WORKED SOLUTIONS

Paper 3

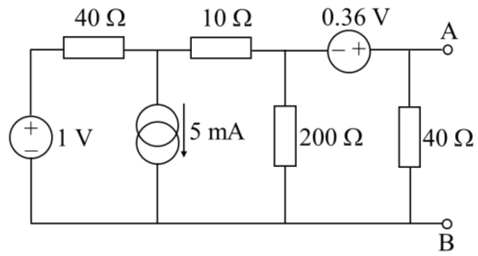
ELECTRICAL & INFORMATION ENGINEERING

Section A – Joyce
Section B – Hasan
Section C – Ferrari/Malliaras

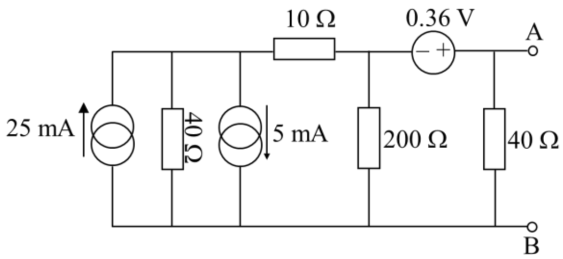
SECTION A

Question 1

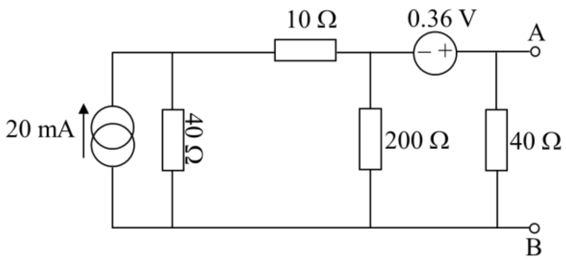
(a) The numbers are chosen so that there is no need for a calculator



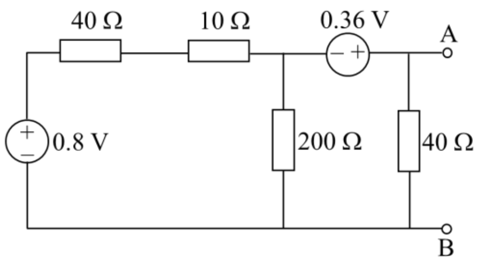
Convert left hand side to Norton equivalent:



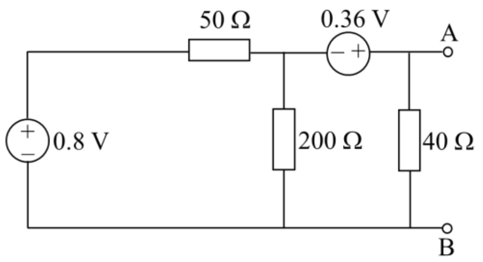
Combine the current sources:



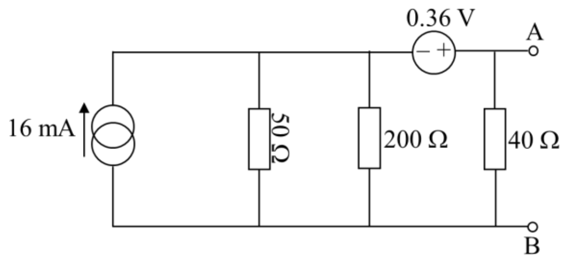
Convert left hand side to Thevenin equivalent:



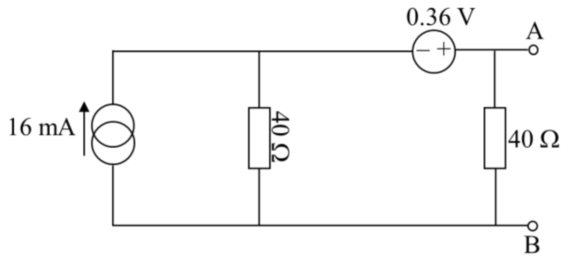
Then, add the 40 Ω and 10 Ω resistors in series:



