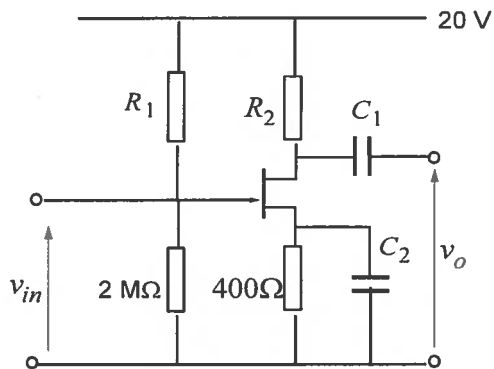


Part 1A paper 3 (Electrical) 2017. Section A Crib

Q1

Corrected 6/6/18 - tdw



$$R1 = 16/4 \cdot 2M\Omega = 8M\Omega$$

$$R2 = 10/2 \cdot 400 = 2k\Omega$$

$$g_m = 5, C2 = \infty, \text{ Gain} = -g_m \cdot \frac{r_d R_L}{r_d + R_L} = -9.52$$

Q2. go from right to left.

$$3\text{ohm} // 6\text{ohm} = 2\text{ohm}, V' = 6V.$$

$$2 + 6 \text{ ohm} = 8 \text{ ohm}.$$

$$8\text{ohm} // 8 \text{ ohm} = 4 \text{ ohm}, V'' = 3V.$$

$$4 \text{ ohm} + 4 \text{ ohm} = 8 \text{ ohm}, 8 \text{ ohm} // 8 \text{ ohm} = 4 \text{ ohm}, V' = 1.5 V.$$

$$4 \text{ ohm} + 16 \text{ ohm} = 20 \text{ ohm}.$$

$$6 V - 1.5 V = 4.5 V, \quad I = 4.5/20 = 0.225 A.$$

note negative feedback is larger covered in pt 1A.

Q3 (a) advantages of negative feedback - **a well-defined gain**, reduced distortion, low output impedance, high input impedance (in one configuration), higher cutoff frequency.

$$\text{b) mid-band gain} = (R1+R2)/R2 = 10$$

output impedance at output of Amp itself ~ 0 ohm

3dB frequency is when the load capacitance starts to short-circuit the load resistance $R4$.

$$f_T = 1/(2 \cdot \pi \cdot 75 \cdot 10^{-9}) = 1/(471 \times 10^{-9}) = 2.12 \text{ MHz}.$$

4(a) Thevenin – replace circuit by voltage source V1 in series with resistor R1,

V1 = open circuit voltage Voc

Short circuit current = Isc, R1 = Voc/Isc.

Norton – replace circuit by current source I1 in parallel with resistance R1,

I1 = Isc, R1 = Voc/Isc.

For the example of fig 4, Voc = 5V, R1 = 10 ohm.

Isc = 0.5 Amp, R1 = 10 ohm.

(b)(i) R, L part -

$$X1 = 1.27 \times 10^{-6} \times 2\pi \times 5 \cdot 10^5 j = 4j, \text{ so } Z1 = 4 + 4j.$$

$$Z2 = R2 // X2. \quad 1/X2 = j 2\pi \cdot 5 \cdot 10^5 \cdot 1.59 \cdot 10^{-7} = j0.5 = j/2.$$

$$Z2 = 2(1/(1+j))[(1-j)/(1-j)] = 2 \times 0.5 \times (1-j) = 1-j.$$

note the values are meant to give nice round numbers.

$$Z_{tot} = 4(1+j) + (1-j) = 5 + 3j$$

INPUT CURRENT

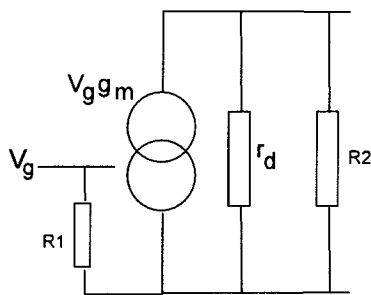
$$i = 6 / (5 + 3j)$$

$$= \frac{3}{17} (5 - 3j) = 0.88 - 0.53j$$

(ii) ~~$f = 1/2\pi \cdot (1.27 \times 10^{-6} \times 1.59 \times 10^{-7})^{1/2}$~~

(iii) ~~$V = 2V$~~

5) a)



input impedance = R1

Gain = $g_m R_2 / (1 + g_m R_2)$ note, should be just less than 1.

ii) iii) Please note there is an error in the question and the circuit is not resonant as its Q factor is too low. As a result of the low parallel resistance of the inductor (2Ω) $R_L \approx r$ and the series to parallel approximation does not hold (P3 of handout #2).

