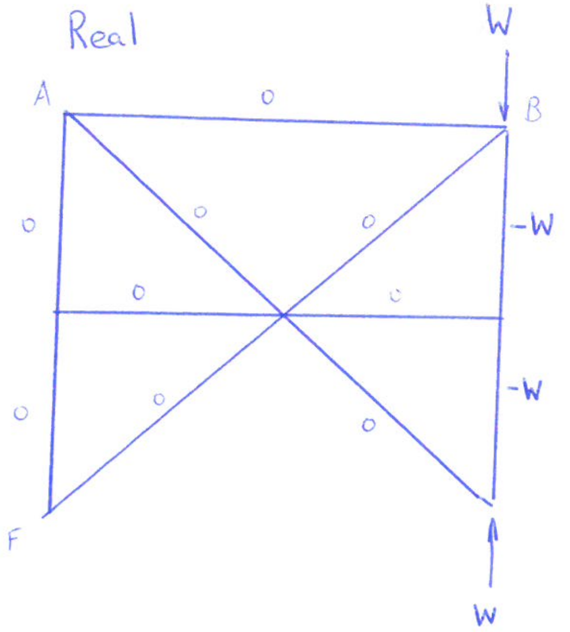
2019 IA Structures

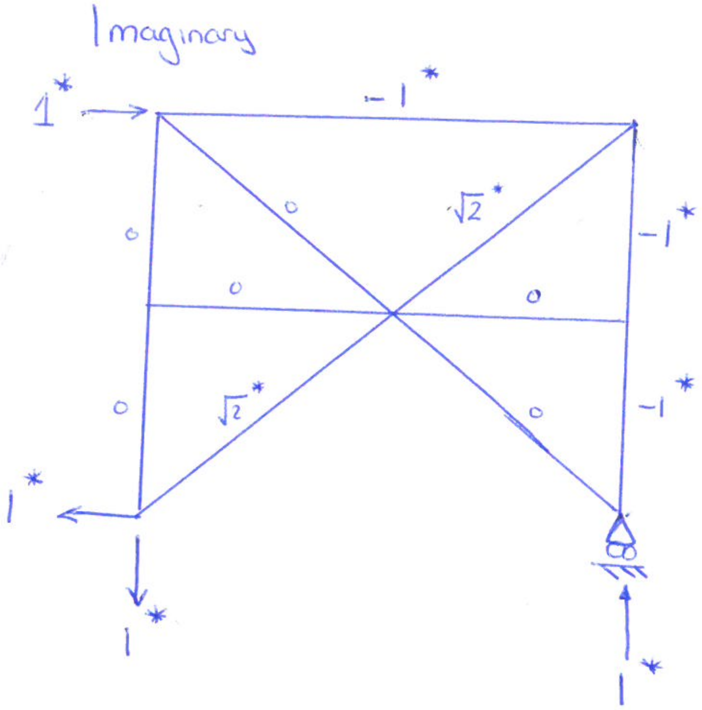
Post-exam cribs

1. Virtual Work

Real



Imaginary

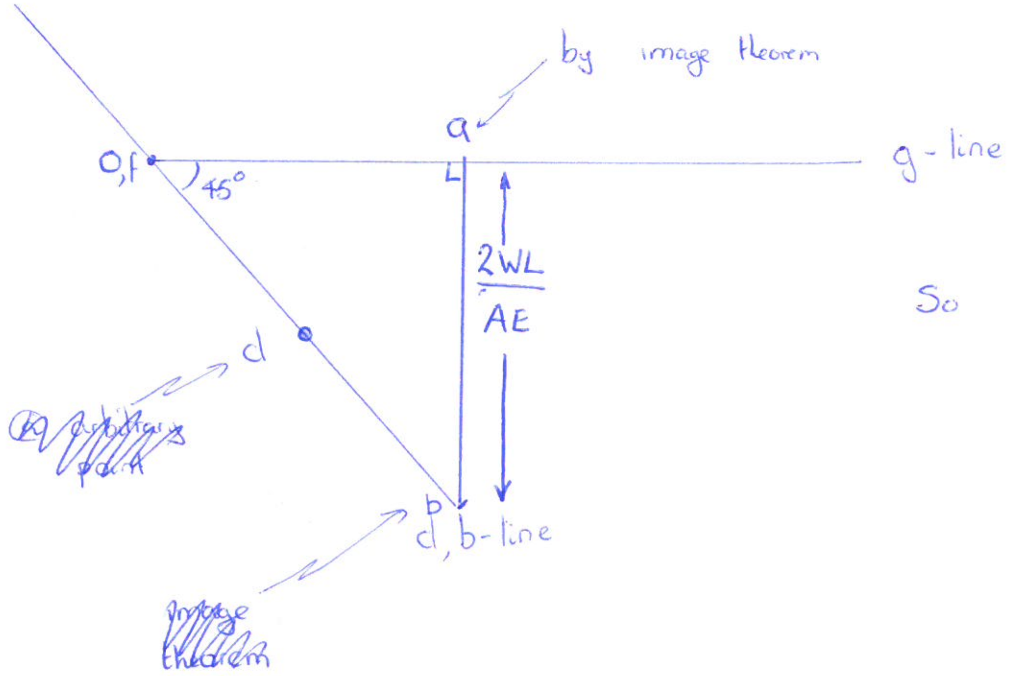


$$1^* \delta = \Sigma T^* e$$

$$= 2(-1^*) \frac{-WL}{AE}$$

$$\delta = \frac{2WL}{AE}$$

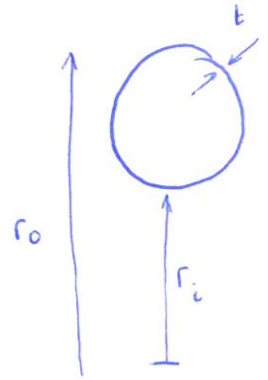
Displacement diagram: Note from above ABF is rigid.