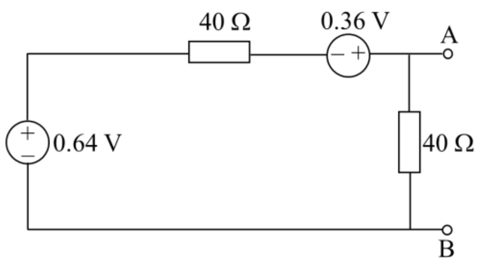
Convert left hand side to Norton equivalent:



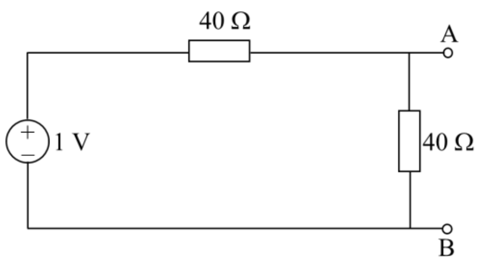
Then, combine the 50 Ω and 200 Ω resistors in parallel:



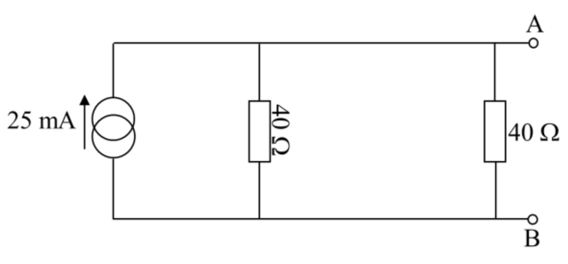
Convert left hand side to Thevenin equivalent:



Add the voltage sources:

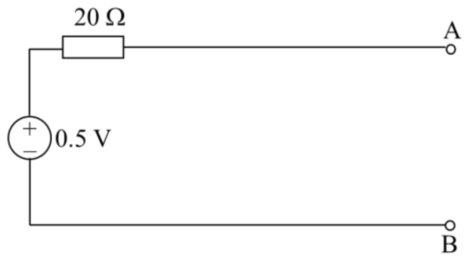


Convert left hand side to Norton equivalent:



Add 40 Ω resistors in parallel to achieve **Norton equivalent**:

Convert to **Thevenin equivalent**:



Examiner's comment: Most students answered this sub-question capably by doing a sequence of Thevenin and Norton simplifications, but many students made errors as a result of taking shortcuts. Common errors were:

- Neglecting the units (especially ohms)
- Summing currents incorrectly into a node (e.g. adding 25 mA and 5 mA when they should actually be subtracted).
- Adding resistances in series when they should be added in parallel.

A number of students started the process from the right hand side (nearest the terminals), which is always the more difficult approach and was more likely to result in errors.
A small number of students attempted to simplify the circuit through nodal or loop analysis. All these students found themselves lost in a sea of algebra and made errors. It is easier to sequentially perform Thevenin and Norton simplifications.

(b) Power to the load is maximised when R_L = **20 Ω**

The current through this load will be ½ × 25 mA.

The voltage across this load will be ½ × 0.5 V.

Power dissipated by load = VI = $I^2R_L = 0.0125^2 \times 20$ = **3.125 mW**

Alternatively,

Power dissipated by load = VI = $V^2/R_L = 0.25^2/20$ = **3.125 mW**

Examiner's comment: Most students correctly identified that maximum power is transferred when the load resistor matches the Thevenin or Norton resistance. A handful tried to derive this from first principles (differentiating), which was actually unnecessary. A fair number of students miscalculated the power lost across the load and got an answer that was 4 times higher than it should be.

Question 2

(a)

$$i = \frac{v_{in}}{R + j\omega L \| 1/j\omega C}$$

Resonance occurs when the impedance of the capacitor negates the impedance of the inductor:

$$\omega L = 1/\omega C$$

$$\therefore C = \frac{1}{\omega^2 L} = \frac{1}{\left(2\pi \times 24 \times 10^3\right)^2 \times 90 \times 10^{-6}}$$ = **489 nF**

(b) At 24 Hz, the impedance of the inductor is:

$Z_L = j\omega L = j2\times\pi\times24\times10^3\times90\times10^{-6} = j13.57\ \Omega$.

