ENGINEERING TRIPOS PART IA 2014

Paper 3 Electrical and Information Engineering

Solutions

Section A : Prof. J. Robertson

Section B : Dr. I.C. Lestas

Section C : Prof. M.J. Kelly

SECTION A

1 (long)

Thevenin – replace any arbitrary linear circuit by a voltage source and an internal impedance, where the voltage source equals the open circuit voltage, and the internal impedance equals the open circuit voltage divided by the short circuit current.
Loop analysis is to work out voltages and currents around a network using a series of circulating currents, obeying nodes.

(b) Z1/Z2 = Z3/Z4 at balance, so no current through the meter.

Put Z3 on bottom line, to make formulae nicer.

Z2/Z1 = Z4/Z3

(jwL+R2)/R1 = R4/Z3

1/Z3 = 1/R3 + jwC !

(jwL/R1) + (R2/R1) = R4.(1/R3 + jwC) = (R4/R3) + jwCR4

Equate real and imaginary parts separately,

R2/R1 = R4/R3, or R1.R4 = R2. R3

jwL/R1 = jwCR4

w cancels ! L/C = R1.R4

(c)

Z2/Z1 = Z4/Z3 ensure Z3 on bottom line

R2/R1 = (R4 + 1/jwC).(1/R3 + jwC)

= (R4/R3) +1 + (1/jwCR3) + jwCR4

so, equate real and imag parts separately,

R2/R1 = (R4+ R3)/R3

1/jwCR3 = - jwCR4

$1 = w^2C^2R3R4$

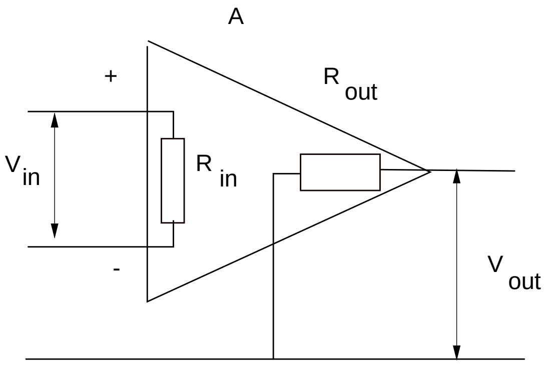
$$w = \frac{1}{C\sqrt{R3.R4}}$$

2 (long)

(a)
neg feedback has the advantages of

1. well defined gain, set by resistors.
2. extended frequency range
3. reduced distortion
4. independence from internal design of the op amp.
5. lowered output resistance (depending on design)
6. increased input resistance (depending on design)
7. opposite of these if you want current amplifier

(b)



(c)
The point here is not to give an involved formula for the gain, but an approximate numerical value.

Gain = (R1 + R2)/R2 = 10.

$A = 10^4$, $\beta = 1/10$, so $A\beta = 10^3$

$R_{out} = 0$ ohm.

For impedance at V2,

1/R = 1/75 + 1/150,000 ~ 1/75

R = 75 ohm.

Frequency response, $f = 1/2\pi CR_3 = 1/(2\pi.10^{-10}.75) = 10^8/(1.5x\pi) \sim 20$ MHz.

3 (short)

4/20 = 10/(R1 + 10)

so R1 = 40 kΩ

I_{ds} = 4, V across R2 = 10 V,

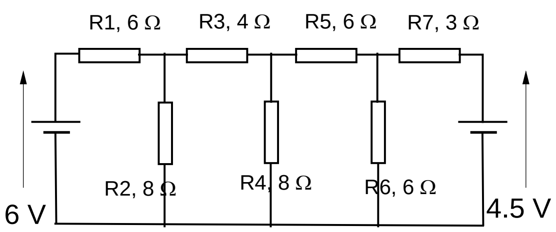
so R2 = 10/4 = 2.5 kΩ.

Gain needs R_out

1/R_out = 1 /2.5 + 1/30 = 13/30. R_out = 30/13 kΩ.

