattern than large cavity in metal mould, and low production rate unimportant as batch sizes low).

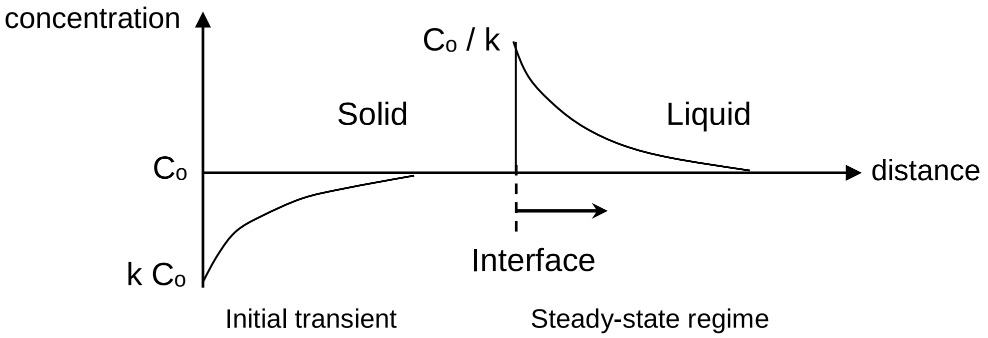
Large production runs: permanent mould (higher mould cost justified, and higher production rate also needed).

(b) (i) Partition coefficient, k:

Any liquid concentration C_L solidifies to a purer solid composition, C_S. Taking the liquidus and solidus as straight lines, the ratio is a constant: $C_S = k\,C_L$.



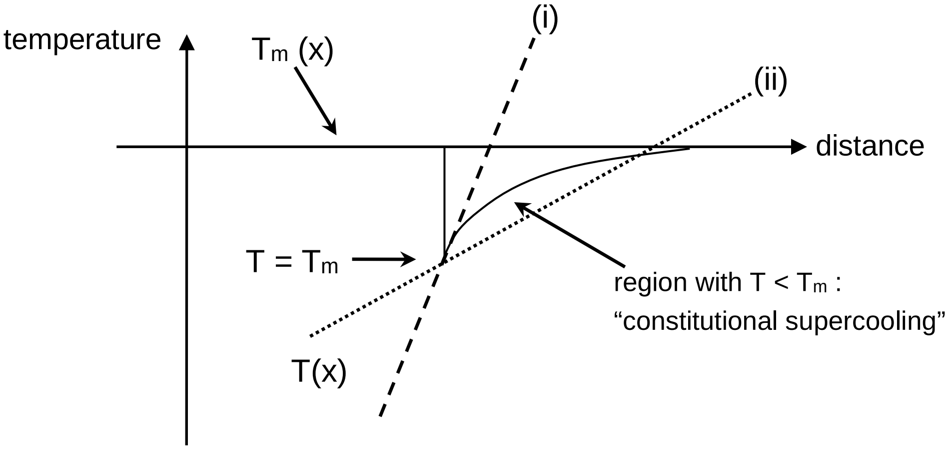
Concentration variation: when casting an alloy of composition C_o, the first solid to form has composition $k\,C_o$ (see figure above). The liquid concentration rises, as the liquid ahead of the interface becomes less pure, until the liquid reaches C_o/k, leaving solid of composition C_o – the steady state regime.



(ii) Casting variables that affect temperature gradient:

- mould material (metal, sand, ceramic)
- pour temperature

Concentration gradient from (i) leads to inverted variation in melting point, with $T=T_m$ at the interface.



In steady-state regime, critical gradient separates regimes with:
(i) $dT/dx > dT_m/dx$; (ii) $dT/dx < dT_m/dx$.

For (i): $T>T_m$ everywhere, leading to columnar growth of a few grains.
For (ii): $T<T_m$ over a region ahead of the interface (constitutional supercooling), leading to the instability of dendritic growth (branched crystal grains) into this region.
Columnar growth leads to macroporosity, as all the gas rejected from solution can be swept to the centre of the casting; dendritic growth traps pockets of liquid between the dendrite arms, dispersing the dissolved gas into many small bubbles (microporosity).

(iii) Modifications to compositions to control macroporosity:
- Vacuum outgassing: stir melt at temperature under vacuum to draw out dissolved gas.
- Add alloying elements that react with dissolved gases to form solid inclusions during solidification (e.g. "killing" steel with Al, to scavenge oxygen in the form of alumina).
- Add inoculants to promote heterogeneous nucleation in the central region of the casting, trapping porosity over a large area of grain boundary over a wider volume of the casting.

(Q2 Casting: high number of attempts, high average mark

Marks were lost for failure to answer all parts of the question (e.g. no sketches as requested), or lack of attention to what was asked (e.g. discussing material variables, rather than process variables).)

3. (a) (i) Problem: permanent softening of cold rolled non-heat-treatable Al alloy in the HAZ by recrystallisation (and weld metal also lower strength than parent plate).
Solution: heat-treatable alloys soften on welding, but natural ageing at room temperature restores strength over a period of days after welding, giving similar strength in the HAZ (and weld metal) as the parent plate.
(ii) Problem: "weld decay" in stainless steel – precipitation of chromium carbide on grain boundaries in HAZ during welding, so boundary regions become susceptible to corrosion.
Solution: Ti and Nb are strong carbide formers, scavenging any carbon in the stainless steel, removing the possibility of forming chromium carbides.
(iii) Problem: risk of martensite formation and embrittlement in HAZ and/or weld metal.
Solution: post-weld heat treatment tempers any martensite, restoring toughness. May also relieve residual stress, improving fatigue resistance.