Please ignore this +dhw 2018 section

$$R' = 2.5k\Omega$$

Output impedance = $R_2 / (1 + g_m R_2)$. Note, should be a lot less than R_2 , that is the point of a source-follower.

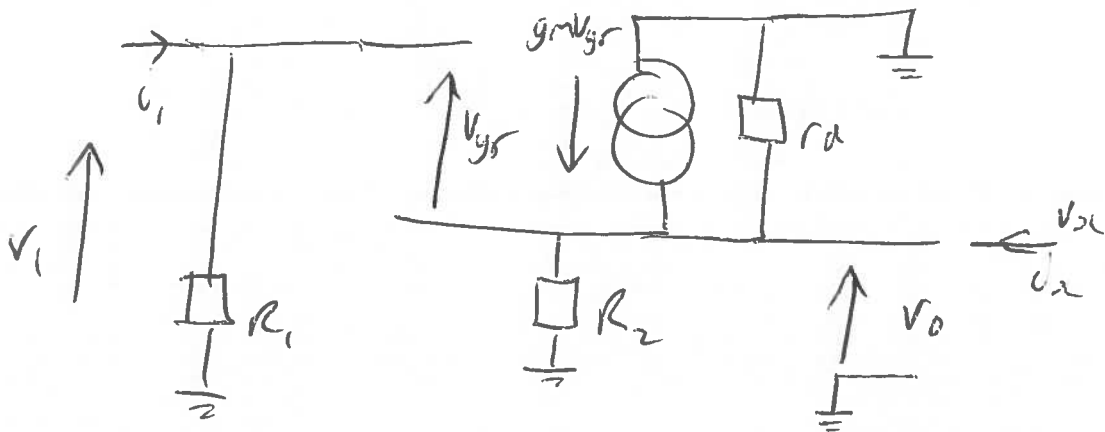
$$Z_{in} = 20 M\Omega, G = 0.978, Z_{out} = 65 \Omega$$

$$G = 0.88$$

$$R_{out} = 294 \Omega$$

Purpose = low output impedance, high input impedance, drive large current.

SSM



$$R_{in} = \frac{v_i}{i_i} = R_1 = 20 M\Omega$$

$$R_{out} = \frac{v_{oc}}{i_{sc}} \Big|_{v_i \text{ short ckt.}}$$

Gain $v_i = v_{gs} + v_o$ (1)

R_2 is in parallel with r_d

$$R' = R_2 \parallel r_d$$

$$\Rightarrow v_o = g_m v_{gs} R' \quad (2)$$

Combine (1) & (2)

$$v_i = \frac{v_o}{g_m R'} + v_o$$

$$\Rightarrow \frac{v_o}{v_i} = \frac{g_m R'}{1 + g_m R'}$$

$$\Rightarrow v_{oc} = v_o$$

$$v_i = 0 \Rightarrow v_{oc} = -v_{gs}$$

Σ currents at o/p

$$i_{sc} + g_m v_{gs} = \frac{v_{oc}}{R'}$$

$$i_{sc} = \frac{v_{oc}}{R'} + g_m v_{oc}$$

$$\frac{v_{oc}}{i_{sc}} = R_{out} = \frac{R'}{1 + g_m R'}$$

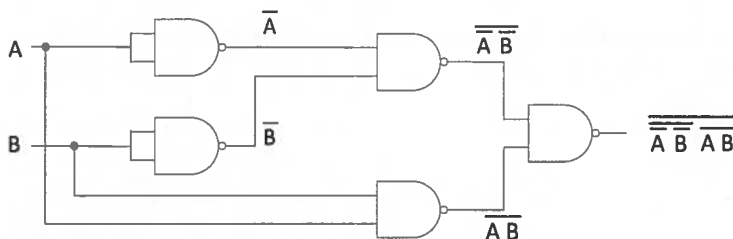
6 (short)

