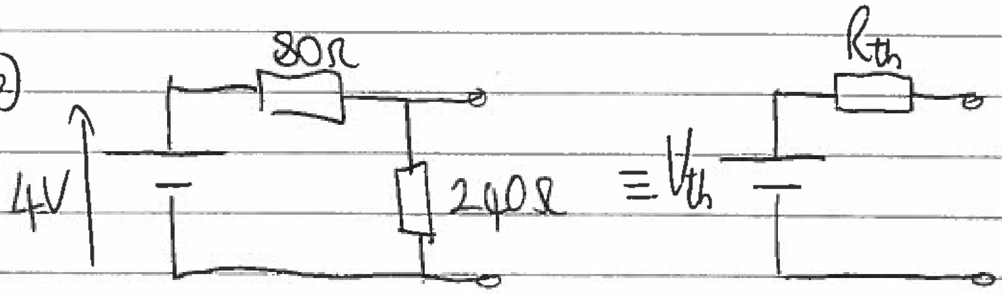
IA Paper 3 2016

1. a) Thevenin's theorem: as far as the load is concerned, a linear circuit may be represented by an ideal voltage source (V_{th}) in series with a resistor (R_{th}) in which $V_{th} = V_{oc}$, $R_{th} = V_{oc}/I_{sc}$

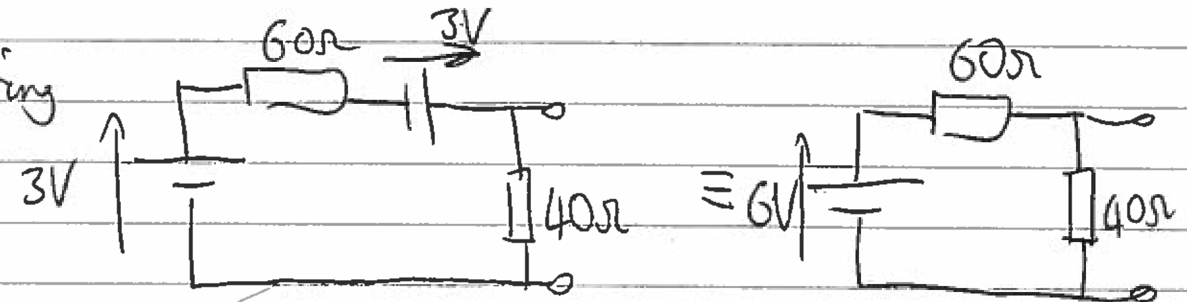
Norton's Theorem: for circuit above, as far as the load is concerned, it can be represented as a current source (I_N) in parallel with a resistor R_N in which $I_N = I_{sc}$, $R_N = R_{th}$. [3]

b)

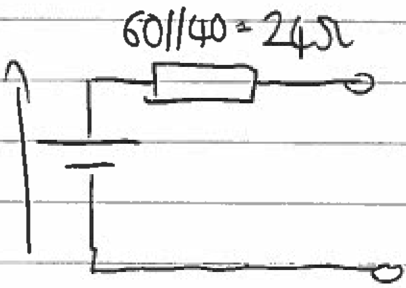


$V_{th} = \frac{240}{240+80} \times 4 = 3V$, $R_{th} = 80\Omega // 240\Omega = 60\Omega$