(iv) Problem: stress concentrations due to shape of weld metal protruding above plate surface, risking fatigue crack initiation from edge of weld.
Solution: grinding weld to make the surface flush and eliminate stress concentration (though even this needs care to align machining marks in least damaging orientation to minimise risk of crack initiation). Also improves appearance.
(v) Problem: fatigue crack initiation from surface, particularly in the presence of residual stresses from welding.
Solution: surface peening plastically deforms a surface layer, eliminating weld stresses at the surface, and leaving a layer in a state of compressive residual stress (due to constrained plastic deformation), enhancing fatigue life.

(b) Laser hardening and PVD produce a hard surface layer for wear resistance.
Laser hardening involves tracking a defocussed laser spot over the surface to produce a thermal cycle to a depth of order 1mm, with the peak temperature needing to be in the austenite region (e.g. 800-1000^{o}C) in order to self-quench to hard martensite. Normalised microstructures can achieve uniform martensite at the surface where the peak temperatures and times are high enough for the carbon to fully re-distribute from former pearlite regions to former ferrite regions. But this is not the case where the temperature only just reaches the austenite field, giving mixed ferrite/martensite on quenching. Quenched/tempered microstructures have a fine-scale ferrite/iron carbide structure to being with, and transform to austenite easily, giving a uniform martensite layer below the surface.
PVD is conducted under vacuum at typically 200-400 ^{o}C, adding a TiN layer to the surface. At these temperatures there is no change in the underlying steel microstructure. The wear properties are largely controlled by the hard TiN layer, but a quenched/tempered microstructure underneath will provide greater wear resistance overall.

(Q3 Weld failure prevention; high number of attempts, high average mark

Candidates were mostly able to identify the right underlying issues and solutions for each class of materials – not something that is always found in this subject. The commonest error was to miss the distinction between plain carbon and stainless steel in relation to Ti/Nb alloying – i.e. microalloying to prevent grain growth and increased hardenability applies in carbon steels, not stainless).

4. (a) Polymer chain alignment enhances the stiffness and strength of products by making more use of the covalent bonds along the chains.
In bottles, stretch blow moulding strains the material in both longitudinal and hoop directions, enhancing the properties in all directions in the wall (more strain is applied hoopwise as pressurised bottles have double the stress in this direction).
Fibres are drawn in tension from fine nozzles using rollers and the winding drum, to apply a large axial strain, with rapid cooling to freeze in the alignment.
Thin films are also blown by extruding a thin cylindrical tube through a die with an air feed to inflate the emerging cylinder – expanding the cylinder aligns chains in the circumferential direction, while drawing the film tube upwards with windup rolls provides axial stretching. The stretched continuous tube is flattened and wound onto a roll for manufacture into plastic bags, or split for use as sheet.

(b) Polymer extrusion would be used with a screw extruder – this can make continuous sections that can be cut to length as required (injection moulding would need long moulds of many sizes – much more expensive). Shape and accuracy are affected by:

(i) melt swell, with chains aligning in the die, but then springing back to a more random orientation after exit from the die. This can be compensated by reshaping the die to give the correct final shape, or rapid cooling at the exit.

(ii) crystallinity: semi-crystalline thermoplastics contract significantly on crystallising, so controlling the cooling history is necessary to produce a consistent level of crystallinity and thus shape.

(c) Pultrusion would be used – bundles of fibres drawn through a resin bath and combined by pulling through a heated die, cooling and curing the resin, and cutting to length. The process maintains good alignment of fibres along the product, giving the maximum stiffness and strength. But the resulting uniaxial composite is susceptible to failure parallel to the fibres, e.g. splitting by delamination (which may be driven by shear stresses in bending). If T-joints are made, then through-thickness tensile stresses may be generated at joints.

(d) (i) Hot plate welding: melts the interface before butting parts together – only suitable for thermoplastics (HDPE to itself, not thermosetting GFRP or dissimilar joints). Also requires good control of alignment in butt or T-joints in 10mm bars, and cannot be disassembled (note that shelter is temporary).

(ii) Adhesive bonding: could join either material and dissimilar joints, but poorly suited to butt or T-joints with 10mm bars (low adhesion area, and not suited to lap configuration in this thickness). Cannot be disassembled (note that shelter is temporary).

(iii) Press-fitting shaped Al joint pieces: the best solution – can handle either material in any combination, and can easily be disassembled (good for temporary shelter, and for separating the materials for final disposal/recycling). An external Al joint piece would also protect the ends of the GFRP and reduce the risk of damage and splitting (as used in many flexible tent poles).

(Q4 Polymer and composite shaping/joining: high number of attempts, high average mark

Candidates again made accurate distinctions between different material-process combinations. An interesting observation was the interpretation by most candidates that a polymer/GFRP frame to support a fabric sheet as a "temporary shelter" had to be cheap, apparently assuming it implied single use and disposal, whereas the intention was to consider ease of disassembly and re-use.)

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