Engineering Tripos, Part IA, 2015

Paper 3 Electrical and Information Engineering

Solutions

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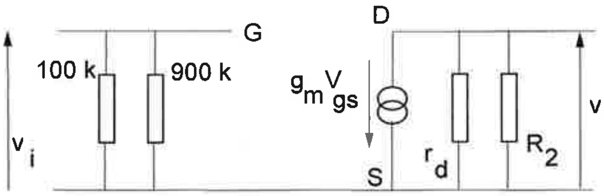
SECTION A

1 (long)

(a) The 900k and 100k act as voltage divider to set the gate voltage. Source-drain current through R1 sets the source voltage, and thus Vgs = Vg – Vs.

(b) Vg = 0.1* 20 = 2V. For operating point of Vgs= - 2 V, need Vs = 4 V. Also, Vds = 8 V means 20-8-4 = 8 V across R2. If Id = 4 mA, R1 =1 kΩ, R2 = 2 kΩ .

(c)



At mid-band capacitors are short circuits.

$$v_0 = -g_m v_{gs}\frac{r_d R_2}{r_d + R_2}, \quad v_i = v_{gs}$$

Hence, gain $\frac{v_o}{v_i} = -g_m \frac{r_d R_2}{r_d + R_2}$

Or gain = -5 x 10 x 2 /(10+2) = -8.33

Output impedance = $R_2 // r_d$ = 1.67 kΩ.

(d) R' = 1.67 + 5 = 6.67

f = 1/2πCR', C = 1/(2 x 3.14 x 6.67.10^3 x 10) = 2.39 x10^{-6} F

(e) V_{ds} =12 V allows +ve swing of 12-20 V = 8, and –ve swing of 12-4 = 8 V, symmetric.

2 (long)

(a) gain = R2/R1 = 300. So R2 = 3 MΩ .

(b) point v_- is virtual earth, so R_{in} = 10 kΩ.

(c) f = 1/(2πR2×C1), so C1 = 1/(2π×3×10^6×10^4) = 5.3 pF.

(d) G=A/(1+A×b), where b= R1/(R1+R2)
So G=2×10^4/(1+2×10^4/300)=296 .

3 (short)

Lump the 6.37 mH inductor and the 2 Ω resistor into a single impedance, Z_1.

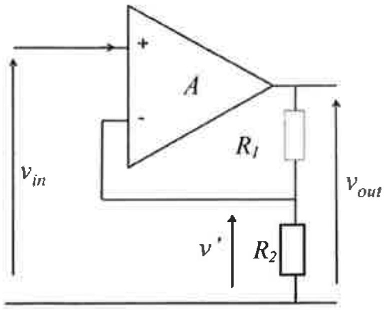
$Z_1 = (2 + j\omega L)\ \Omega = 2 + j\times 2\pi\times 50\times 6.37\times 10^{-3}\ \Omega = 2 + 2j\ \Omega$

The second set of impedances, the 4 Ω resistor and the 796 μF capacitor are in parallel, and can be combined into a single impedance, Z_2.

$1/Z_2 = (1/4 + j\omega C)\ \Omega \Rightarrow Z_2 = 2\text{-}2j\ \Omega$
Therefore, the total impedance, $Z_{tot} = Z_1 + Z_2 = 4\ \Omega$
The Input current is the input voltage divided by the circuit impedance,
i.e. I= V/ Z_{tot} = 60 A .

4 (short)

The circuit we are dealing with is as follows:



Now, $v_{out} = A(v_{in} - v')$.

We can treat the resistors R_1 & R_2 as a potential divider, as no current flows through the op-amp input (it is ideal with an infinite input resistance).

=> $v' = \frac{R_2}{R_1 + R_2} \times v_{out}$

Using the expression for v_{out} from above, we find that:

$$v_{out} = A(v_{in} - \frac{R_2}{R_1 + R_2} \times v_{out})$$

$$= \frac{A((R_1 + R_2) \times v_{in} - R_2 \times v_{out})}{R_1 + R_2}$$

=> $v_{out} \times (R_1 + R_2) = Av_{in} \times (R_1 + R_2)$ - $Av_{out}R_2$

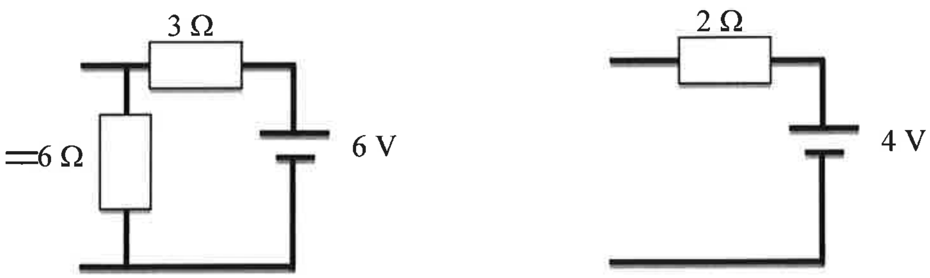
$v_{out} \times (AR_2 + R_1 + R_2) = Av_{in} \times (R_1 + R_2)$

=> the voltage gain, $\frac{v_{out}}{v_{in}} = \frac{A(R_1 + R_2)}{AR_2 + R_1 + R_2}$

As A -> ∞, then the gain tends towards $\frac{A(R_1 + R_2)}{AR_2} = 1 + \frac{R_1}{R_2}$

5 (short)

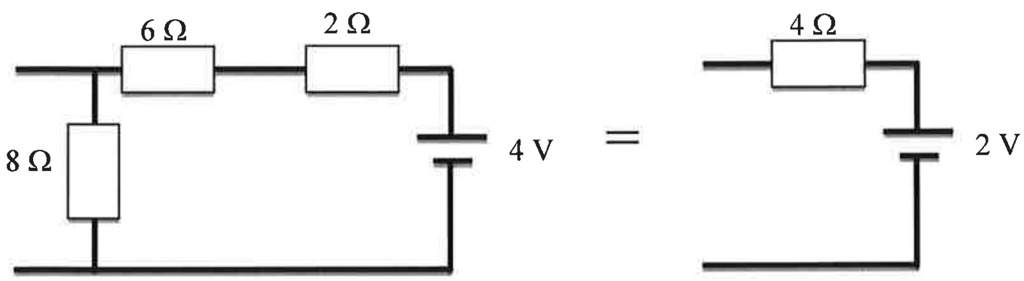
Starting at the right hand side, convert the first loop into its Thévenin equivalent:



(The open circuit voltage from the circuit on the left $= 6 \times \frac{6}{6+3} = 4\,V$

The equivalent resistance is the open circuit voltage divided by the short-circuit current.
The short-circuit current = 6V/3Ω = 2 A => resistance = 4V/2A = 2 Ω.

The next part of the circuit then becomes:



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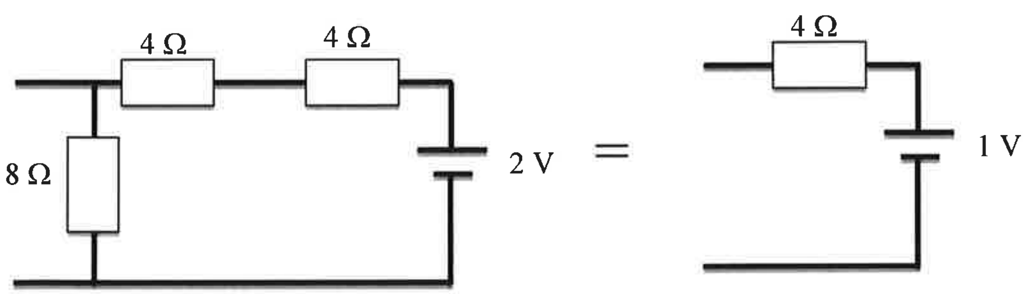
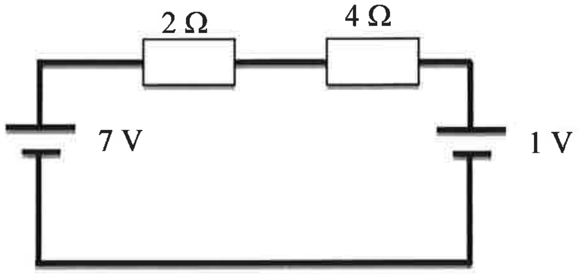


Finally, adding in the last bit:



The current flowing in the circuit is the total potential difference divided by the total resistance around the circuit

$= \frac{7\ V - 1\ V}{2\Omega + 4\Omega} = 1$ A.