So $\delta = \frac{2WL}{AE}$

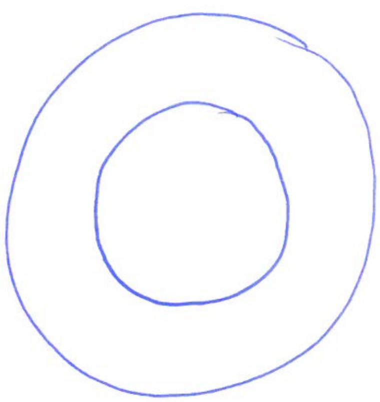
to right.

2 Cut in 2 directions and use equilibrium



$$p \cancel{\pi} \frac{(r_o - r_i)^2}{4} = \cancel{\pi} \underline{(r_o - r_i)} t \, \sigma_L$$

$$\sigma_L = \frac{p (r_o - r_i)}{4 t}$$

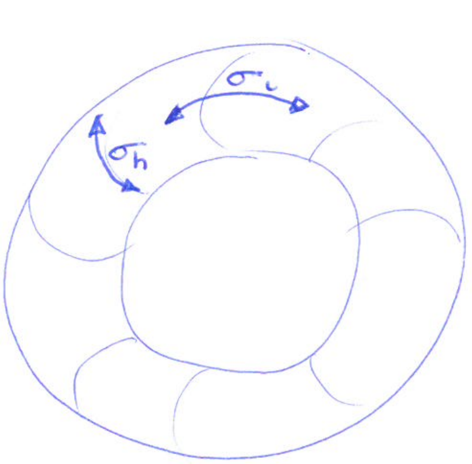


$$p \cancel{\pi} (r_o^2 - r_i^2) = 2 \sigma_h \pi (r_o + r_i) t$$

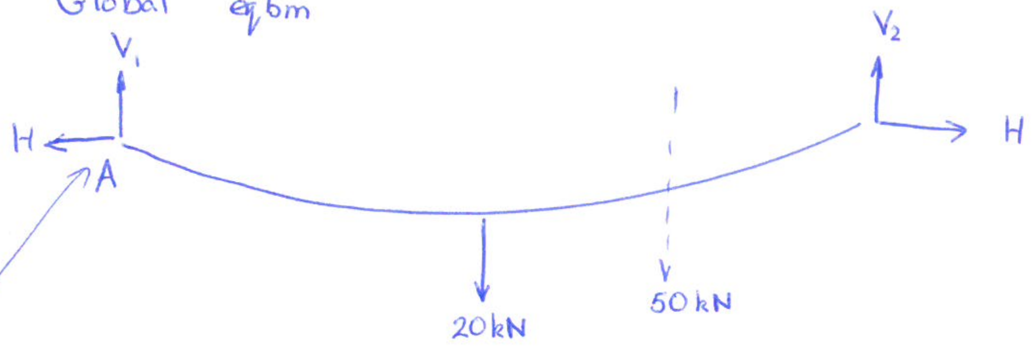
$$p (r_o - r_i)\cancel{(r_o + r_i)} = 2 \sigma_h \cancel{(r_o + r_i)} t$$

$$\sigma_h = \frac{p (r_o - r_i)}{2 t}$$

Clearly indicate directions:



3 Global eqbm



no moment for cables despite built-in support

$$\circlearrowright A \qquad V_2 \times 100 = 20 \times 50 + 50 \times 75$$

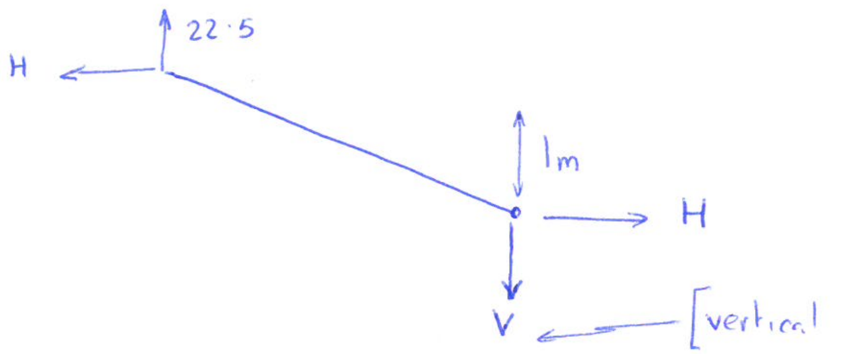
$$V_2 = 47.5 \text{ kN}$$

$$\uparrow \qquad V_1 + V_2 = 70$$

$$V_1 = 22.5 \text{ kN}$$

Cut @ known dip: [to left of load]

[Free body diagram]

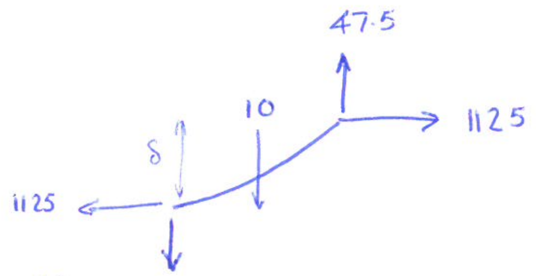


[vertical component of tension]