(a) The Boolean expression for exclusive-NOR gate can be written as:

$$\begin{aligned} & A \cdot \bar{B} + \bar{A} \cdot B \\ &= \overline{(A \cdot \bar{B}) \cdot (\bar{A} \cdot B)} \\ &= \overline{(\bar{A} + \bar{B}) \cdot (\bar{A} + B)} \\ &= \overline{(\bar{A} + B) \cdot (A + \bar{B})} \\ &= \bar{A} \cdot \bar{B} + A \cdot B \end{aligned}$$

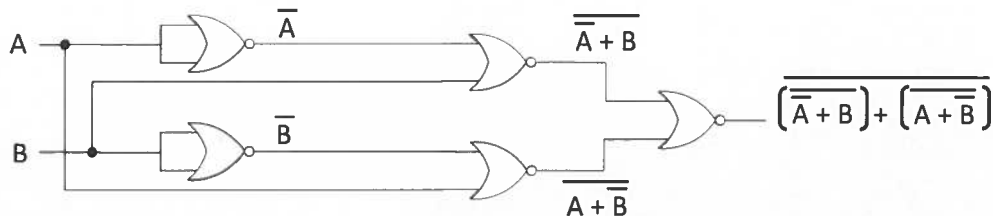
Implementation using NAND gates:

$$\begin{aligned} & \bar{A} \cdot \bar{B} + A \cdot B \\ &= \overline{\overline{\bar{A} \cdot \bar{B}} + \overline{A \cdot B}} \\ &= \overline{(\overline{\bar{A} \cdot \bar{B}}) \cdot (\overline{A \cdot B})} \end{aligned}$$



Implementation using NOR gates:

$$(\bar{A} + B) \cdot (A + \bar{B})$$



7 (short)

(a) Considering ideal OP-Amp, the inverting terminal is a virtual ground. Hence, the voltages at the inputs D_0 - D_4 contribute to currents to the summing junction and is inversely proportional to the corresponding resistor values. Thus, V_{out} is also inversely proportional to these resistors.

Drawbacks are:

- Resistor accuracy requirement is very high.
- Difficult to implement the resistors because of their size, especially for DACs with higher bit count.
- May require the use of different materials for different resistors due to their wide variation in values. This may result in drifting of values with temperature.

(b) When all inputs are set, the resolution is $1/(2^4 - 1) = 1/15 = 6.67\%$.

$$V_{out[1001]} = - \left[\frac{10 \times 5}{25} + \frac{10 \times 5}{200} \right] V = -2.25V$$

$$V_{out[1011]} = - \left[\frac{10 \times 5}{25} + \frac{10 \times 5}{100} + \frac{10 \times 5}{200} \right] V = -2.75V$$

Change is $V_{out} = -0.50 V$.

8 (short)

<p>(a) main movlw 111 ; movwf 0x33 ; movf 0x30, W ; call label ; movwf 0x32 ; btfsc 0x32, 2 ; clrf 0x33 ; </p>	<p>moves decimal 111 to w moves content of w (111) into loc. 0x33 move content of 0x30 (unknown) into w calls subroutine 'label' moves content of w (10) to 0x32 test bit2 (Z), if zero, skips next instruction clears content of 0x33</p>
--	--

<p>label subwf 0x31, W ; return ;</p>	<p>subtracts w from contents of 0x31 returns to main program</p>
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(b) Contents:

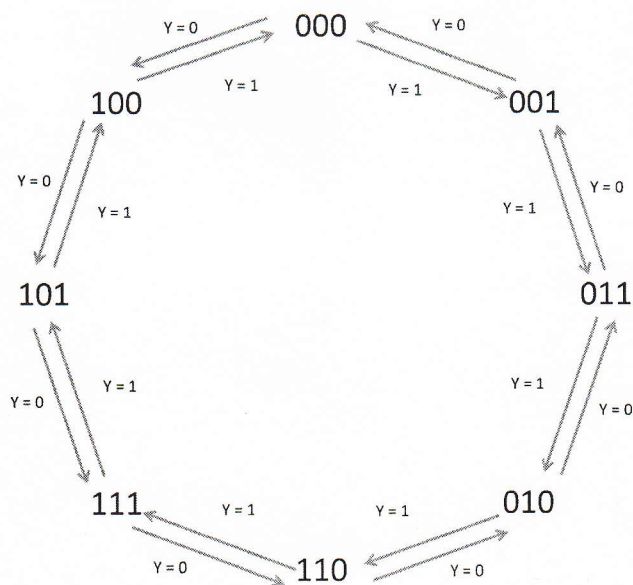
0x32 = 10 (decimal)