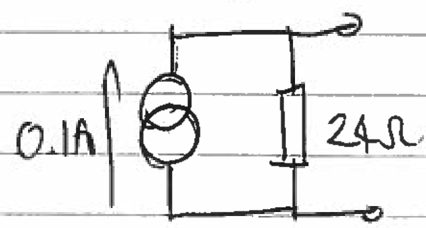
giving



$\equiv \frac{6 \times 40}{40+60} = 2.4V$



$I_{sc} = \frac{2.4}{24} = 0.1A = I_N$, $R_N = R_{th}$

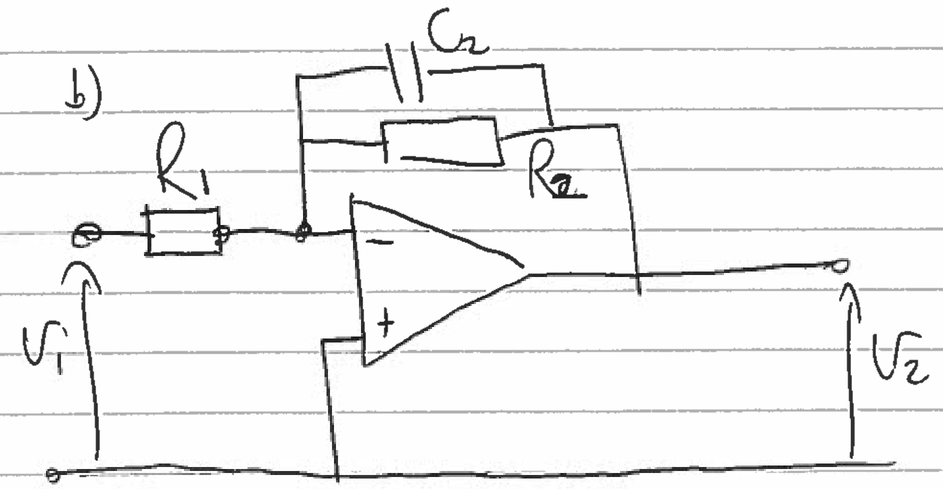


[7]

a) Ideal op-amp assumed to have infinite open loop gain and input resistance, and zero output resistance.

[3]

b)



At low frequency such that $1/\omega C_2 \gg R_2$:

$$\frac{V_2}{R_2} + \frac{V_1}{R_1} = 0 \qquad \frac{V_2}{V_1} = -\frac{R_2}{R_1}$$

Increasing frequency will decrease gain as capacitor impedance falls

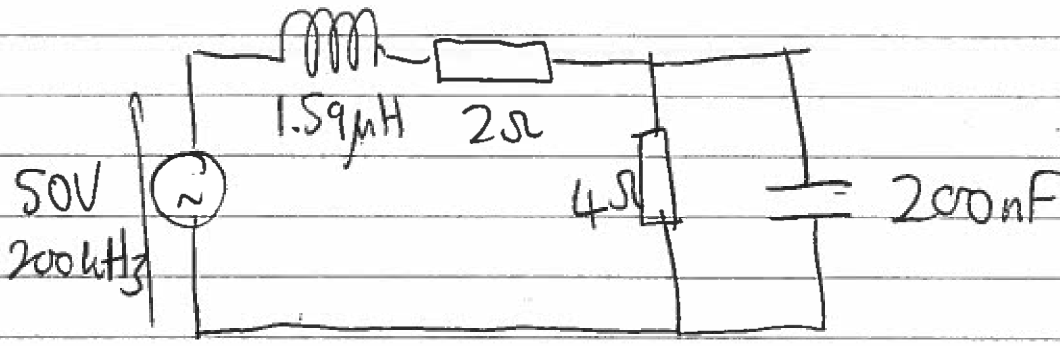
$$R_2 // C_2 = \frac{R_2 \times 1/j\omega C_2}{R_2 + 1/j\omega C_2} = \frac{R_2}{1+j\omega C_2 R_2}$$

so $$\frac{V_2}{V_1} = \frac{-R_2}{R_1(1+j\omega C_2 R_2)}$$

−3dB frequency when real & imaginary parts of denominator are equal

$$\therefore \omega_{-3dB} C_2 R_2 = 1 \rightarrow \omega_{-3dB} = \frac{1}{R_2 C_2}.$$

3.



$$\bar{Z}_L = j\omega L = j2\pi \times 200 \times 10^3 \times 1.59 \times 10^{-6} = j2\,\Omega$$

$$\bar{Z}_C = \frac{1}{j\omega C} = -j \times \frac{1}{2\pi \times 200 \times 10^3 \times 200 \times 10^{-9}} = -j4\,\Omega$$

$$4\Omega \,//\, -j4\Omega = \frac{4 \times (-j4)}{4 - j4} = \frac{-j4}{1-j} = \frac{-j4(1+j)}{2}$$

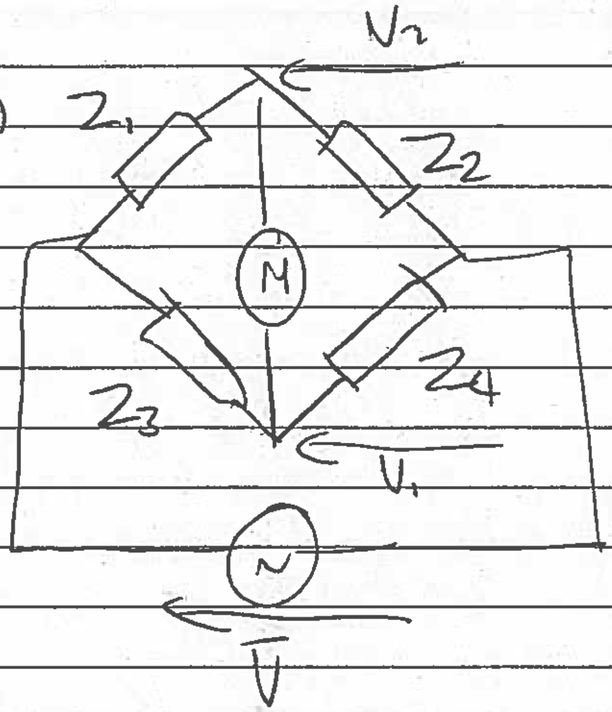
$$= (2 - j2)\,\Omega$$

$$\therefore \bar{Z} = j2 + 2 + 2 - j2 = 4\,\Omega$$

$$I = \frac{50}{4} = 12.5\text{A}\angle 0^\circ$$

4/

a)