$$\circlearrowright \text{cut} \qquad H \times 1 = 22.5 \times 50$$

$$H = 1125 \text{ kN}$$

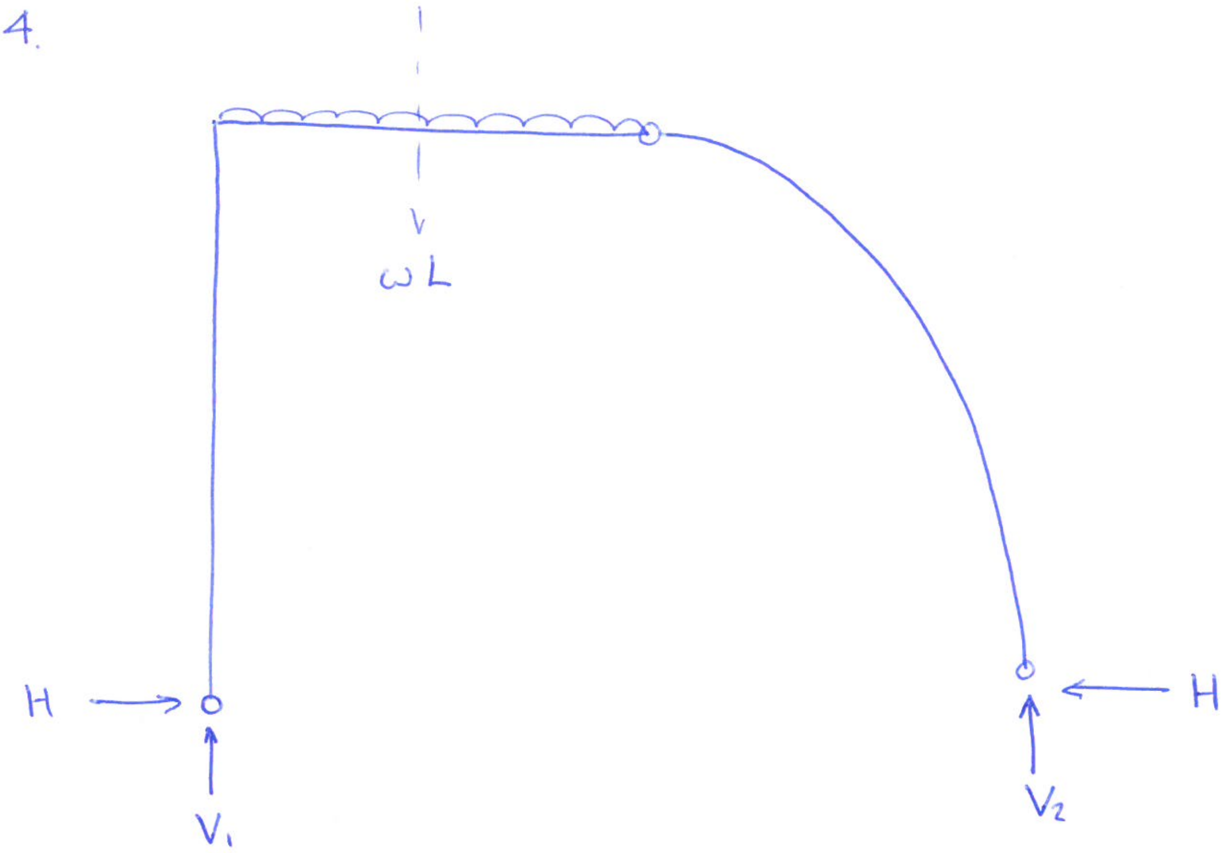
Cut 10m from support



$$\circlearrowright \text{cut} \qquad 1125\,\delta + 10 \times 5 = 47.5 \times 10$$

$$\delta = \underline{0.38 \text{ m}}$$

4.



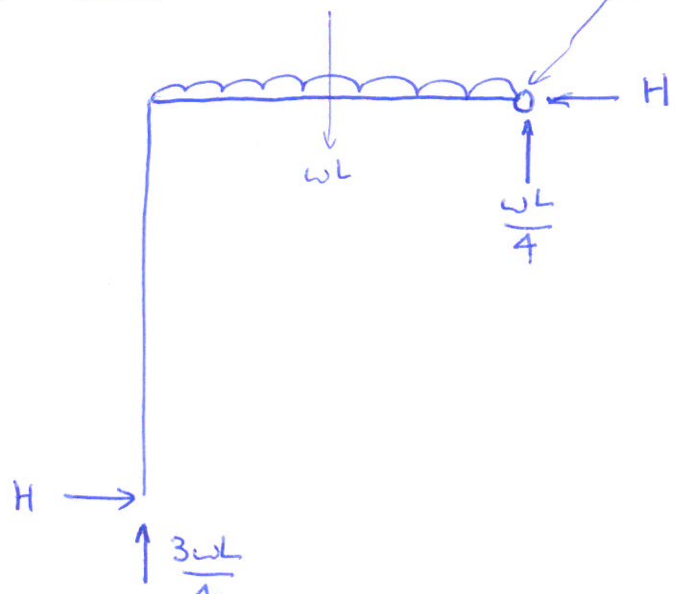
Global eqbm

$$V_2 = \frac{\omega L}{4}$$

$$V_1 = \frac{3\omega L}{4}$$

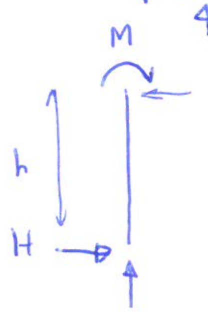
Free body

zero moment but reactions.



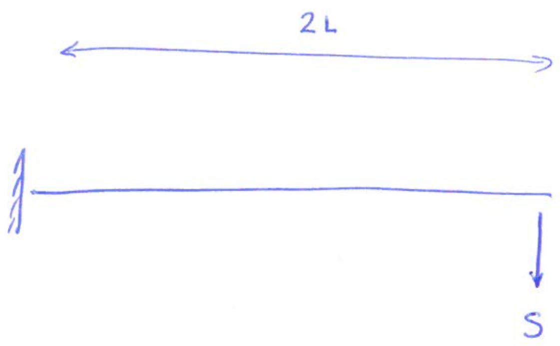
$$\frac{\omega L^2}{2} + Hh = \frac{3\omega L^2}{4}$$

$$H = \frac{\omega L^2}{4h}$$

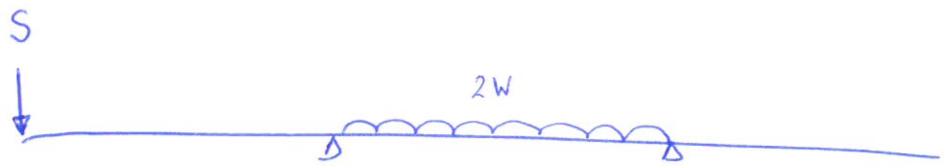


Moment at $A = Hh = \frac{\omega L^2}{4}$

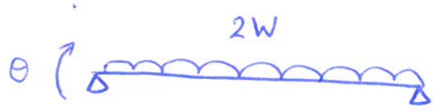
5.