At 24 Hz, the impedance of the capacitor is:

$Z_C = 1/j\omega C = 1/(j2\times\pi\times24\times10^3\times100\times10^{-9}) = -j66.31\ \Omega$.

Total impedance of the circuit $Z_{total} = R + Z_L \| Z_C$:

$Z_{total} = 10 + j13.57 \| -j66.31 = 10 + j\,17.06 = 19.8\ \Omega \angle 59.6°$

$I_{rms} = V_{rms}/Z_{total} = 200/(19.8\ \Omega \angle 59.6°) = 10.1 \angle -59.6°$

$I_{peak} = \sqrt{2}\ I_{rms} =$ **14.3 A $\angle -59.6°$**

In this case the current lags the voltage, because the load is predominately inductive.

Examiner's comment: A small number of students made errors in calculating the impedance of the circuit, some adding the reactances in series rather than in parallel. Many students only gave the rms current, but not the peak current. The strongest students explicitly stated that the current lags the voltage. Some students gave the phase in radians, which was marked correctly but degrees would be preferable.

Question 3

(a) The impedance of the load referred to the primary is:

$Z_L' = (N_1/N_2)^2(0.3 + j0.2) = (10)^2(0.3 + j0.2) =$ **$30 + j20\ \Omega$**

(b) The load current referred to the primary, I_L' is:

$I_L' = 240/(Z_L' + R_1 + R_2' + X_1 + X_2') = 240/(40 + j30)$

$|I_L'| = 4.8\text{A}$

The actual load current is:

$|I_L| = (N_1/N_2)\times 4.8$

$=$ **48 A**

(c) The real power dissipated in the load is:

$P = I_L'^2 R_L' = 4.8^2\times 30$

$=$ **691.2 W**

Or alternatively:

$P = I_L^2 R_L = 48^2\times 0.3$

$= 691.2$ W

The reactive power is:

$P = I_L^2 X_L = 48^2\times 0.2$

$=$ **460.8 VAR**

(d) The power loss $P_{loss} = I_L'^2 (R_1 + R_2') = 4.8^2 (6 + 4) =$ **230.4 W**

$P_{in,\ total} = 230.4 + 691.2 = 921.6$ W

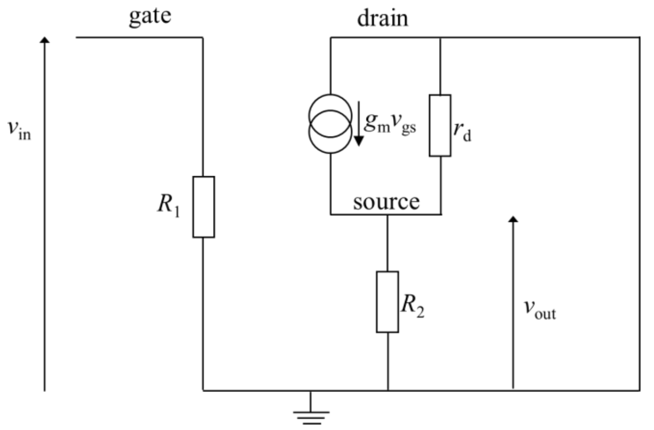
Efficiency $= 230.4/921.6$

$=0.75$ (i.e. **75%**)