$V_1 = V_2$ for balance $\Rightarrow \dfrac{Z_4}{Z_4+Z_3} = \dfrac{Z_2}{Z_2+Z_1}$

$$Z_4(Z_2+Z_1) = Z_2(Z_4+Z_3)$$

$$\underline{Z_1 Z_4 = Z_2 Z_3}$$

b) $Z_1 = R_1$, $Z_2 = R_2$, $Z_3 = \dfrac{R_3 \times 1/j\omega C}{R_3 + 1/j\omega C}$

$$= \frac{R_3}{1+j\omega C R_3}$$

$$Z_4 = R_4 + \frac{1}{j\omega C} = \frac{1+j\omega C R_4}{j\omega C}$$

Balance: $R_1 Z_4 = R_2 Z_3$

$$\Rightarrow R_1\left(\frac{1+j\omega C R_4}{j\omega C}\right) = \frac{R_2 R_3}{1+j\omega C R_3}$$

$$R_1(1+j\omega C R_4)(1+j\omega C R_3) = j\omega C R_2 R_3$$

$$R_1\left[(1-\omega^2 C^2 R_3 R_4) + j\omega C(R_3+R_1)\right] = j\omega C R_2 R_3$$

equate real parts $\quad R_1(1-\omega^2 C^2 R_3 R_4) = 0$

$$\omega = \frac{1}{C}\sqrt{\frac{1}{R_3 R_4}}$$

equate imaginary parts

$$R_1(R_3+R_4) = R_2 R_3$$

$$\frac{R_2}{R_1} = 1 + \frac{R_4}{R_3}$$

c) $\quad Z_1 = R_1 + \frac{1}{j\omega C_1} \qquad Z_2 = \frac{1}{j\omega C_2}$

$\quad Z_3 = R_3 \qquad Z_4 = \frac{R_4}{1+j\omega C_4 R_4}$

$$\frac{(1+j\omega C_1 R_1)}{j\omega C_1} \times \frac{R_4}{1+j\omega C_4 R_4} = \frac{R_3}{j\omega C_2}$$

$$(1+j\omega C_1 R_1) R_4 C_2 = R_3 C_1 (1+j\omega C_4 R_4)$$

real parts: $\quad R_4 C_2 = R_3 C_1$

imag parts: $\quad C_1 R_1 R_4 C_2 = R_3 C_1 C_4 R_4$

or $\quad R_1 C_2 = R_3 C_4$

$$\boxed{\begin{aligned} R_4 C_2 &= R_3 C_1 \\ R_1 C_2 &= R_3 C_4 \end{aligned}}$$ independent of frequency

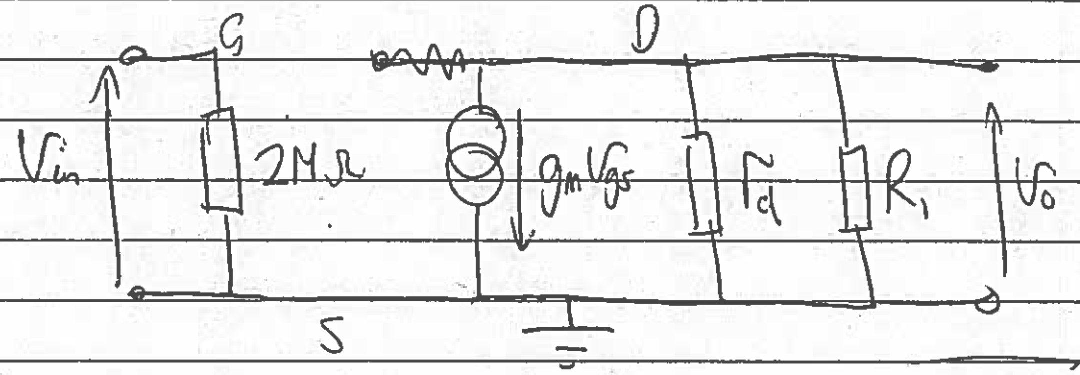
5/a) $R_2 I_D = +2V$ (2MΩ holds $V_G = 0V$)