G = - 5.30/13 = -150/13 = - 11.53.

4 (short)



Want current thru 6V battery, so work from right.

Using Thevenin repeatedly, replace 4.5V and R6 // R7 with Ra and Va

1/Ra = 1/3 + 1/6 = 3/6, so Ra = 2.

Va = 6/(3 + 6). 4.5 = 3V.

R5’ = 6+2 = 8

Replace R5’ // R4 and 3 V with Rb and Vb

1/Rb = 1/8 + 1/8 = 2/8 Rb = 4

Vb = 8/(8+8) x 3 = 1.5V

R3’ = 4 + 4 = 8

Replace R3’ and R2 by Rc

1/Rc = 1/8 + 1/8 = ¼, Rc = 4

Vc = 0.5 x 1.5 = 0.75 V

Finally I = (6 – 0.75)/ (R1 + Rc) = 5.25 / 10 = 0.525 A

5 (short)

(a)
240 = I(R + jX), but power factor of 0.707 means X = R.

so, IR = 240 / (√2)

P = I^2 R = 24,000, so I = 100√2, R = 240/200 = 1.2ohm

P_line = I^2. 0.02 = **400 W.**

V = $(1.22^2 + 1.24^2)^{1/2}$. 100. √2 = 246V

Q_factory = 24,000. Q_line = 800. So Q_tot = 24,800 VAR

(b)
putting C across factory

Q_fac = V^2/X = 24x10^3 V = 240V, X = 1/jwC = 2.35 ohm

so C = Q / [2πfV^2] = 1.35x10^{-3} F.

Section B

6 (a) true for $n=1$

for $n=2$

A_1	A_2	$A_1 + A_2$	$\overline{\bar{A}_1 . \bar{A}_2}$
0	0	0	0
0	1	1	1
1	0	1	1
1	1	1	1

So true for $n=2$

Prove by induction that this is true for any n

Assume true for $n=k$

$$A_1 + A_2 + \dots + A_k = \overline{\bar{A}_1 . \bar{A}_2 \dots \bar{A}_k} \qquad (1)$$

Show that the expression is true for $n=k+1$

$$A_1 + A_2 + \dots + A_k + A_{k+1} = \overline{\bar{A}_1 . \bar{A}_2 \dots \bar{A}_k} + A_{k+1} \quad (\text{using } (1))$$

$$= \overline{\bar{A}_1 . \bar{A}_2 \dots \bar{A}_k \bar{A}_{k+1}}$$

(using expression for $n=2$)

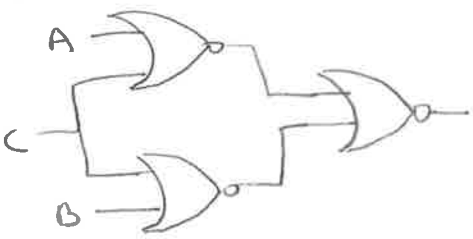
(b) Let $Y = A.B + C$

C \ AB	00	01	11	10
0	0	0	1	0
1	1	1	1	1

$$\bar{Y} = \bar{A}.\bar{C} + \bar{B}.\bar{C}$$

$$= \overline{A+C} + \overline{B+C}$$

$$\Rightarrow Y = \overline{\overline{A+C} + \overline{B+C}}$$



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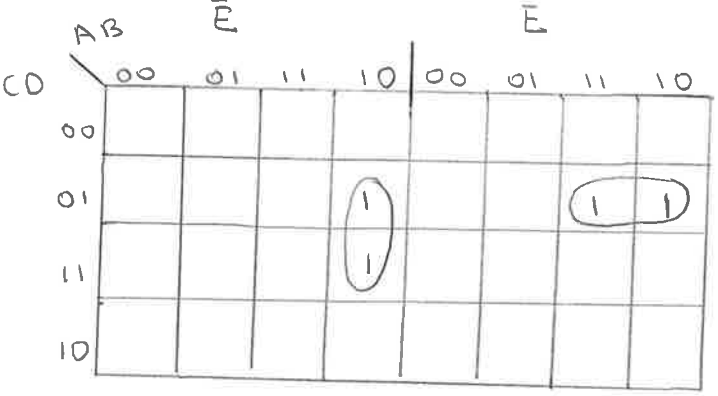
(a) (bookwork)