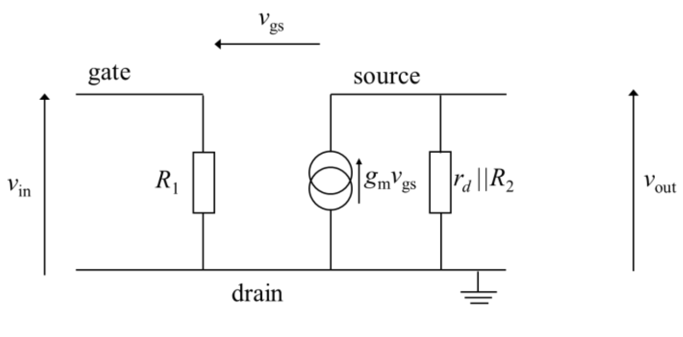
Examiner's comment: Common errors were:
- Misunderstanding that X denotes an imaginary component (reactance).
- In (a), neglecting the units (ohms).
- In (b), only calculating the load current referred to the primary. The full, correct answer is the load current referred to the secondary winding, which is the actual load current.
- In (c), calculating the total real and reactive power dissipated, rather than the power dissipated in the load.
- In (d), misunderstanding the definition of transformer efficiency. Many students calculated efficiency incorrectly as the ratio of apparent power out/in. The correct definition is the ratio of real power out/in.

Question 4

(a)



which can be re-drawn as:



(b) and (c)

<u>Input impedance</u> $R_{in} = R_1 =$ **10 MΩ**

<u>Gain</u>

$$v_{out} = g_m v_{gs} \left(R_2 \| r_d \right) \qquad \text{equation 1}$$

$$v_{gs} = v_{in} - v_{out} \qquad \text{equation 2}$$

Using equations 1 and 2 and eliminating v_{gs} gives

$$v_{in} - v_{out} = \frac{v_{out}}{g_m (R_2 \| r_d)}$$

$$\therefore v_{in} = v_{out} \left(\frac{1}{g_m (R_2 \| r_d)} + 1 \right)$$

$$\therefore \frac{v_{out}}{v_{in}} = \frac{1}{\left(\frac{1}{g_m (R_2 \| r_d)} + 1 \right)}$$

$$\therefore \frac{v_{out}}{v_{in}} = \frac{g_m (R_2 \| r_d)}{1 + g_m (R_2 \| r_d)}$$

Knowing that $R_2 \| r_d$ = 5k||20k = 4 kΩ, we can calculate the gain as

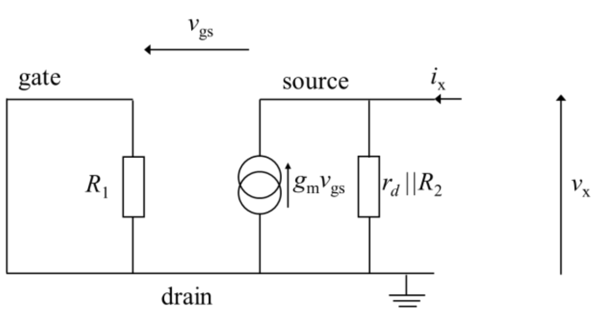
$$\frac{v_{out}}{v_{in}} = \frac{10 \times 10^{-3} (4 \times 10^3)}{1 + 10 \times 10^{-3} (4 \times 10^3)} = 40/41$$

= **0.976**

≈ **1**

This value (unity!) is expected for a source-follower (also known as a unity gain buffer)

<u>Output impedance</u>: Short circuit the input. Apply a test current i_x and test voltage v_x to the output terminal.



$$\begin{aligned} v_x &= (i_x + g_m v_{gs})(R_2 \| r_d) \\ &= (i_x - g_m v_x)(R_2 \| r_d) \end{aligned}$$

noting that v_{in}=0, so v_{gs} = -v_x

$$\therefore v_x + g_m v_x \left(R_2 \| r_d\right) = i_x \left(R_2 \| r_d\right)$$

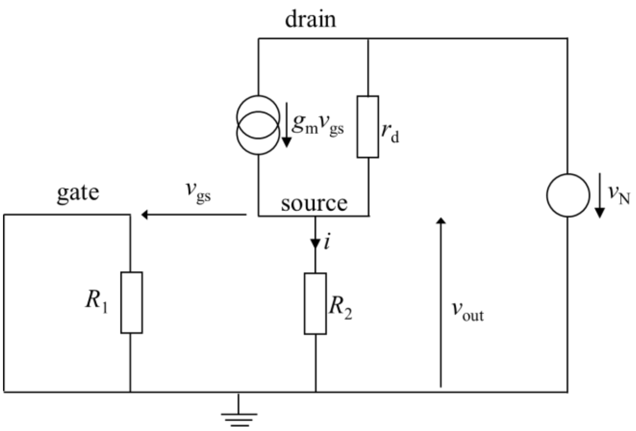
$$R_{out} = \frac{v_x}{i_x} = \frac{\left(R_2 \| r_d\right)}{1 + g_m \left(R_2 \| r_d\right)}$$

$$= \frac{\left(4 \times 10^3\right)}{1 + 10 \times 10^{-3} \left(4 \times 10^3\right)}$$