$R_2 = \frac{+2V}{0.2mA} = \underline{10k\Omega}$

$V_{DS} = 8V$ so $V_{D\to 0} = 2 + 8 = 10V$ so $\frac{10}{R_1} = I_D$

$R_1 = \frac{10}{0.2mA} = \underline{50k\Omega}$ $\boxed{10}$

b)



$\boxed{5}$

c) $V_{gs} = V_{in}$

$V_o = -g_m V_{gs}\, r_d // R_1$ so $\frac{V_o}{V_{in}} = -g_m \frac{r_d R_1}{r_d + R_1}$

$R_{out} = \frac{V_o}{i_o}$ with $V_{in} = 0$ giving $r_d // R_1$

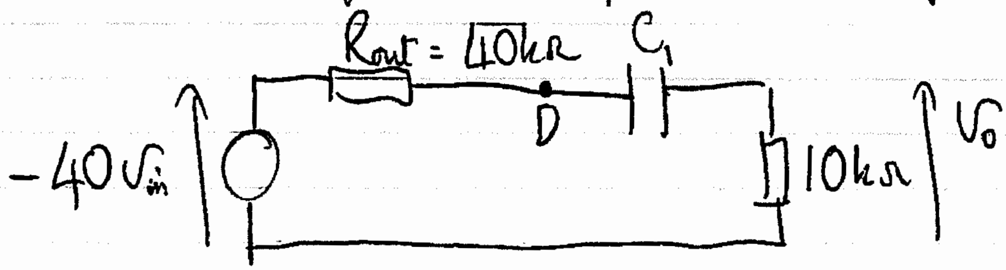
$= \frac{R_1 r_d}{R_1 + r_d}$

$r_d = 200k\Omega$, $R_1 = 50k\Omega$ so $r_d // R_1 = 40k\Omega$

Gain $= -1\times10^{-3} \times 40\times10^{3} = \underline{-40}$

$R_{out} = \underline{40k\Omega}$

d) Output of cct can be put into Thevenin form:-



$$V_0 = -40V_{in} \times \frac{10k\Omega}{40k\Omega + 10k\Omega + 1/j\omega C_1}$$

so $-3dB$ frequency given by

$$50k\Omega = \frac{1}{2\pi \times 15 \times C_1} \rightarrow C_1 = \underline{212\,nF}$$

e) $1\,kHz \gg 3dB$ frequency, so C_1 can be treated as a short-circuit.

$$\therefore V_0 = \frac{10}{10+40} \times 40 \times 0.1 = \underline{0.8V} \quad (0.4V\,peak)$$

Since the capacitor C_1 gets rid of any d.c. voltage across the load, the maximum voltage with be ~~0.8V~~ 0.4V, minimum ~~-0.8V~~ -0.4V.

The small-signal voltage at the drain will also be ~~0.8V~~, ±0.4V but there is a ~~bias~~ bias voltage of 10V there. (DC)

$\therefore$ ~~Peak~~ Maximum drain voltage = ~~10.8V~~ 10.4V, minimum = ~~9.2V~~ 9.6V

Section B Digital circuits June 2016 Dr TJ Flack

6. a) If the LHS and RHS of the expressions are equal for all 4 combinations of inputs A and B then the theorems are proved.
(Short)

A	B	$A.B$	$\overline{A.B}$	$\overline{A}$	$\overline{B}$	$\overline{A}+\overline{B}$
0	0	0	1	1	1	1
0	1	0	1	1	0	1
1	0	0	1	0	1	1
1	1	1	0	0	0	0

Columns $\overline{A.B}$ and $\overline{A}+\overline{B}$ are the same so $\overline{A.B} = \overline{A}+\overline{B}$

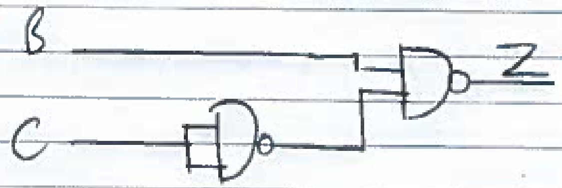
A	B	$A+B$	$\overline{A+B}$	$\overline{A}$	$\overline{B}$	$\overline{A}.\overline{B}$
0	0	0	1	1	1	1
0	1	1	0	1	0	0
1	0	1	0	0	1	0
1	1	1	0	0	0	0