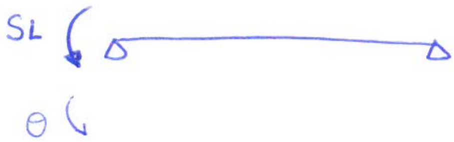
From databook $\quad \delta = \dfrac{S(2L)^3}{3EI} = \dfrac{8}{3}\dfrac{SL^3}{EI}$ downwards



① $\delta = \dfrac{SL^3}{3EI}$ down



② $\theta = \dfrac{2WL^2}{24EI}$ clockwise



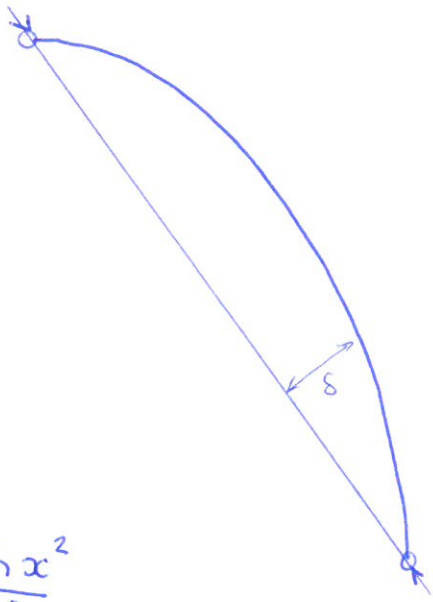
③ $\theta = + \dfrac{SL\,L}{3EI}$ anticlockwise.

With rigid pointer :

$$\delta = \frac{SL^3}{3EI} + \left[\frac{SL^2}{3EI} - \frac{WL^2}{12EI}\right] L$$

$$= \frac{2SL^3}{3EI} - \frac{WL^3}{12EI} \quad \text{down}$$

b) RHS



2 forces so co-linear

max BM at maximum δ

i.e. lines are parallel

$$\frac{dy}{dx} = \frac{h}{L} = \frac{3hx^2}{L^3}$$

$$x = \frac{L}{\sqrt{3}} \qquad y = \frac{h}{3\sqrt{3}}$$

c) Superpose a and b

Both have same deflection at pin where S is shear force.

S is no longer an external force + shouldn't appear in answer!

$$\delta = \frac{8SL^3}{3EI} \quad \text{upwards}$$

$$=$$

$$\delta = \frac{(W - 8S)L^3}{12EI} \quad \text{upwards}$$

$$32S = W - 8S$$

$$\underline{S = \frac{W}{40}}$$

$$\delta = \frac{8WL^3}{120EI} = \frac{WL^3}{15EI} \quad \underline{\text{up}}$$

6. a) Euler length $= L/2$

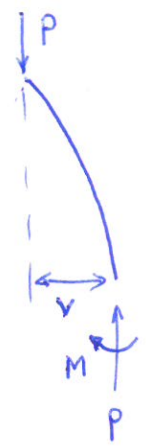
$$P_E = \frac{\pi^2 EI}{\left(\frac{L}{2}\right)^2} = \frac{4\pi^2 EI}{L^2}$$

$$I = \frac{bd^3}{12} = \frac{25^4}{12} = 32.55 \times 10^{-9} \ m^4$$

$E = 210$ GPa [in data book - marks still given for other similar values from materials D.B]

$$P_E = 270 \text{ kN}$$

b) Draw free body + define variables



$$M = Pv = -EI\left(\frac{\partial^2 v}{\partial x^2} - \frac{\partial^2 v_0}{\partial x^2}\right)$$

$$EI\frac{\partial^2 v}{\partial x^2} + Pv = -EI\delta\frac{4\pi^2}{L^2}\sin\frac{2\pi x}{L}$$

Solve to give: $\underbrace{v = A\sin\sqrt{\frac{EI}{P}}\,x + B\cos\sqrt{\frac{EI}{P}}\,x}_{\text{gives buckling load}}$

$$+ \quad v = \alpha \sin\frac{2\pi x}{L}$$

$$P\alpha - \frac{4\pi^2}{L^2}\alpha EI = -EI\delta\frac{4\pi^2}{L^2}$$

$$\alpha = \frac{\delta EI \frac{4\pi^2}{L^2}}{\frac{4\pi^2 EI}{L^2} - P} = \frac{P_E\delta}{P_E - P}$$

Pre - buckling only α matters

c) Compression due to P

Bending due to $P\delta$ effects.

$$P = \frac{P_E}{2} \qquad \alpha = 2\delta \qquad (\text{max } 4\text{mm})$$

Compression $\sigma = \frac{135}{25^2} = 0.216 \text{ GPa}$

Bending $\sigma = \frac{My}{I} = \frac{P\delta\, y_{max}}{I} = \frac{135 \times 10^{3} \times 4 \times 10^{-3} \times 12.5 \times 10^{-3}}{32.55 \times 10^{-9}}$

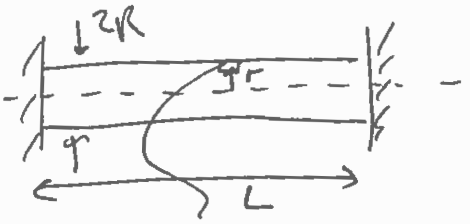
$= 0.207 \text{ GPa}$

So max stress (bending + compression) is approx twice average stress (compression only)

$\boxed{Q7}$ (a) Larger volume of material => higher probability that a flaw exists of critical size for a given applied stress, σ.

$$P_S(V = nV_0) = P_S(V_0) \times P_S(V_0) \times \ldots = \left[P_S(V_0)\right]^n, \quad n = \frac{V}{V_0}$$