=97Ω

High input impedance, low output impedance, unity gain.

(d)



$v_{out} = iR_2$ equation 1

$v_{out} = -v_{gs}$ equation 2

$v_{out} = -v_N + \left(g_m v_{gs} - i\right) r_d$ equation 3

Substitute equations 1 and 2 into equation 3:

$$v_{out} = -v_N + \left(-g_m v_{out} - \frac{v_{out}}{R_2}\right) r_d$$

$$\therefore v_{out} \left(1 + g_m r_d + \frac{r_d}{R_2}\right) = -v_N$$

$$\therefore \frac{v_{out}}{v_N} = -\frac{1}{\left(1 + g_m r_d + \dfrac{r_d}{R_2}\right)}$$

$$\therefore \frac{v_{out}}{v_N} = -\frac{1}{\left(1 + 10 \times 20 + \dfrac{20}{5}\right)}$$

= **-0.005**

Examiner's comment: This question was answered reasonably well. Common mistakes in parts (a – c) were:

- Failing to ground the FET drain in the small signal model, leaving it at 10 V.
- Calculating the output impedance as r_d in parallel with R_2. This is incorrect, and the correct impedance is found by applying a test current and test voltage (i_x, v_x).
- Incorrectly drawing $v_{in} = v_{gs}$ in the small signal model.
- Incorrectly assuming $v_{in} = v_{gs}$ in the calculation of gain.

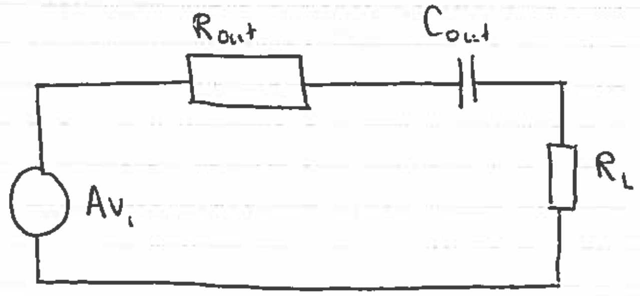
The best students noted that, as a source follower, you would expect unity gain, which agreed with their calculated gain. Some of these students also noted the low output impedance and high input impedance, which are desirable in a buffer.

Common mistakes in part (d) were:

- Drawing v_N across r_d only.
- Combining r_d in parallel with R_2, which is not correct as they do not share the same two nodes.

No students noted how little the noise is amplified (gain = 0.005), which is a pity!
In part (e), the vast majority of students correctly answered that C_{out} should be placed in series with R_L to create a high pass filter. Some students didn't explain why. In the calculation of C_{out}, some students used the wrong values of resistance. E.g. they added R_L and R_{out} in parallel rather than in series, or they added R_L and r_d.
Strangely, some students did not answer part 4(e) but could capably answer question 5(c-ii), which is a closely related question.

(e) C_{out} should be placed in series with the load resistor, as the impedance of C_{out} is $\frac{1}{j\omega C_{out}}$ which increases as ω decreases