Columns $\overline{A+B}$ and $\overline{A}.\overline{B}$ are the same so $\overline{A+B} = \overline{A}.\overline{B}$

b) $Z = B.C + A.\overline{B} + \overline{A}.\overline{B}$

C \ AB	00	01	11	10
0	1	0	0	1
1	1	1	1	1

$\overline{Z} = B.\overline{C}$ ∴ $Z = \overline{B.\overline{C}}$



7 (short) a) 8 data lines $\Rightarrow$ data stored in bytes

$\therefore$ Nº bytes = 2^{13} = 8192 bytes = 8 kbytes

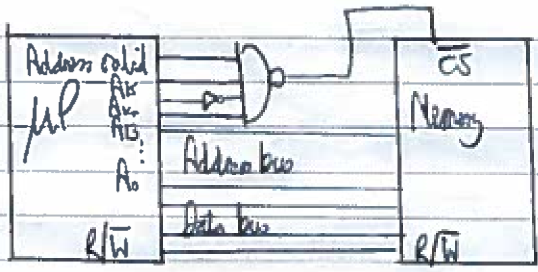
A 32 bit processor can address 2^{32} bytes = 4 Gbytes.

Point of interest: First PCs were 16 bit and could address 2^{16} = 64 kbytes of memory. When 32 bit computers came out it was believed that it would never be possible to connect 4 Gbytes of memory (you can now buy a 64 Gbyte memory stick for ~ £15!) A CD holds ~ 1 Gbyte, a DVD ~ 4 Gbytes, Blu-ray ~ 25 GBytes and we now have 64 bit computers to facilitate manipulating this order of data.

b) $\overline{CS}$ input allows one of a number of memory devices connected to the μP to be accessed. R/$\overline{W}$ line dictates whether data is being written to or read from memory

	A_{15}	A_0
A000	1 0 1 0 0000 0000 0000	
BFFF	1 0 1 1 1111 1111 1111	

$\overline{CS} = \overline{A_{15} \cdot \overline{A_{14}} \cdot A_{13} \cdot \text{Address valid}}$



8 (start) a) Literal instructions operate directly on the supplied 8 bit integer argument. ~~Literal~~ File Register Instructions have the memory location from which to obtain the data to be operated on as their argument.

b)
```
movlw 15;      15 = 0F hex into W
movwf 0x20;    0F hex into 0x20
movlw 0xF1;    F1 hex into W
addwf 0x20,W;  F1 added to 0F, result into W
movlw 0xAA ;   AA hex into W
movwf 0x20 ;   AA copied into 0x20
movlw 255  ;   FF hex into W
xorwf 0x20 ;   XOR of AA and FF into 0x20;
sleep ;
```

i)
```
   W   1111 0001
0x20   0000 1111
   W  (1)0000 0000
```

∴ Half-carry (DC) set (DC = 1)
Carry (C) set (C = 1)
Result is zero so Z set (Z = 1)

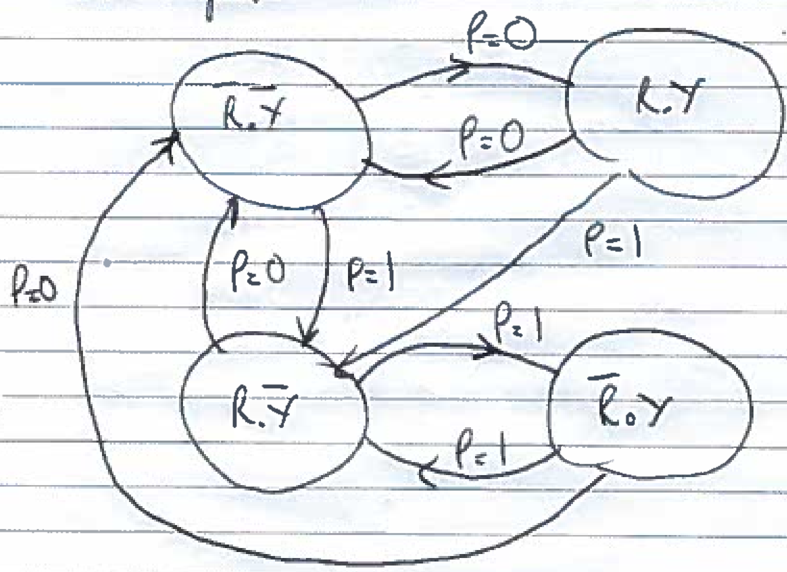
0x20 contains 0F hex, W contains 00 hex

ii) 1 0 1 0 1 0 1 0   0x20
| | | | | | | | W
0 1 0 1 0 1 0 1   0x20

∴ C=0, DC=0, Z=0 since result ≠ 0,
0x20 contains 55hex, W remains FFhex.