Section B

6 (short)

(a) 2's complement is a coding of a binary number that allows the representation of negative numbers

For an 8 bit word

$0_H \rightarrow 7F_H$ represent $0 \rightarrow 127$ (stay the same in 2s complement form)

$80_H \rightarrow FF_H$ represent $-128 \rightarrow -1$
(are converted to 2s complement form by inverting the number and adding 1)

Significance: normal binary arithmetic can be used (no need for separate circuits to add negative numbers)

(b)

	X			−X			
X(decimal)	Y_3	Y_2	Y_1	Z_4 Z_3		Z_2	Z_1
0	0	0	0	0	0	0	0
1	0	0	1	1	1	1	1
2	0	1	0	1	1	1	0
3	0	1	1	1	1	0	1
−4	1	0	0	0	1	0	0 ←
−3	1	0	1	0	0	1	1
−2	1	1	0	0	0	1	0
−1	1	1	1	0	0	0	1

Answers that perceived the negation of −4 as an invalid operation, and thus used only three bits, were also marked as correct

Z_1 (Y_2 over the middle two columns, Y_1 under the middle two columns, Y_3 on the first row)

0	1	1	0
0	1	1	0

$Z_1 = Y_1$

Z_2 (Y_2 over the first two columns, Y_1 under the middle two columns, Y_3 on the first row)

1	0	1	0
1	0	1	0

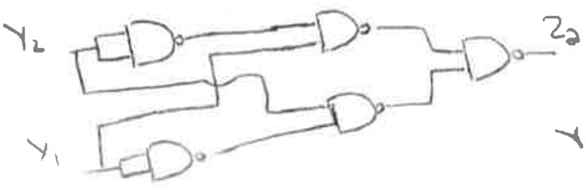
$Z_2 = Y_2\bar{Y}_1 + \bar{Y}_2 Y_1$

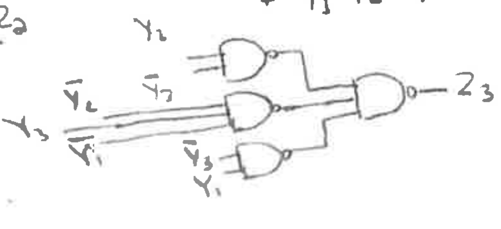
Z_3 (Y_2 over the first two columns, Y_1 under the middle two columns, Y_3 on the first row)

0	0	0	1
1	1	1	0

$Z_3 = Y_2\bar{Y}_3 + Y_1\bar{Y}_3 + Y_3\bar{Y}_2\bar{Y}_1$

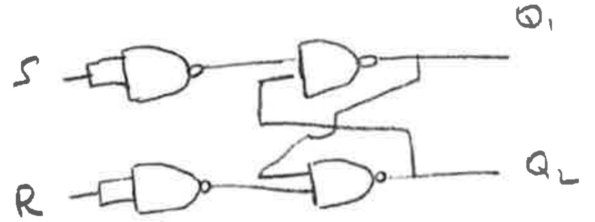
$Z_4 = Y_2\bar{Y}_3 + Y_1\bar{Y}_3$





7 (short)

(a)



$Q_1 = \overline{\bar{S}\, Q_2} = S + \overline{Q_2}$

$Q_2 = \overline{\bar{R}\, Q_1} = R + \overline{Q_1}$

(b) - master-slave configuration with clock (clock must switch from low to high for input to affect output)
This avoids propagation of hazards

- $S = R = 1$ forbidden state in SR bistable
$J = K = 1$ toggles the output in JK bistable (additional functionality)
- presence of $\overline{\text{preset}}$ and $\overline{\text{clear}}$:
output can be changed irrespective of inputs

(c) $S = R = 1 \Rightarrow Q_1 = Q_2 = 1$ (irrespective of the initial values of Q_1, Q_2)

$S = R = 0$, $Q_1 = Q_2 = 1 \Rightarrow Q_1 = Q_2 = 0$

$S = R = 0$, $Q_1 = Q_2 = 0 \Rightarrow Q_1 = Q_2 = 1$

Hence when $S = R = 0$ and $Q_1 = Q_2$ the Q_1, Q_2 oscillate between 1 and 0

In practice one of Q_1, Q_2 will switch first

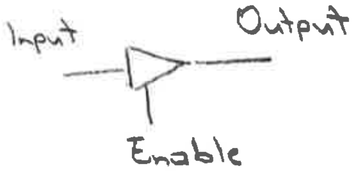
$S = R = 0$, $Q_1 = 1$, $Q_2 = 0 \Rightarrow Q_1 = 1$, $Q_2 = 0$