$$\omega_{3dB} = \frac{1}{C(R_L + R_{out})}$$

$$\therefore \quad C = \frac{1}{\omega_{3dB}(R_L + R_{out})}$$

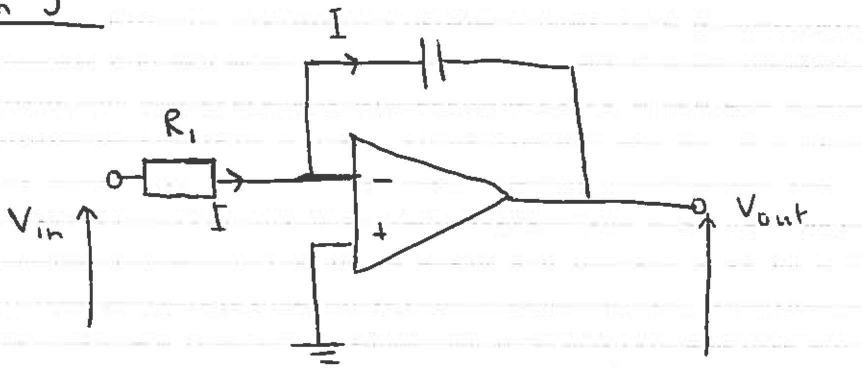
$$= \frac{1}{2\pi \times 200 \times (100 + 97)}$$

$$= 4.04 \times 10^{-6} \text{ F}$$

$$= 4.04 \ \mu\text{F}$$

Question 5

(a)



$$\left.\begin{aligned} V_- &= 0 \\ V_+ &= 0 \end{aligned}\right\} \text{ground}$$

Sum currents into node at negative terminal:

$$I = -C \frac{dV_{out}}{dt}$$

$$I = \frac{V_{in}}{R_1}$$

$$\therefore \frac{V_{in}}{R_1} = -C \frac{dv_{out}}{dt}$$

$$\therefore V_{out} = -\frac{1}{R_1 C} \int_0^t V_{in} \, dt$$

$$\beta = -\frac{1}{R_1 C}$$

(b) (i) $V_{in} = 0.1 \sin(2\pi \times 1000t)$

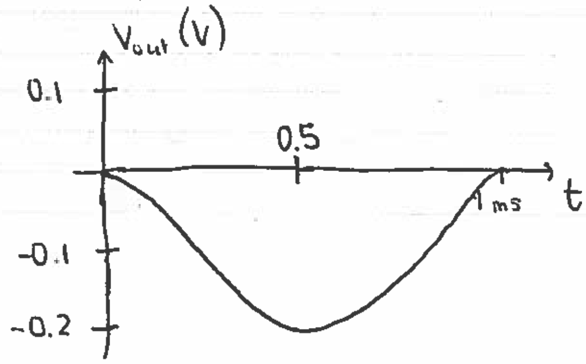
$$V_{out} = +\beta \int_0^t V_{in} \, dt$$

$$= -6.3 \times 10^3 \int_0^t 0.1 \sin(2\pi \times 1000t) \, dt$$

$$= \frac{-6.3 \times 10^3}{2\pi \times 1000} \, 0.1 \left[\cos(2\pi \times 1000t)\right]_0^t$$