Static hazard - signal undergoes momentary change when it is supposed to remain unchanged

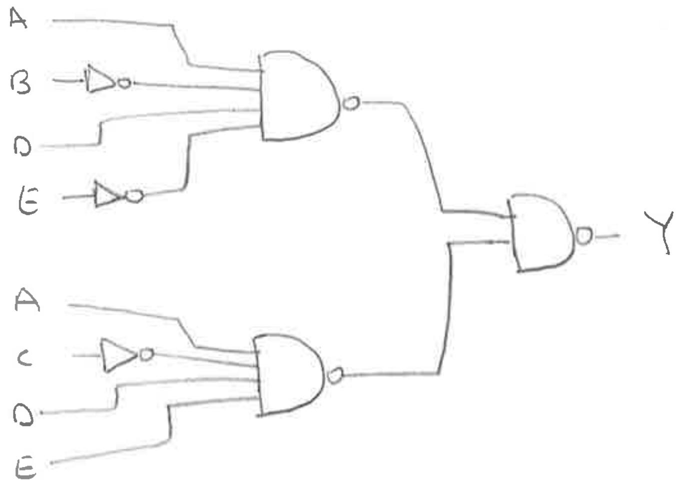
Dynamic hazard - signal changes more than once when it is supposed to change only once

(b)



$$Y = A.\bar{B}.D.\bar{E} + A.\bar{C}.D.E$$

$$= \overline{\overline{A.\bar{B}.D.\bar{E}} \,.\, \overline{A.\bar{C}.D.E}}$$



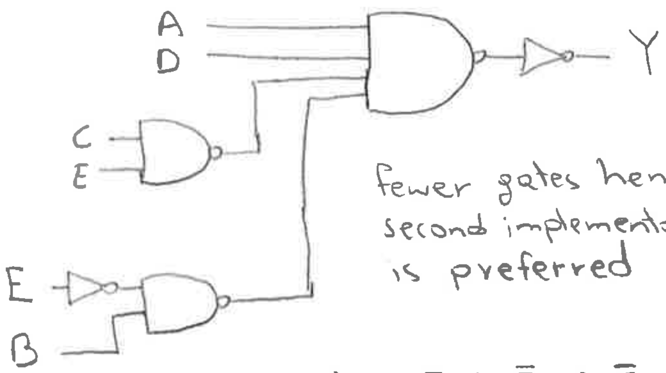
Mapping the inverse of the output

$$\bar{Y} = \bar{A} + \bar{D} + C.E + B.\bar{E}$$

$$= \overline{A.D} + C.E + B.\bar{E}$$

$$= \overline{A.D \,.\, \overline{C.E} \,.\, \overline{B.\bar{E}}}$$

So $Y = \overline{\overline{A.D.\overline{C.E}.\overline{B.\bar{E}}}}$



Fewer gates hence second implementation is preferred

(c) Add the additional term $A.\bar{B}.\bar{C}.D$ in the first expression, i.e. $Y = A.\bar{B}.D.\bar{E} + A.\bar{C}.D.E + A.\bar{B}.\bar{C}.D$

8 (a) a : number of address wires

d : number of data wires

Capacity in bits = $d \times 2^a$

$= 8 \times 2^{30} = 2^{33}$

1 Mbyte = 10^6 bytes (2^{20} was also perceived as acceptable)

So 8×2^{30} bits $= \frac{2^{30}}{10^6}$ Mbytes $\approx$ 1074 Mbytes

(↑ 8 = 1 byte)

(b) (i)

	movlw	0x02	load 2 into register W
	movwf	0x20	load W into memory location 0x20
lb	decfsz	0x20	decrement the contents of mem. location 0x20. Skip next line if 0
	goto lb		go to lb

Value 2 is loaded into 0x20 via first 2 lines. This is then decremented twice in the loop. After this answer is zero so final line is skipped.