$S = R = 0$, $Q_1 = 0$, $Q_2 = 1 \Rightarrow Q_2 = 1$, $Q_1 = 0$

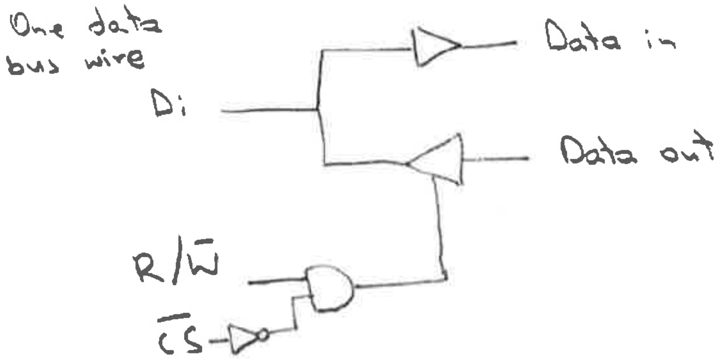
and the bistable stays in either of the two states without changing.

8 (short)

(a) Tristate buffer



E	I	Output
1	0	0
1	1	1
0	0	floating
0	1	floating



When $\overline{CS}$ is low and R/$\overline{W}$ is low the memory writes the data on the data bus

(b)

	movlw	0x AA	load 170 into W	(1 cycle)
	movwf	0x35	load W into 0x35	(1 ~)
	decf	0x35	decrement 0x35	(1 ~)
label	dec fsz	0x 35	decrement 0x 35 skip if zero	(1~ for first 168, 2~ for final)
	goto label		jump to label	(2~)

Run time : $1+1+1+168(1+2)+2=509$

Second case

Run time : $1+1+(170/2-1)(1+1+2)+(1+2)=341$

9 (long)

(a)

X		Y		P			
X_2	X_1	Y_2	Y_1	P_4	P_3	P_2	P_1
0	0	1	0	0	0	1	0
		1	1	0	0	1	1
0	1	1	0	0	0	1	1
		1	1	0	1	0	0
1	0	1	0	0	1	1	0
		1	1	0	1	1	1
1	1	1	0	1	0	1	1
		1	1	1	1	0	0

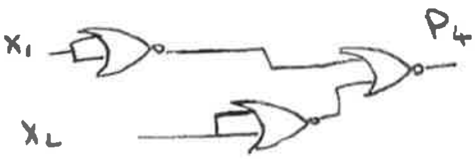
P_4 (X_1 over columns 1–2, Y_1 beside the rows, X_2 under columns 2–3)

0	1	0	0
0	1	0	0

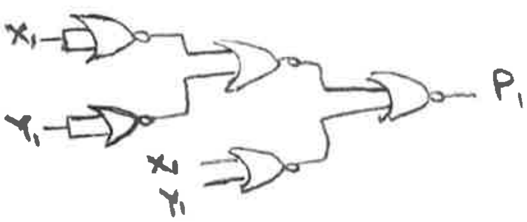
P_1

0	0	1	1
1	1	0	0

$\overline{P_4} = \overline{X_1} + \overline{X_2}$

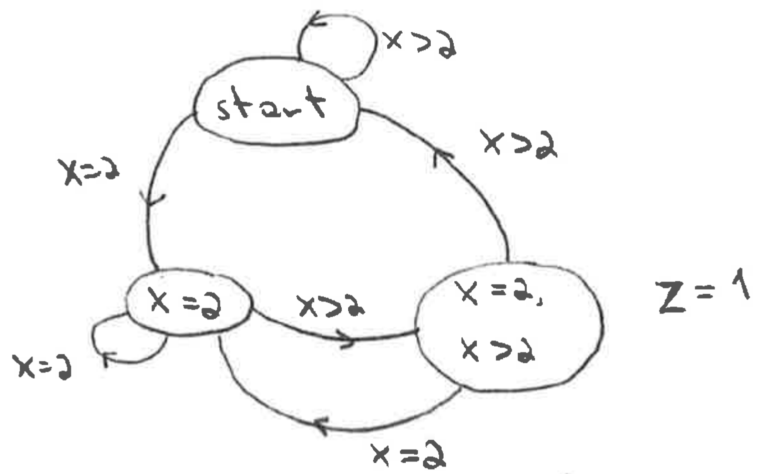


$\overline{P_1} = X_1 Y_1 + \overline{Y_1}\,\overline{X_1} = \overline{\overline{X_1} + \overline{Y_1}} + \overline{X_1 + Y_1}$