0x33 = 111 (decimal)

Bit 2 in status register (Z flag) = 0

9 (long)

(a) Consider: control input = Y. The state diagram will be:



(b) The state transition table is:

Q ₂	Q ₁	Q ₀	Q ₂₊	Q ₁₊	Q ₀₊	Q ₂₊	Q ₁₊	Q ₀₊
Present State			Next State Y=0			Next State Y = 1		
0	0	0	1	0	0	0	0	1
0	0	1	0	0	0	0	1	1
0	1	1	0	0	1	0	1	0
0	1	0	0	1	1	1	1	0
1	1	0	0	1	0	1	1	1
1	1	1	1	1	0	1	0	1
1	0	1	1	1	1	1	0	0
1	0	0	1	0	1	0	0	0

(c) Resulting Karnaugh maps:

Q ₂ Q ₁ \ Q ₀ Y	00	01	11	10
00	1	0	0	0
01	0	1	0	0
11	X	X	X	X
10	X	X	X	X

J2 Map

Q ₂ Q ₁ \ Q ₀ Y	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	1	0	0	0
10	0	1	0	0

K2 Map

Q ₂ Q ₁ \ Q ₀ Y	00	01	11	10
00	0	0	1	0
01	X	X	X	X
11	X	X	X	X
10	0	0	0	1

J1 Map

Q ₂ Q ₁ \ Q ₀ Y	00	01	11	10
00	X	X	X	X
01	0	0	0	1
11	0	0	1	0
10	X	X	X	X

K1 Map

Q ₂ Q ₁ \ Q ₀ Y	00	01	11	10
00	0	1	X	X
01	1	0	X	X
11	0	1	X	X
10	1	0	X	X

J0 Map

Q ₂ Q ₁ \ Q ₀ Y	00	01	11	10
00	X	X	0	1
01	X	X	1	0
11	X	X	0	1
10	X	X	1	0

K0 Map

Thus, the Boolean expressions are:

$$J_2 = \bar{Q}_1 \bar{Q}_0 \bar{Y} + Q_1 \bar{Q}_0 Y$$

$$K_2 = Q_1 \bar{Q}_0 \bar{Y} + \bar{Q}_1 \bar{Q}_0 Y$$

$$J_1 = \bar{Q}_2 Q_0 Y + Q_2 Q_0 \bar{Y}$$

$$K_1 = Q_2 Q_0 Y + \bar{Q}_2 Q_0 \bar{Y}$$

$$J_0 = \bar{Q}_2 \bar{Q}_1 Y + \bar{Q}_2 Q_1 \bar{Y} + Q_2 \bar{Q}_1 \bar{Y} + Q_2 Q_1 Y$$

$$K_0 = \bar{Q}_2 \bar{Q}_1 \bar{Y} + \bar{Q}_2 Q_1 Y + Q_2 Q_1 \bar{Y} + Q_2 \bar{Q}_1 Y$$

(c) Gates required for this design:

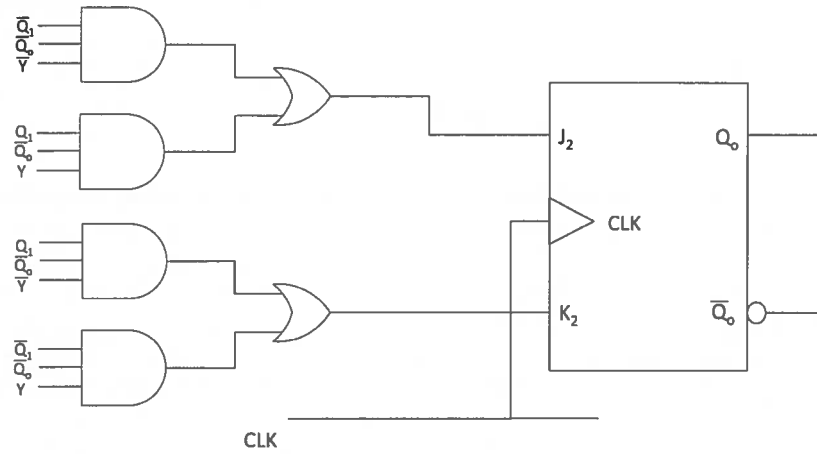
1 inverter (for control input Y)