9 (long) a) There are 4 different states: $\overline{R}.\overline{Y}$, $R.Y$, $\overline{R}.Y$, $R.\overline{Y}$

State diagram:-



b) $2^N \geq 4$ where N = N° of bistables so N=2 i.e. 2 bistables are needed.

c) Denote one bistable R, with output Q_R, the other one Y with output Q_Y. To avoid output logic let $R = Q_R$, $Y = Q_Y$

P	Present Q_R	Present Q_Y	Next Q_R	Next Q_Y	J_R	K_R	J_Y	K_Y
0	0	0	1	1	1	X	1	X
1	0	0	1	0	1	X	0	X
0	1	1	0	0	X	1	X	1
1	1	1	1	0	X	0	X	1
0	1	0	0	0	X	1	0	X
1	1	0	0	1	X	1	1	X
0	0	1	1	0	0	X	X	1
1	0	1	1	1	1	X	X	1

J_R

P \ $Q_R Q_Y$	00	01	11	10
0	1	0	X	X
1	1	1	X	X

$J_R = P + \overline{Q}_Y$

K_R

P \ $Q_R Q_Y$	00	01	11	10
0	X	X	1	1
1	X	X	0	1

$K_R = \overline{P} + \overline{Q}_Y$

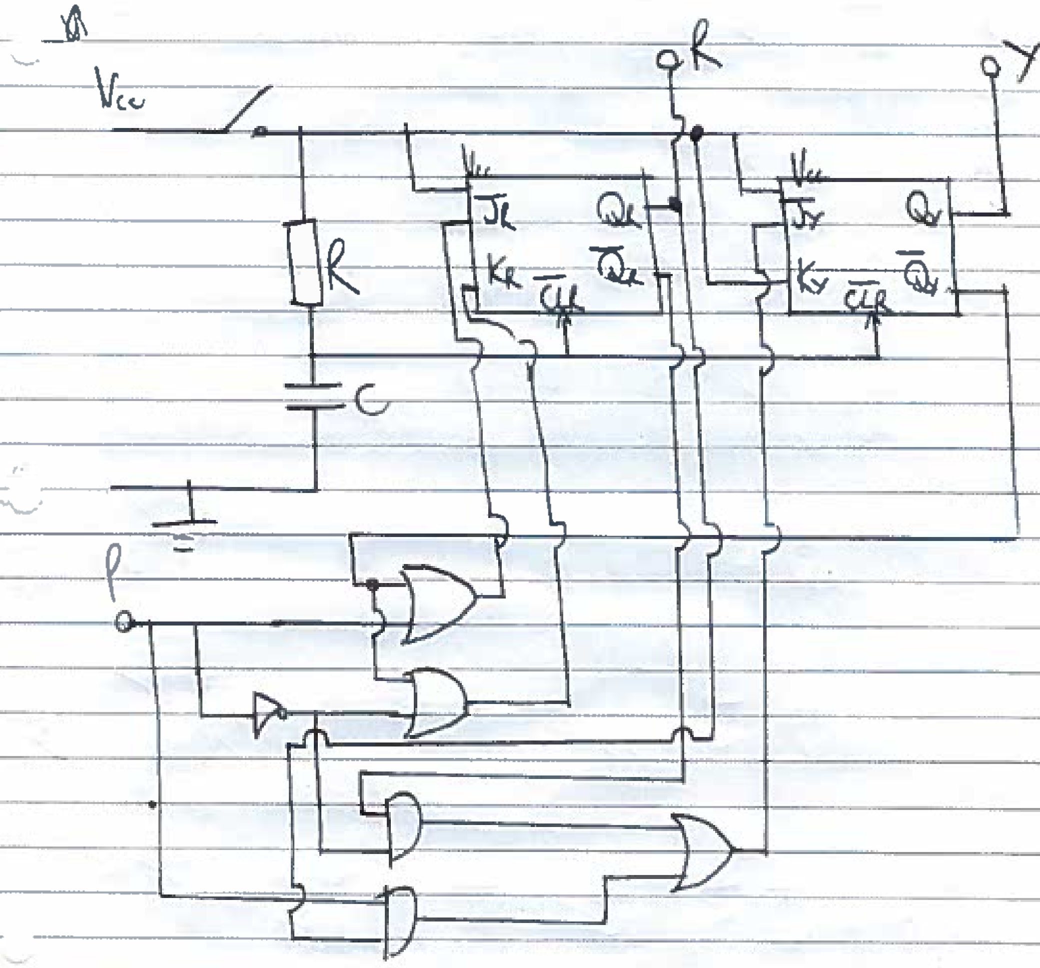
J_Y

P \ $Q_R Q_Y$	00	01	11	10
0	1	X	X	0
1	0	X	X	1

$J_Y = \overline{P} . \overline{Q}_R + P . Q_R$

By inspection $K_Y = 1$

d) To set initial state to $\overline{R} . \overline{Y}$ use $\overline{CLR}$ input by making sure $\overline{CLR}$ is zero ~~to zy~~ when power is applied using an R-C circuit