(b)

states	bistable allocation	
	Q_A	Q_B
start	0	0
$X=2$	0	1
$X=2, X>2$	1	0



$X>2$ $(X_1 = 1)$

$X=2$ $(X_1 = 0)$

Current state Q_A	Current state Q_B	Input X_1	Next State Q_A	Next State Q_B	J_A	K_A	J_B	K_B
0	0	0	0	1	0	X	1	X
0	0	1	0	0	0	X	0	X
0	1	0	0	1	0	X	X	0
0	1	1	1	0	1	X	X	1
1	0	0	0	1	X	1	1	X
1	0	1	0	0	X	1	0	X

J_A (columns: Q_A over the first two, Q_B under the middle two; rows: X_1 on the first)

X_1	X	X	1	0
	X	X	0	0

$J_A = X_1 Q_B$

J_B (columns: Q_A over the first two, Q_B under the middle two; rows: X_1 on the first)

X_1	0	X	X	0
	1	X	1	1

$J_B = \overline{X}_1$

SECTION C

10 (short)

(a) $C=\varepsilon_r\varepsilon_0 A/d$ is book-work, and straight out of the lecture notes.

(b) If the electric field is E in the dielectric, then the force on the charge Q is the lower electrode when attached to the dielectric is $F=(1/2)V^2 dC/dx|_{x=d}$ $dC/Dx=eA/d^2$

Weight of plate $=\rho gA < (1/2)\, V^2\, \varepsilon_r\varepsilon_0\, A/d^2$ So $V^2> 2\rho gd^2/\varepsilon_r\varepsilon_0$

i.e. $V > d\,[2\rho g/\varepsilon_0]^{1/2}$

11 (long)

(a) The only thing that matters is the number of ampere turns, by Ampere's law, as the field lines are strongly confined inside the material, which has a very high permeability.

(b) From NI=HL we have H=1000A m^{-1} at B=1T, and so [Note: some used

NI= 1000 x 0.6 = 600 A turns.

Note this could be 100 turns with 6A or 1000 turns at 0.6A.

[Note: some used H=1100 A m-1 or 1200 A m-1, within accuracy of drawing: no marks deducted for this, and the other numbers changed appropriately.]

(c) With the gap, we now have $NI= H_iL + H_g\, g$

or $NI = H_iL + Bg/\mu 0 = 1000 \times 0.598 + 1 \times 0.002 / 4\pi \times 10^{-7} = 2188$ A turns.

The very high permeability of the steel means a small H in the steel, but a very large H in the air-gap.

(d) In opening up the gap an extra energy is required being the volume of the gap times the change in energy density in the iron and in the gap.

$\Delta E = A\, g\, [B^2/2\mu_0 - B^2/2\mu_0\mu r] = Fg$ the work force by the force at the pole pieces.

$F = 10^{-3}\,(0.5)\,(1\text{-}1/795]/[4\pi x\ 10^{-7}] = 398$ N.

Note one can do this because the B-H curve is still essentially linear in the regimes we are working in.

(e) One uses a strengthened steel rather than soft cast steel to support the gap, or a frame to support the magnet of the material is too soft.

12 (short)

(a) $B=\mu_0 I/2\pi a$ by bookwork

(b) The magnitude of the magnetic field a distance d from one wire is $B_d=\mu_0 I/2\pi d$, and the direction of the field is given by the right hand rule with respect to the direction of the current and the shortest distance to the wire.

This for the four points asked for the magnetic field is:

(d,d,d)	$\mathbf{B}= B_d\,(0, 0, 0)$	Magnitude: 0
(-d,-d,-d)	$\mathbf{B}= B_d\,(0, 0, 0)$	Magnitude: 0
(d,d,0)	$\mathbf{B}= B_d\,(-1/\sqrt{2}, 1/\sqrt{2}, 0)$	Magnitude: $B_d/\sqrt{2}$
(-d,-d,0)	$\mathbf{B}= B_d\,(1/\sqrt{2}, -1/\sqrt{2}, 0)$	Magnitude: $B_d/\sqrt{2}$