(ii) Value 2 loaded into 0x20 via first two lines.
Next line decrements 0x20 (i.e. contents = 1)

goto lb

lb movwf 0x20

will load the contents of W (i.e. 2) into 0x20
Hence the loop will keep repeating itself without terminating.

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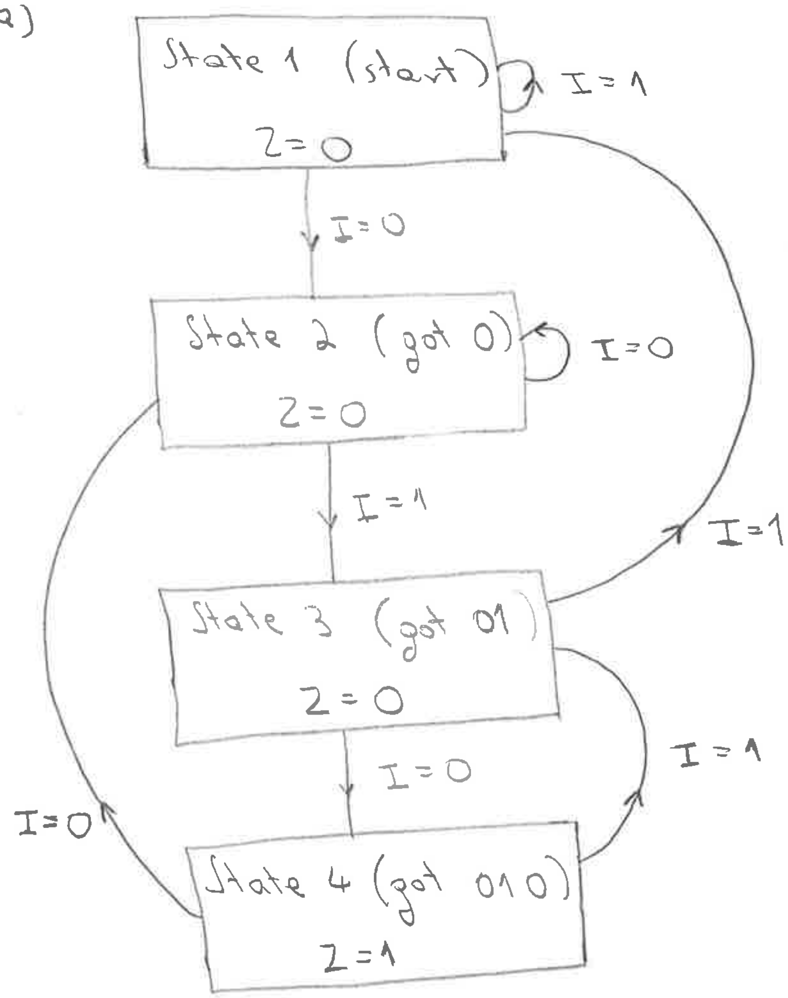
(a)



(b)

Input I	Current State Q_A	Current State Q_B	Next State Q_A	Next State Q_B	Bistable Inputs J_A	K_A	J_B	K_B
0	0	0	0	1	0	X	1	X
1	0	0	0	0	0	X	0	X
0	0	1	0	1	0	X	X	0
1	0	1	1	0	1	X	X	1
0	1	0	1	1	X	0	1	X
1	1	0	0	0	X	1	0	X
0	1	1	0	1	X	1	X	0
1	1	1	1	0	X	0	X	1