(b)



$$\sigma(r) = \frac{\alpha r}{R^4}$$

<u>note</u>: residual stress magnitude <u>reduces</u> for larger R, though volume <u>increases</u> → effects trade off

$$P_S(V) = \exp\left[\int_0^R -\left(\overset{\sigma/\sigma_0}{\frac{\alpha r}{R^4 \sigma_0}}\right)^m \frac{\overbrace{L 2\pi r\, dr}^{dV}}{V_0}\right]$$

$$= \exp\left[-\left(\frac{\alpha}{R^4 \sigma_0}\right)^m \left(\frac{2\pi L}{V_0}\right) \int_0^R r^{m+1}\, dr\right]$$

$$= \exp\left[-\left(\frac{\alpha}{R^4 \sigma_0}\right)^m \left(\frac{2\pi L}{V_0}\right)\left(\frac{R^{m+2}}{m+2}\right)\right]$$

$$\therefore \ln P_S(V) \propto R^{-3m+2}$$

Taking ratios:

$$\frac{\ln P_S(V_2)}{\ln P_S(V_1)} = \left(\frac{R_2}{R_1}\right)^{-3m+2}$$

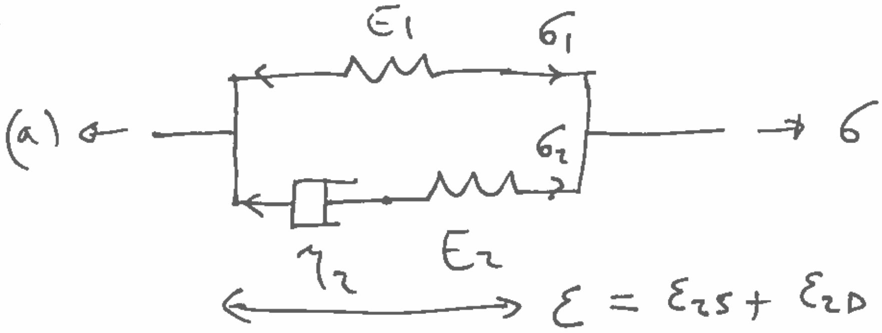
$$\therefore \ln P_S(V_2) = \ln(0.9)\left(\frac{17}{15}\right)^{-7}$$

$$\therefore P_S(V_2) = 0.96$$

∴ net <u>increase</u>, due to this trade-off

Q8

(a)



$$\sigma_1 = E_1 \varepsilon \quad (1)$$

$$\sigma_2 = E_2 \varepsilon_{2S} \quad (2)$$

$$\sigma_2 = \eta_2 \dot{\varepsilon}_{2D} \quad (3)$$

Total strain: $\varepsilon = \varepsilon_{2S} + \varepsilon_{2D}$ (4)

Total stress: $\sigma = \sigma_1 + \sigma_2$ (5)

(2), (3) → (4): $\dot{\varepsilon} = \dfrac{\dot{\sigma}_2}{E_2} + \dfrac{\sigma_2}{\eta_2}$

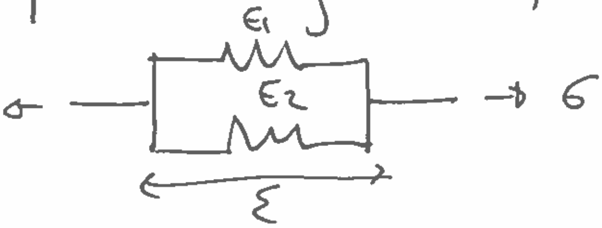
(1) → (5): $\sigma_2 = \sigma - E_1 \varepsilon \quad \therefore \quad \dot{\sigma}_2 = \dot{\sigma} - E_1 \dot{\varepsilon}$

$$\therefore \quad \dot{\varepsilon} = \frac{\dot{\sigma}}{E_2} - \frac{E_1}{E_2}\dot{\varepsilon} + \frac{\sigma}{\eta_2} - \frac{E_1}{\eta_2}\varepsilon$$

Rearranging: $\boxed{\sigma + \left(\dfrac{\eta_2}{E_2}\right)\dot{\sigma} = \eta_2\left(1 + \dfrac{E_1}{E_2}\right)\dot{\varepsilon} + E_1 \varepsilon}$

(b) (i) High frequencies: dashpot doesn't have time to respond to changes in σ, so $\varepsilon_{2D} \to 0$

$\therefore$

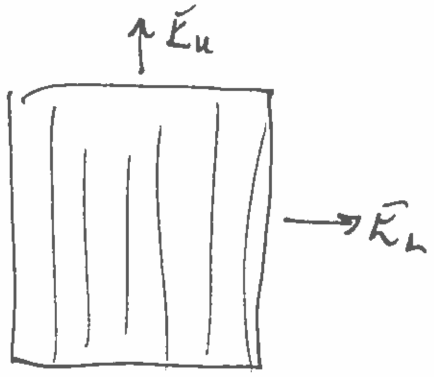


(ii) Low frequencies: dashpot relaxes, i.e. $\dot{\varepsilon}_{2D} = \sigma_2 \to 0$

$\therefore$ lower branch is unloaded

$\therefore$

9 (a)



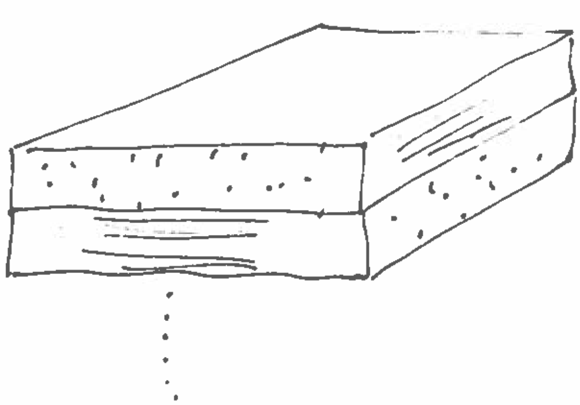
Unidirectional layers/fibres.
moduli E_1 and E_2
volume fraction V_f of E_1

E_u : same strain in 1 and 2

E_L : same stress in 1 and 2

1: fibres, 2: matrix in UD fibre composite.