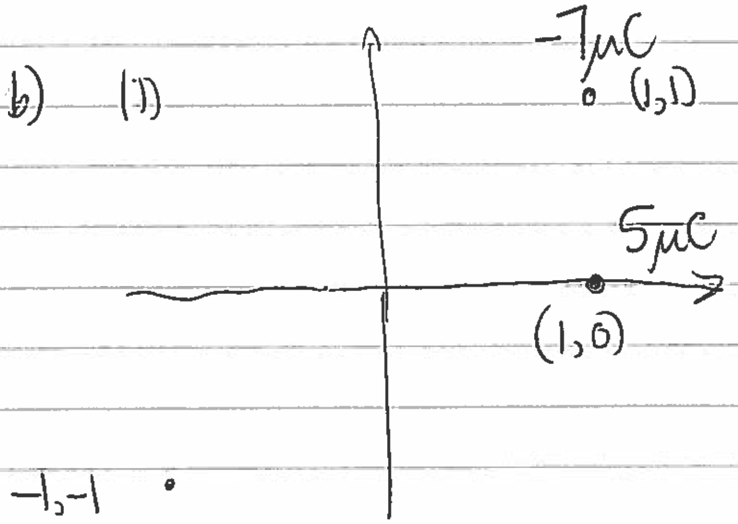


A
Vcc
R
Y
R
C
P

10 (a) By Gauss' law, any internal net charge would cause an internal electric field, compelling such charges to repel each other to the surface of the conductor, such that the internal net electric field becomes zero.

[2]

b) (i)



Field due to $5\mu C$ charge: $\bar{E} = \dfrac{q}{4\pi\epsilon_0 r^2}\hat{r}$

$r = \sqrt{2^2 + 1^2} = \sqrt{5}$ cm $\quad \hat{r} = \dfrac{-\underset{\sim}{i}2 - \underset{\sim}{j}1}{\sqrt{5}}$

So $\bar{E}_1 = \dfrac{5\times10^{-6}}{4\pi\times8.85\times10^{-12}\times(\sqrt{5}\times10^{-2})^2}\left(\dfrac{-\underset{\sim}{i}2-\underset{\sim}{j}1}{\sqrt{5}}\right)$

$= 4.02\times10^7(-2\underset{\sim}{i}-\underset{\sim}{j})$

$7\mu C$ charge $r = 2\sqrt{2}$ cm, $\hat{r} = \dfrac{\underset{\sim}{i}+\underset{\sim}{j}}{\sqrt{2}}$

$\bar{E}_2 = \dfrac{7\times10^{-6}}{4\pi\epsilon_0(2\sqrt{2}\times10^{-2})^2}\cdot\dfrac{\underset{\sim}{i}+\underset{\sim}{j}}{\sqrt{2}} = 5.56\times10^7(\underset{\sim}{i}+\underset{\sim}{j})$

$\bar{E}_T = \bar{E}_1 + \bar{E}_2 = -\underset{\sim}{i}\,2.48\times10^7 + \underset{\sim}{j}\,1.54\times10^7$ Vm^{-1} [6]

ii) $\underset{\sim}{F} = q\underset{\sim}{E} = 10\times10^{-6}\left(-\underset{\sim}{i}\,2.48\times10^{7} + \underset{\sim}{j}\,1.54\times10^{7}\right)$

$= -\underset{\sim}{i}\,248 + \underset{\sim}{j}\,154$

$|\underset{\sim}{F}| = \underline{292\ \mathrm{N}}$

11 a) Self inductance = Flux linkages ~~of~~ the circuit concerned / I

[2]

b)

Amperes law $\oint \underset{\sim}{H}\cdot d\underset{\sim}{l} = I_{enc}$

Assume l very large so $\oint \underset{\sim}{H}\cdot d\underset{\sim}{l} = Hl = Inl$

$B = \mu_0 H \Rightarrow B = \mu_0 I n$

$\phi = BA = \mu_0 I n A \quad \phi' = nl\phi = \mu_0 I n^2 l A$

so $\frac{\phi'}{I} = L = \mu_0 n^2 A l$ [3]

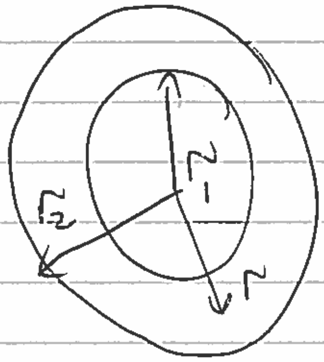
c) $0 = RI + L\frac{dI}{dt}$ subject to $I(0) = I_0$

$\cancel{\Rightarrow I(t) = I_0(1-e^{-t/\tau}),\ \tau = L/R}$ [5]

$\int \frac{dI}{I} = -\frac{R}{L}dt \qquad \ln kI = -\frac{Rt}{L} \qquad kI = e^{-t/\tau} \qquad \tau = L/R$

$t=0,\ I = I_0 \Rightarrow 1/k = I_0$ so $I = I_0 e^{-t/(L/R)}$ [5]

12 a) i)



Assume charge q on inner shell, $-q$ on outer shell.