(c) J_A

I \ Q_AQ_B	00	01	11	10
0	0	0	X	X
1	0	1	X	X

$J_A = I.Q_B$

K_A

I \ Q_AQ_B	00	01	11	10
0	X	X	1	0
1	X	X	0	1

$K_A = \bar{I}.Q_B + \bar{Q}_B.I$

J_B

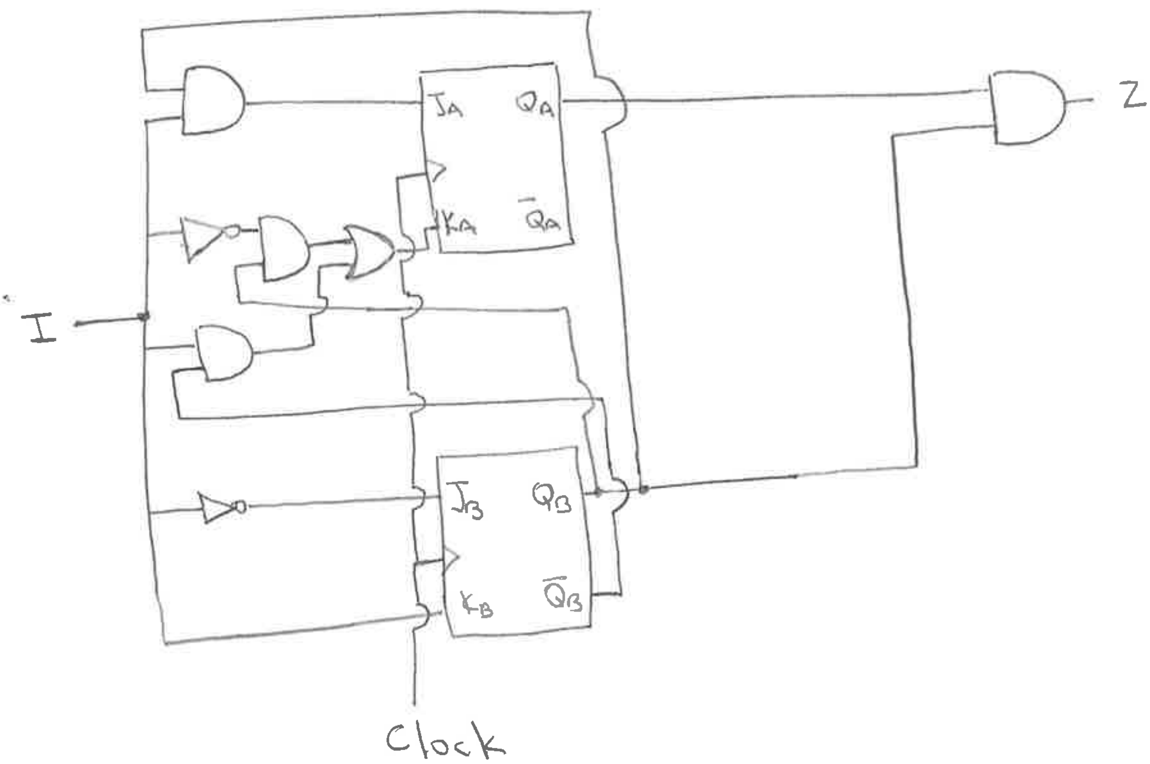
I \ Q_AQ_B	00	01	11	10
0	1	X	X	1
1	0	X	X	0