(b)



Moduli of a layer:

$$E_{\parallel} = 0.5\left(E_f + E_m\right)$$

$$E_{\perp} = \left(\frac{0.5}{E_f} + \frac{0.5}{E_m}\right)^{-1}$$

Laminate has 50% of each modulus material, loaded at constant strain → upper bound.

$$E_{lam} \simeq 0.25\left(E_f + E_m\right) + 0.5\left(\frac{0.5}{E_f} + \frac{0.5}{E_m}\right)^{-1}$$

$$\simeq 0.25\left(E_f + E_m\right) + \left(\frac{E_f E_m}{E_f + E_m}\right)$$

$$\simeq 0.25 E_f\left(1 + \frac{E_m}{E_f}\right) + E_f\left(\frac{1}{\frac{E_f}{E_m} + 1}\right)$$

$$= 0.25\left(1.01\right) E_f + \frac{E_f}{101} \simeq 0.262\, E_f$$

$$\left(\simeq 0.25 E_f\right)$$

(c) $E_{comp} = 120 \times 0.262 \approx 31$ GPa.

$\rho_{comp} = 0.95 \ Mg/m^3$

Material located near top of envelope for woods (parallel to grain): nearest competition is bamboo.

(10)

(a) Hot rolled Cu: $\sigma_y = 90$ MPa, $\rho_d = 10^{13}$ m/m^3

Cold rolled Cu: $\sigma_y = 360$ MPa, $\rho_d = 10^{15}$ m/m^3

$$\sigma_y \simeq \sigma_0 + C_1\sqrt{\rho_d}$$

$$90 = \sigma_0 + C_1\sqrt{10^{13}}$$ σ_0 : σ_y for negligible ρ_d

$$360 = \sigma_0 + C_1\sqrt{10^{15}}$$

$$\frac{360-\sigma_0}{90-\sigma_0} = \frac{\sqrt{10^{15}}}{\sqrt{10^{13}}} = 10 \quad \therefore \sigma_0 = 60 \text{ MPa}$$

annealed pure Cu

(b) Cu–10%Ni : $\sigma_y = 110$ MPa, $\therefore \Delta_{ss} = 50$ MPa

Cu–20%Ni : $\sigma_y = 131$ MPa $\therefore \Delta_{ss} = 71$ MPa

$$\sigma_y = \sigma_0 + \Delta\sigma_{ss} \qquad \Delta\sigma_{ss} = C_2 C_{ss}^{\,n}$$

$\log \Delta\sigma_{ss}$ vs. $\log C_{ss}$ → slope n

or $$\frac{(\Delta_{ss})_1}{(\Delta_{ss})_2} = \left[\frac{(C_{ss})_1}{(C_{ss})_2}\right]^n$$

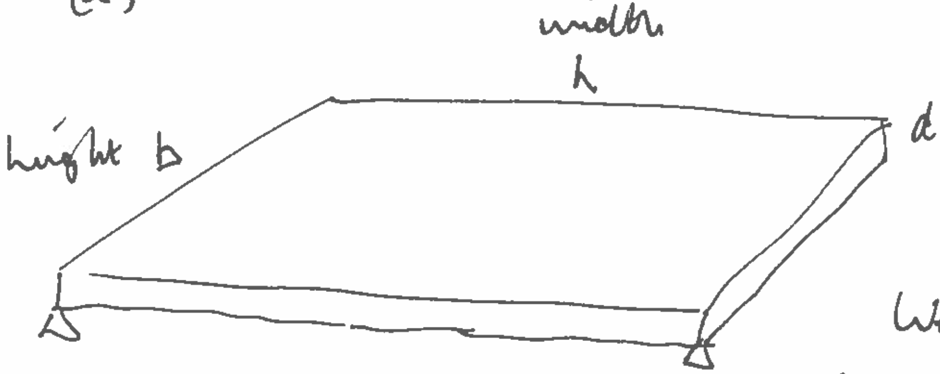
$$\Rightarrow \frac{71}{50} = \left(\frac{20}{10}\right)^n \quad \therefore n \simeq 1/2 \qquad \Delta\sigma_{ss} \propto \sqrt{C_{ss}}.$$

(c) Cu–15%Ni : $\frac{\Delta\sigma_{ss}}{50} = \sqrt{1.5} \quad \therefore \Delta\sigma_{ss} = 61.2$ MPa

Cu – rolled to $\rho_d = 10^{14}$ m/m^3 : $\frac{\Delta\sigma_{wh}}{30} = \frac{\sqrt{10^{14}}}{\sqrt{10^{13}}} = \sqrt{10} =$ 94.9 MPa

Addition σ_y : $\sigma_y = \sigma_0 + \Delta\sigma_{ss} + \Delta\sigma_{wh} = 60 + 61.2 + 94.9 \simeq 216$ MPa ✓

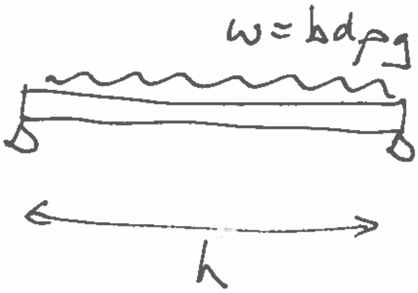
(11) (a)



$\underline{b < h}$

Weight $= bhd\rho g$

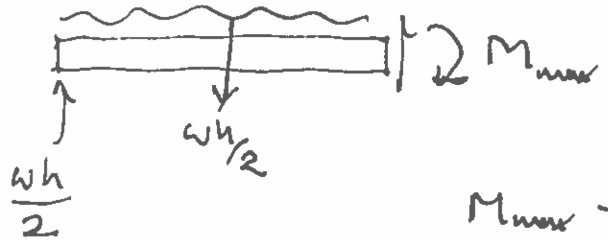
Self-weight/unit length $= bd\rho g$



Structures Databook : $\delta = \dfrac{5wh^4}{384EI}$

where $I = \dfrac{bd^3}{12}$

Hence central deflection $\delta = \dfrac{5.\, bd\rho g.\, h^4}{384E.\, bd^3/12} = \dfrac{5\rho g h^4}{32Ed^2}$



$$M_{max} + \frac{wh}{2}\left(\frac{h}{2} - \frac{h}{4}\right) = 0$$

$$\therefore M_{max} = -\frac{wh^2}{8}$$

$\dfrac{\sigma}{y} = \dfrac{M}{I}$ $\quad \therefore \sigma_{max} = \dfrac{(wh^2/8).(d/2)}{(bd^3/12)} = \dfrac{bd^2h^2\rho g/16}{bd^3/12}$

$$= \frac{3\rho g h^2}{4d}$$

(b) $b = 2m$, $h = 3m$

d : 5–20mm.