3 JK bistables

16 3-input NAND gates

4 2-input NOR gates and 2 4-input NOR gates

Implementation:



Numerical answers:

7(b)

6.67%, - 0.5V

8(b)

0x32 = 10 (decimal), 0x33 = 111 (decimal), Z flag = 0.

9(d)

1 x inverter (for control input Y)

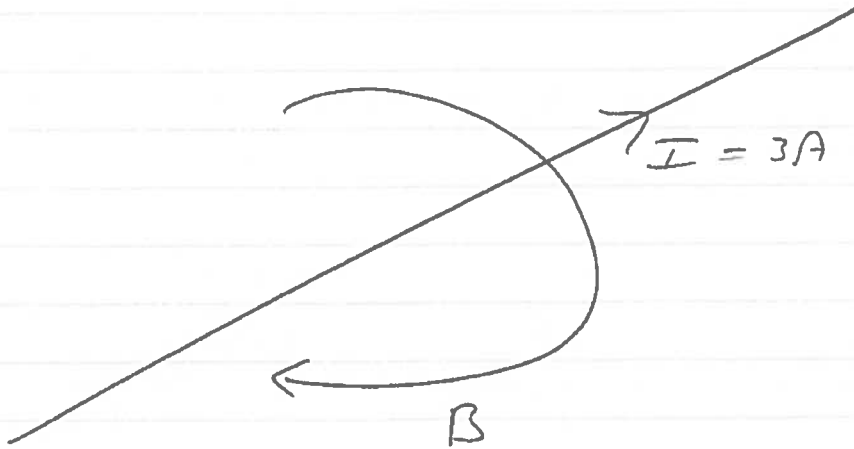
3 x JK bistables

16 x 3-input NAND gates

4 x 2-input NOR gates and 2 x 4-input NOR gates

Note: Other combinations possible.

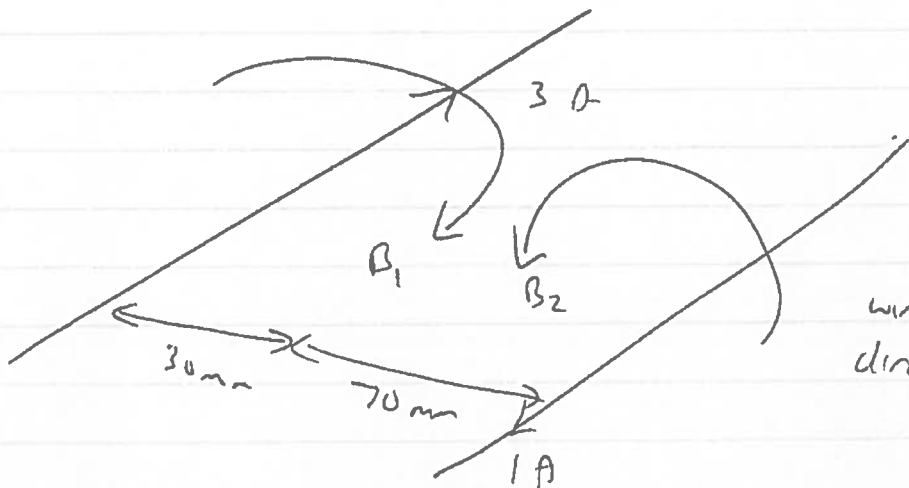
10) a)



\vec{B} circulates the wire $|B| = \frac{\mu_0 I}{2\pi r}$

$$= \frac{2 \times 10^{-7} \times 3}{2\pi \times (100 \times 10^{-3})} = 6 \times 10^{-6} \text{ T}$$