$J_B = \bar{I}$

K_B

I \ Q_AQ_B	00	01	11	10
0	X	0	0	X
1	X	1	1	X

$K_B = I$



(d) (i) 2 bistables

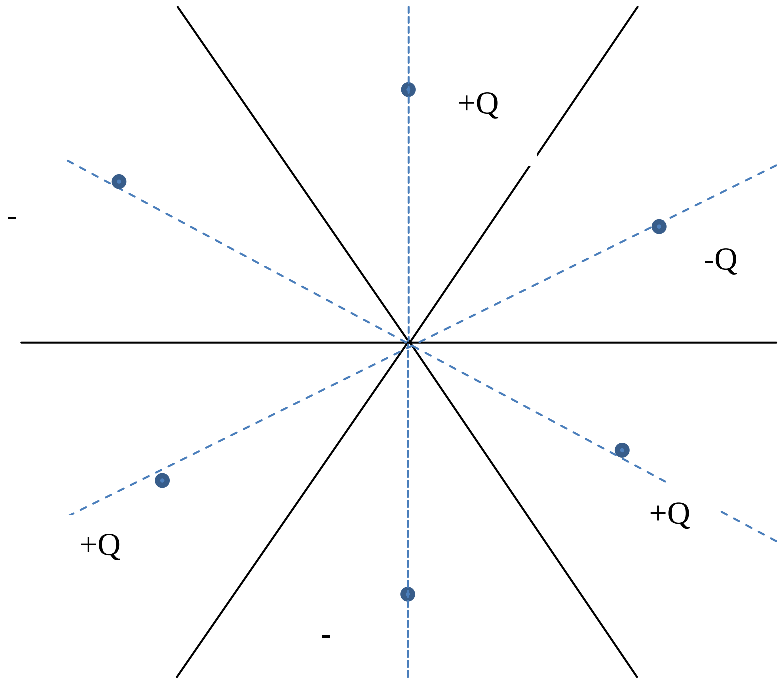
(ii) 2 bistables

SECTION C

10 (short)

(a) Method of images: given that the solution to Poisson's equation is unique, one can use fictitious charges (image changes) to calculate the electric potential and electric forces anywhere if the resulting potential is shown to match all the boundary conditions. For example, a metal sheet is an equipotential, and for a single charge in front of a metal sheet, the image charge of the opposite sign but equal magnitude at the mirror position with respect to the front edge of the sheet produces a zero equipotential at the sheet.

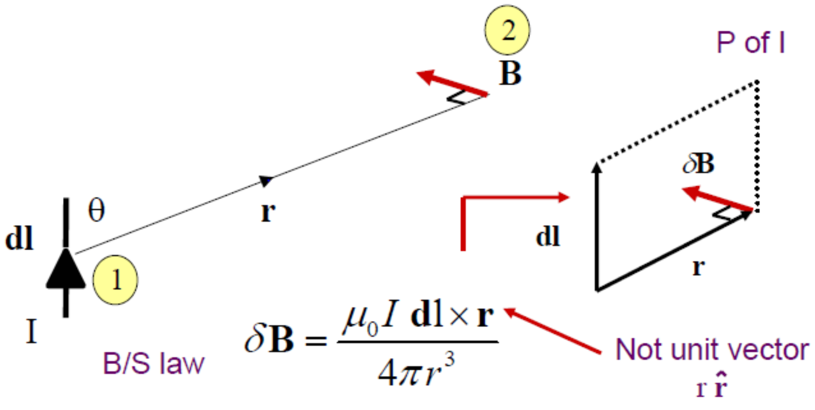
(b) Three sheets at 60 degrees, in pairs, the six charges ensure that the three lines are all equipotentials and so the six charges give the potential profile within the wedge.



(c) The charge will move towards the nearest point where the two sheets meet – i.e. vertically down in Figure 5.

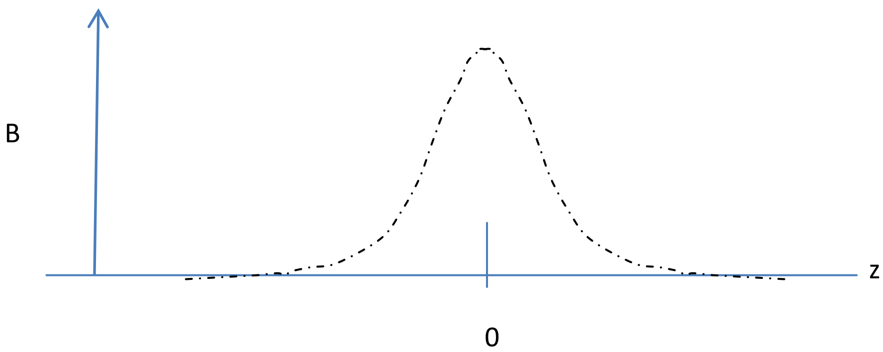
11 (short) Bio-Savart Law:

(a)



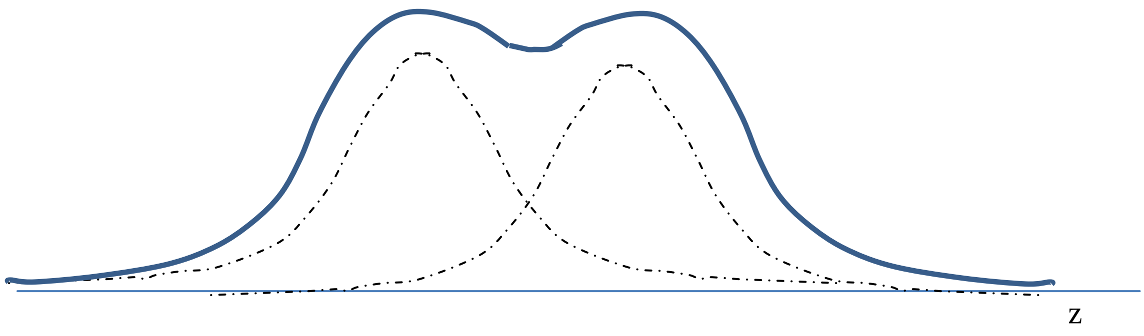
And: $B(z) = \mu_o I a^2 / 2[z^2+a^2]^{3/2}$

Sketch



Note that B(z) is even about z: if you make an error in calculating this result and get B(z) odd in z, the rest of the question makes no sense!

(b)



(c) The field is symmetrical about the central point, so all odd derivatives are zero, and since B'' is zero, this means that the field is very uniform: $B=B_o + O(z^4)$ near the central point. This is needed when measuring the magnetic properties of materials to ensure that the materials sample sees exactly the same field. It also means that the positioning of the sample is not critical.

12 (long)

(a) Suppose the aluminium bar moves down a distance *dx*.
If the force is *F* then the work done is $dW=Fdx$.
A volume of Adx is converted from the field $\mu_0 H$ in the vacuum to $1.000023\mu_0 H$ in aluminium.
The change in energy density is therefore $B^2/2(1.00023\mu_0) - B^2/2\mu_0$
or $-[2.3\ 10^{-5}]B^2/2\mu_0$.
Thus $Fdx = -[2.3\ 10^{-5}]\ AB^2dx/(2\mu_0)$.
There is an upward force of $[2.3\ 10^{-5}]10^{-4}\ (0.5)^2[2 \text{ X } 4\pi 10^{-7}] = 2.28\text{x}10^{-4}$ N.
[Note many candidates did not take off the energy that was already in the field without the Al being present and got a very large number!]

(b) The current though a resistor during a change of flux because a coil is removed from a magnetic field is given by $I = -[1/R]\ d\Phi/dt$
If we integrate this over time, the total charge is $Q=\Delta\Phi/R = AB/R$ Coulombs
For typical values: $AB= 10^{-4}\ 0.5/R = 5\text{x}10^{-3}/R = 0.05\mu C$ for $R=1k\Omega$
This is a very large charge. Store the change in a capacitor and measure the voltage. Electrolyse material or use a dividing electroscope.

(c) $\text{Emf} = -d\Phi/dt = (d/dt)(NAB \sin\omega t) = AB\omega = 10^{-4}\ 0.5\ 2\pi\ 60 = 0.018$V amplitude

(d) The emf is much easier to measure precisely, and the coil can be moved while rotating so the process is much more effective in mapping out the field. For the charge process, one has to be certain that B=0, or B is known precisely at the end point of the motion.