Gauss:- $\oint \underline{D} \cdot d\underline{S} = q \Rightarrow D \times 4\pi r^2 = q = \epsilon_0 E \times 4\pi r$

$$E = \frac{q}{4\pi\epsilon_0 r^2} \qquad r_1 < r < r_2 \;,\; E = 0 \text{ otherwise}$$

$$V = \int \underline{E} \cdot \underline{dl} = \frac{q}{4\pi\epsilon_0} \int_{r_1}^{r_2} \frac{1}{r^2}\, dr = \frac{q}{4\pi\epsilon_0} \left[-r^{-1}\right]_{r_1}^{r_2}$$

$$= \frac{q}{4\pi\epsilon_0} \left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$C = \frac{q}{V} = \frac{4\pi\epsilon_0}{\left(\frac{1}{r_1} - \frac{1}{r_2}\right)}$$

[9]

ii) Largest electric field will be at $r = r_1$ $= \dfrac{q}{4\pi\epsilon_0 r_1^2} = 3\times10^6$
at breakdown.

$q = 4\pi\epsilon_0 r_1^2 \times 3\times10^6$ with $r_1 = 1$ cm giving $q = \underline{33.3\,nC}$

[3]

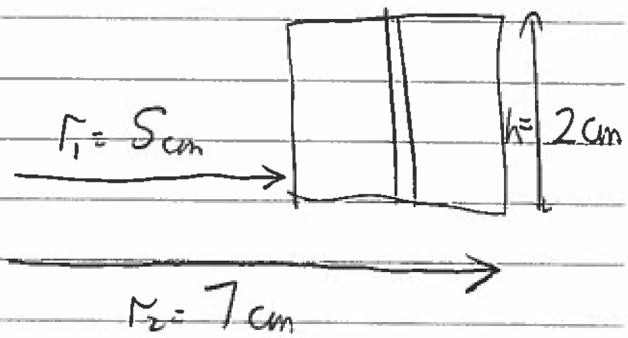
b) i) $\oint \underset{\sim}{H} \cdot \underset{\sim}{dl} = NI$

$H \times 2\pi r = NI$

$H = \dfrac{NI}{2\pi r}$ so $B = \dfrac{\mu_0 \mu_r NI}{2\pi r}$ in ferrite [4]

ii) $B = \dfrac{\mu_0 NI}{2\pi r}$ in air [4]

iii) $\delta\phi = B h \, \delta r$



$= \dfrac{\mu_0 NI}{2\pi r} h \, \delta r$

$\phi_{air} = \displaystyle\int_{r_1}^{r_2} \dfrac{\mu_0 NI}{2\pi r} h \, dr$

$= \dfrac{\mu_0 NIh}{2\pi} \ln \dfrac{r_2}{r_1} = \dfrac{4\pi \times 10^{-7} \times 200 \times 1 \times 2\times10^{-2}}{2\pi} \ln 7/5$

$= 2.7 \times 10^{-7}$ Wb

For $\phi_{ferrite}$ need to divide region of integration into three :-

$$\phi_{ferrite} = \frac{\mu_0 \mu_r NI}{2\pi} \left[0.03 \int_{0.04}^{0.05} \frac{1}{r} dr + 2 \times 0.005 \int_{0.05}^{0.07} \frac{1}{r} dr + 0.03 \int_{0.07}^{0.08} \frac{1}{r} dr \right]$$

$$= \frac{\mu_0 \mu_r N I}{2\pi} \left[0.03 \left(\ln 5/4 + \ln 8/7 \right) + 0.01 \ln 7/5 \right]$$

$$= \frac{4\pi \times 10^{-7} \times 500 \times 200 \times 1}{2\pi} \times 0.01406$$

$$= 2.81 \times 10^{-4} \text{ Wb}$$

iv) $\phi_{total} = \phi_{air} + \phi_{ferrite} = 2.82 \times 10^{-4} \text{ Wb}$

$$L = \frac{N \phi_{total}}{I} = 200 \times 2.82 \times 10^{-4} = 56.3 \text{ mH}.$$