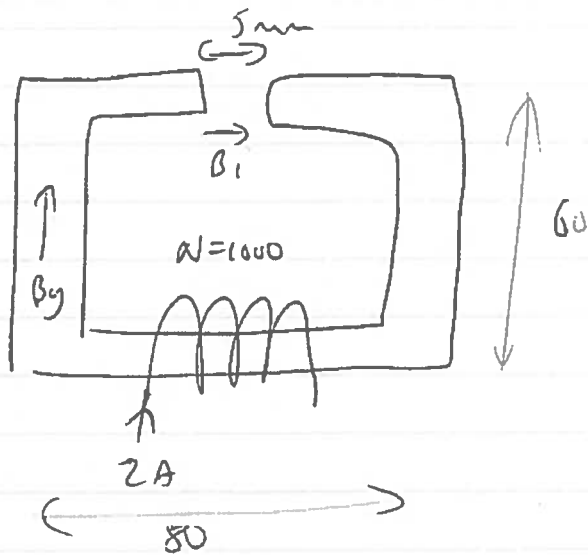
b)



Between the wires B in same direction \Rightarrow add

$$B_T = B_1 + B_2 = \frac{2 \times 10^{-7}}{2\pi} \left(\frac{3}{180 \times 10^{-3}} + \frac{1}{70 \times 10^{-3}} \right) = 2.3 \times 10^{-5} \text{ T}$$

ii) a)



$$A = 100 \text{ mm}^2$$

Ampere's Law $\sum Hl = NI$ $l = 1 \text{ m}$
 $g = \text{gap}$

$$\Rightarrow H_i l_i + H_y l_y = NI$$

In the air gap $B_g = \mu_0 H_g$

Flux conservation (no fringing fields)

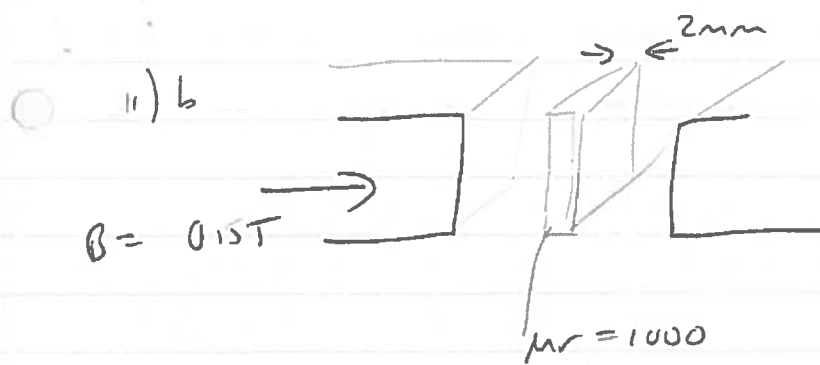
$$B_g = B_i$$

$$\Rightarrow H_i l_i + \frac{B_i l_g}{\mu_0} = NI$$

$$l_i = 80 + 80 + 60 + 60 - 5 = 275 \text{ mm} \quad \left(\text{Could be est at } 255 \text{ mm} \right)$$

$$\Rightarrow 0.275 H_i = 2000 - 4000 B_i$$

From data book graph $B = 0.5 \text{ T}$



A/C $H_i l_i + H_m l_m + H_g l_g = \mathcal{N} I$ $m \equiv \text{metal sheet}$

From flux conservation $B_i = B_g = B_m$

$$\Rightarrow H_i l_i + \frac{B_i l_m}{\mu_0 \mu_r} + \frac{B_i l'_g}{\mu_0} = \mathcal{N} I$$

$$H_i l_i + \frac{B_i}{\mu_0} \left[\frac{l_m}{\mu_r} + l'_g \right] = \mathcal{N} I$$