(b) Stiffness constraints: $d\_deflection = \left(\frac{5\rho g L^4}{32E\delta}\right)^{1/2}$

Strength constrains: $d_{strength} = \frac{3\rho g L^2}{4\sigma_{max}} = \frac{3\rho g L^2}{4(\sigma_f/2)} = \frac{3\rho g L^2}{2\sigma_f}$

	d_strength (mm)	d_deflection (mm)	min d (m)	*Pass or Reject for part b*	Mass (kg)	Total cost (£)	*Pass or Reject for part c*	Embodied energies (MJ)	Transportation energy (MJ)	Total energy (MJ)
Al foam	36.64	15.75		Reject						
Biocomposite	7.01	10.47	0.0105	Pass	33.39	26.71	Pass	233.73	31.39	265.12
Pine	1.98	6.10	0.0061	Pass	21.96	17.57	Pass	263.52	20.64	284.16
Rigid polymer foam	7.06	25.72	0.026	Reject						
Fibreboard	9.92	10.78	0.011	Pass	49.5	14.85	Reject (too heavy)			

From the table above, Al foam and rigid polymer foam do not meet the target range of thickness 5-20 mm.

(c) Fibreboard doesn't meet the constraint.

By evaluating the embodied energies, biocomposite performs better than pine.

(d) Considering the total energy, the ranking between biocomposite and pine doesn't change.

12 (**long**)

(a) In low cycle fatigue, high amplitude cyclic stresses ($\sigma_y < \sigma < \sigma_{ts}$) induce plastic deformation in a component. The fatigue life of the component is markedly shortened, $N_f < 10^4$ cycles. In high cycle fatigue, low amplitude cyclic stresses ($\sigma < \sigma_y$) cause elastic deformation in a component. Nonetheless though cracks develop and cause failure, just takes more cycles to do so $N_f > 10^4$ cycles.

The low cycle part of the stress life curve (Fig.1) can be expressed using Coffin-Manson's law. In this case, the number of cycles to failure correlates with plastic strain $\Delta\varepsilon^{pl}$ (= total strain – elastic strain (usually v. small) $\simeq$ total strain)

$$\Delta\varepsilon^{pl}\ N_f^{\beta} = C_2$$

where C_2 and β are constants.

The high cycle part of the stress life curve (Fig.1) can be expressed using Basquin's law

$\Delta\sigma\ N_f^{\alpha} = C_1$ where C_1 and α are constants.

"Endurance (or fatigue) limit" of a (nominally defect-free) material is the applied stress amplitude e, about zero mean stress, below which fracture does not occur at all, or occurs only after a very large number of cycles ($N_f > 10^7$) – see Fig.1.

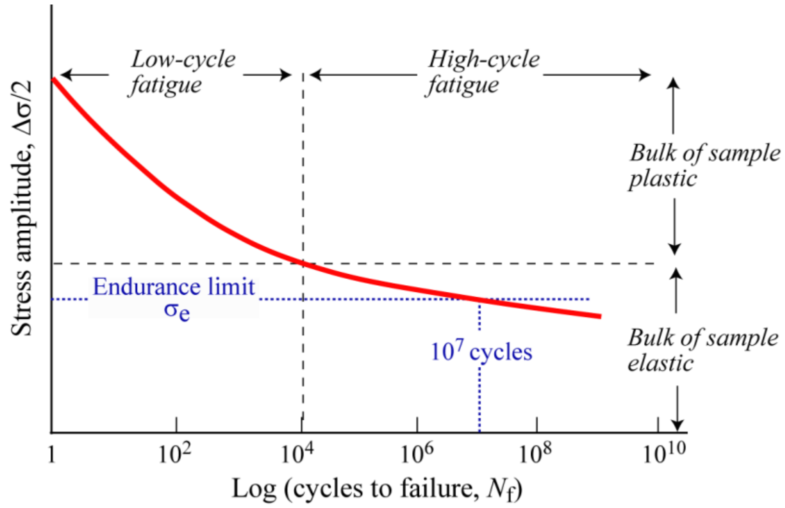
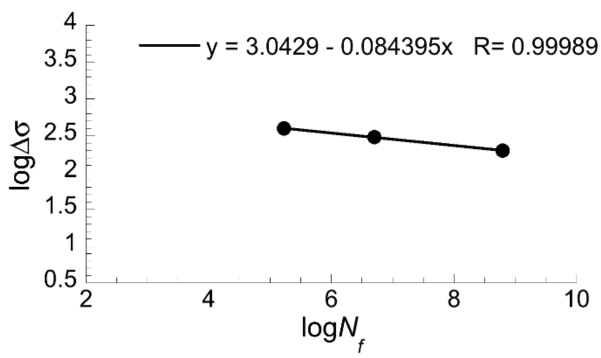


Fig.1

(b) Basquin's law $\Delta\sigma\ N_f^{\alpha} = C_1$

Taking the logs: $\log\Delta\sigma + \alpha\log N_f = \log C_1 \Rightarrow \log\Delta\sigma = -\alpha\log N_f + \log C_1$

$\Delta\sigma$	$\log\Delta\sigma$	N_f	$\log N_f$
200	2.30	6.15×10^8	8.79
300	2.48	5.06×10^6	6.70
400	2.60	1.68×10^5	5.23



Data straight line, hence Basquin's law is applicable.

$$\text{slope} = -\alpha = \frac{2.30-2.60}{8.79-5.23} = -0.0845 \Rightarrow \alpha = 0.0845$$

$$\log C_1 = \log\Delta\sigma + \alpha\log N_f = 2.60 + 5.23\alpha = 3.04 \Rightarrow C_1 = 1105$$