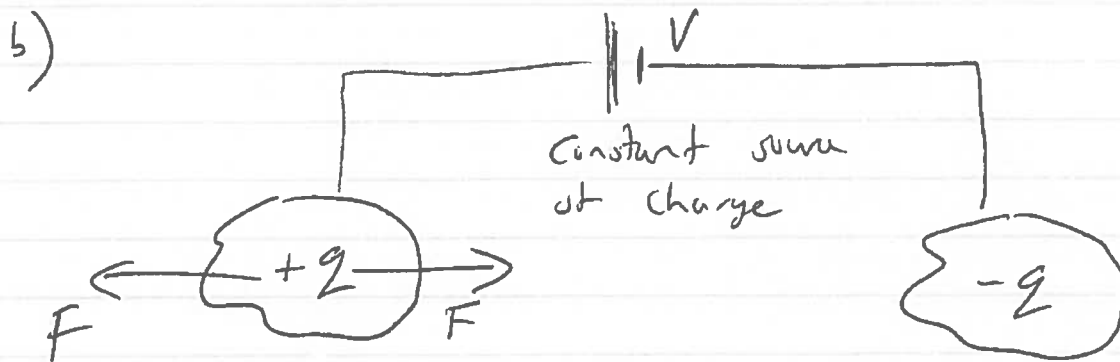
negligible

$$l'_g = 5 - 2 = 3 \text{ mm}$$

$$\Rightarrow 0.275 H_i = 2000 - 2400 B_i$$

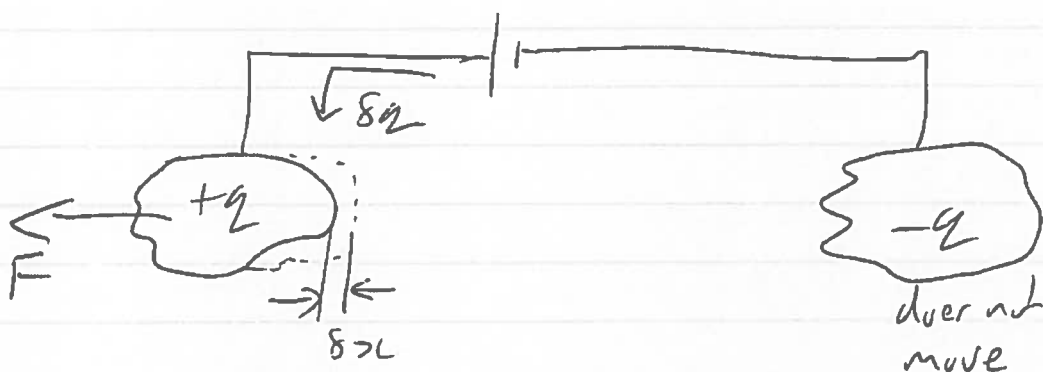
From graph $B_g = B_i = 0.18 \text{ T}$

12) a) Electrostatic equilibrium is maintained between two charge objects. In order to do this a restoring force must act on the objects. If one of the objects then moves a distance δx then work is done ($-F\delta x$) mechanically. To maintain equilibrium, the battery with constant voltage must do work ($\delta q V$) and there is also a change in the electrostatic energy in the complete system ($\frac{1}{2} \delta q V$).



Attractive force F is balanced by restoring force ($-F$) to maintain equilibrium

If Left hand object moves a distance δx



\Rightarrow Mechanical work = $-F\delta x$

Work done by battery = $\delta q V$

Change in electrostatic energy = $\frac{1}{2} \delta q V$ (only one charge moves)

⇒ These energy changes must balance ~~the~~
~~the~~ assuming constant V

⇒ Mech work + work by battery = change in electrostatic energy

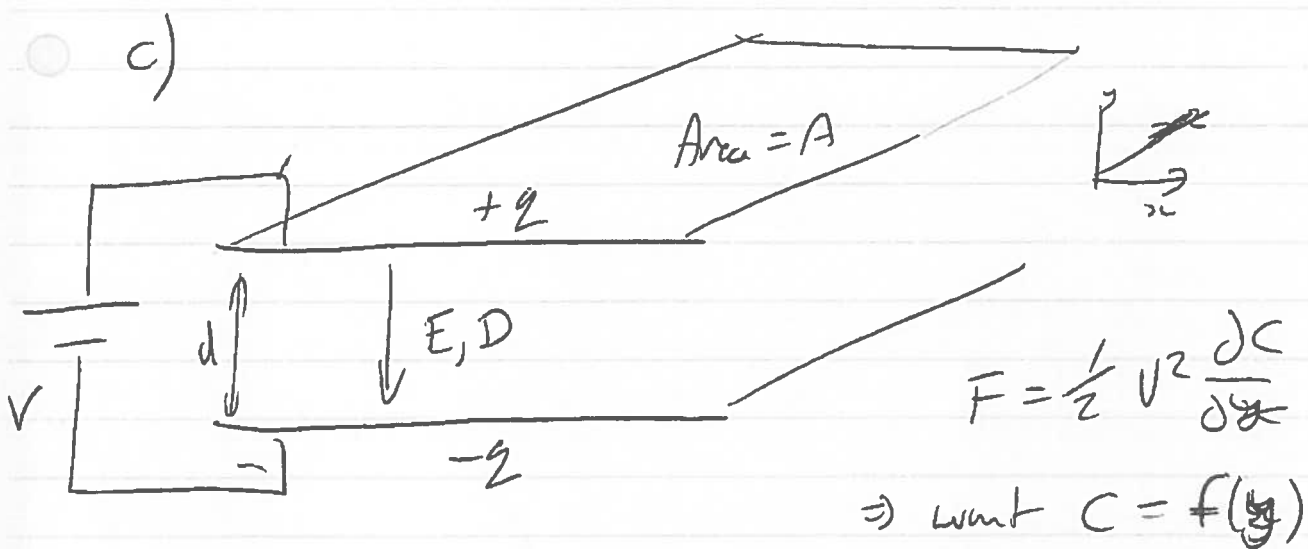
$$-F \delta x + \delta q V = \frac{1}{2} \delta q V$$

$$\Rightarrow -F \delta x = -\frac{1}{2} \delta q V$$

$$F = \frac{1}{2} V \frac{\delta q}{\delta x}$$

$$q = CV \Rightarrow \frac{\delta q}{\delta x} = V \frac{\delta C}{\delta x}$$

$$\Rightarrow F = \frac{1}{2} V^2 \frac{\delta C}{\delta x}$$

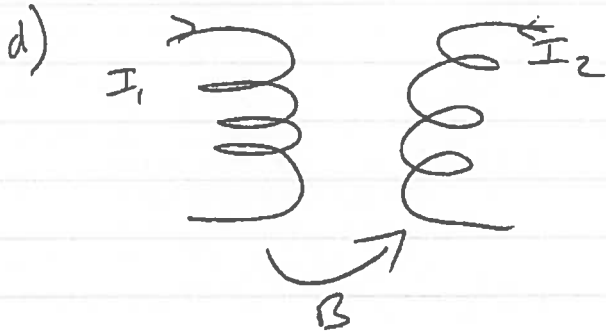


Parallel plate capacitor $C = \frac{\epsilon_0 A}{d} = \frac{\epsilon_0 A}{y}$

$$\frac{\partial C}{\partial y} = -\frac{\epsilon_0 A}{y^2}$$

$$\Rightarrow F = -\frac{1}{2} V^2 \frac{\epsilon_0 A}{y^2} = -\frac{1}{2} V^2 \frac{\epsilon_0 A}{d^2}$$

$$C = \frac{\epsilon_0 A}{d} \Rightarrow F = -\frac{1}{2} V^2 \frac{C}{d}$$



$$\phi_1 = L_1 I_1 + M I_2$$

$$\phi_2 = L_2 I_2 + M I_1$$

$$V_1 = \frac{d\phi_1}{dt} = L_1 \frac{dI_1}{dt} + M \frac{dI_2}{dt}$$

$$V_2 = \frac{d\phi_2}{dt} = L_2 \frac{dI_2}{dt} + M \frac{dI_1}{dt}$$

$$P_{\text{avg}} = \overline{VI} = \frac{dW}{dt} = V_1 I_1 + V_2 I_2$$

$$= L_1 I_1 \frac{dI_1}{dt} + M I_1 \frac{dI_2}{dt} + L_2 I_2 \frac{dI_2}{dt} + M I_2 \frac{dI_1}{dt}$$

$$= \frac{L_1}{2} \frac{d(I_1^2)}{dt} + \frac{L_2}{2} \frac{d(I_2^2)}{dt} + M \frac{d(I_1 I_2)}{dt}$$

$$\Rightarrow W = \underbrace{\frac{1}{2} L_1 I_1^2}_{\text{self}_1} + \underbrace{\frac{1}{2} L_2 I_2^2}_{\text{self}_2} + \underbrace{M I_1 I_2}_{\text{mutual}}$$

When evaluating virtual work, there is a restoring force between the coils which means that when a coil moves a distance δx the only change in energy between the coils is due to their mutual inductance M . The self inductance terms remain constant when the coils move with respect to each other, hence the force between them must only depend on their mutual inductance.