Or straight from the equation

$$(200)\left(6.15\times10^{8}\right)^{\alpha}=(400)\left(1.68\times10^{5}\right)^{\alpha}\Rightarrow\alpha=0.0845$$

$$C_1=(200)\left(6.15\times10^{8}\right)^{0.0845}=1105$$

For $\Delta\sigma = 350$ (mean stress is zero)

$$1105=350\ N_f^{0.0845}\Rightarrow N_f=3.16^{1/0.0845}$$

$$\therefore N_f\sim 8.1\times10^{5}\text{ cycles}$$

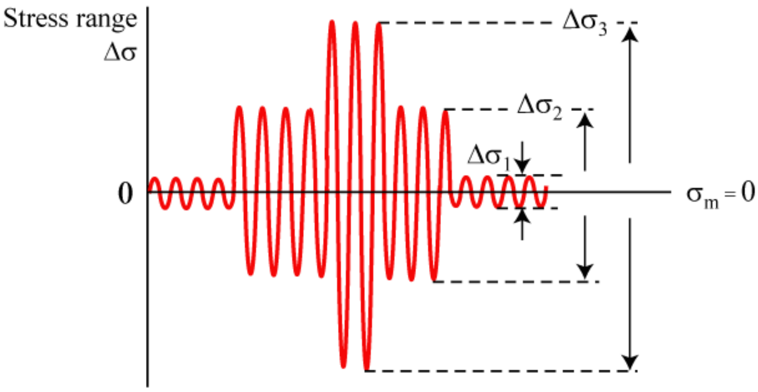
(c) (i) Goodman's empirical rule allows for the effect of mean stress on stress life data. If you have a stress range $\Delta\sigma_{(\sigma m)}$ under a non-zero mean stress σ_m, the equivalent stress range (giving the same N_f) for failure at a stress range $\Delta\sigma_o$ with a zero mean stress is given by

$$\Delta\sigma_o=\frac{\Delta\sigma_{(\sigma m)}}{\left(1-\frac{\sigma_m}{\sigma_{ts}}\right)}$$ where σ_{ts} is the tensile strength.

(ii) The fatigue life for each stress cycle will be calculated using Basquin or Coffin-Manson's laws and then use Miner's rule will be used to determine the life time. Miner's rule states that the specimen fails when the proportion of the life time used up by each block adds up to 1.

$$\sum_i\frac{N_i}{N_{fi}}=1$$

where N_i is the number of cycles corresponding to the ith block of constant stress range $\Delta\sigma_i$, and N_{fi} is the number of cycles to failure at that stress range.



(d) $\Delta\sigma_o=\frac{\Delta\sigma_{(\sigma m)}}{\left(1-\frac{\sigma_m}{\sigma_{ts}}\right)}$, σ_{ts} =600 MPa and Basquin's law $\Delta\sigma_o\ N_f^{0.0845}=1105$

σ_{max}	σ_{min}	$\Delta\sigma$	σ_m	σ_m/σ_{ts}	$\Delta\sigma_o$	N_f
300	250	50	275	0.458	92.3	5.81×10^{12}
300	200	100	250	0.417	171.4	3.81×10^{9}
300	150	150	225	0.375	240.0	7.10×10^{7}
300	100	200	200	0.333	300.0	5.06×10^{6}

Cycles per day

$$\frac{120000\ \text{m}}{\pi \cdot 0.64\ \text{m/cycle}} = 59683\ \text{cycles}$$

Miner's rule

$$\left(\frac{0.80}{5.81\times10^{12}} + \frac{0.12}{3.81\times10^{9}} + \frac{0.06}{7.10\times10^{7}} + \frac{0.02}{5.06\times10^{6}}\right) \cdot 59683 \cdot \text{life} = 1$$

$\Rightarrow$ life = 3471 days

Comments

Q7 In part (a), several candidates focused on the number of flaws and not on the critical flaw size. In part (b), a surprising large number of candidates made mistakes in the algebra. In addition, a few candidates replaced V_0 (volume of the test sample) with the cylinder volume. In (c), several candidates made numerical errors when calculating the survival probability and several candidates were confused as to why the survival probability increased with volume and attributed this to numerical errors.

Q8 In part (a), many candidates lost significant marks in the derivation of the differentiation equation for the spring-dashpot network. Many could not correctly define the strain, stress, and the rate of strain/ stress relationships for the parallel versus series connection of the network.

Q9 In part (a), most candidates focused on assumptions such the strong fibre/matrix bonding and perfect fibre alignment instead of the equal stress and equal strain assumptions. Parts (b) and (c) were generally well answered.

Q10 In part (a), marks were lost for failing to realise that the dislocation density associated with work hardening is additive to the baseline yield stress of fully annealed pure Cu. In part (b), many candidates lost marks because of numerical mistakes. In part (c), candidates who answered well parts (a) & (b) easily scored full marks here; whereas those who failed to fully answer (a) & (b) did not complete this part well.

Q11 This question was generally well answered. In part (a), the derivation of the bending moment was required in order to obtain full marks. In parts (b-d), missing a factor of ½ for the failure strength and calculation errors were the most common mistakes resulting in mark deduction.

Q12 Many complete or near-complete answers, but also many candidates run out of time. In part (a), marks were lost because of lack of details and inaccurate sketch of the cyclic stress amplitude against the fatigue life plot. Several candidates thought that the endurance limit refers to the number of cycles rather than the stress amplitude. In part (b), a significant number of candidates didn't plot the data using a suitable graph and several of them used $\Delta\sigma$ = 175 MPa instead of 350 MPa for estimating the fatigue life. Part (c) was answered well. In (d), a surprising high number of candidates made errors in calculating $\Delta\sigma_0$ and hence in estimating the number of cycles for the different loading regimes. Also, several candidates didn't estimate the Basquin's law constants and hence didn't calculate the number of cycles accurately. Those who roughly estimated the fatigue life using Table 2 received reduced marks