$$= -0.1\left(1 - \cos(2\pi \times 1000t)\right)$$

$$= 0.1\left(\cos(2\pi \times 1000t) - 1\right)$$

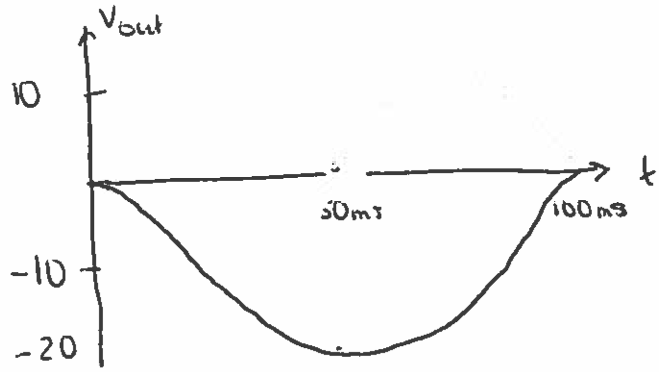


$$V_{out} = -\beta \int_0^t V_{in}\, dt$$

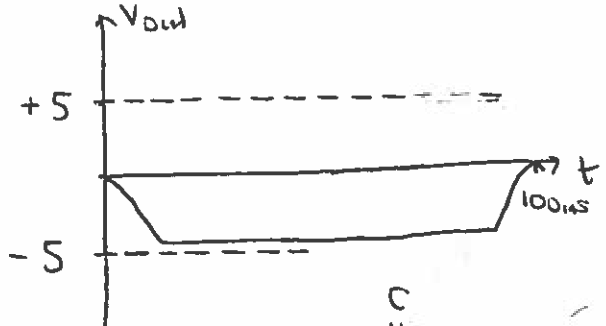
$$= -6.3 \times 10^3 \int_0^t 0.1 \sin(2\pi \times 10t)\, dt$$

$$= -\frac{\cancel{6.3} \times 10^{\cancel{3}2}}{\cancel{2\pi} \times \cancel{10}}\, 0.1 \left[\cos(2\pi \times 10t)\right]_0^t$$

$$= -10(1 - \cos(2\pi \times 10t))$$

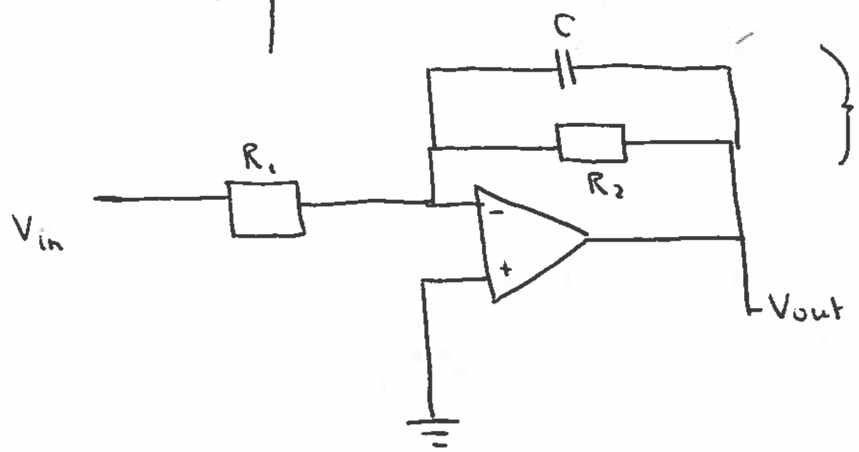


But op-amp is only drawing power from $\pm 5V$ rails. $\rightarrow$ V_{out} will be clipped.



In reality, clipping will be at slightly less than $\pm 5V$, but students still will obtain full marks if they show clipping at $\pm 5V$.

(c) (i)



$$Z = R_2 \| C = \frac{R_2}{1 + j\omega C R_2}$$

Summing currents into node at negative terminal:

$$\frac{V_{in} - 0}{R_1} = \frac{0 - V_{out}}{Z} \Rightarrow \frac{V_{out}}{V_{in}} = -\frac{Z}{R_1} = -\frac{R_2}{R_1(1 + j\omega C R_2)}$$

(ii) $$\frac{V_{out}}{V_{in}} = \frac{-R_2}{R_1(1 + j\omega R_2 C)}$$

3 dB cut off when real = imaginary in denominator :

$$1 = \omega C R_2$$

$$\therefore R_2 = \frac{1}{\omega C}$$

$$= \frac{1}{2 \times \pi \times 10 \times 10^3 \times 3.18 \times 10^{-9}}$$

$$= 5000\,\Omega$$

$$= 5\ k\Omega$$

6 (a) K-map

OP \ MN	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

Sum of products

$$Z = MN + OP + PN + MP + NO + MO$$

$$\bar{Z} = \overline{MN + OP + PN + MP + NO + MO}$$

$$\bar{Z} = \overline{MN} \cdot \overline{OP} \cdot \overline{PN} \cdot \overline{MP} \cdot \overline{NO} \cdot \overline{MO}$$

$$Z = \overline{\overline{MN} \cdot \overline{OP} \cdot \overline{PN} \cdot \overline{MP} \cdot \overline{NO} \cdot \overline{MO}}$$

Total number of gates required: 6 2input NAND & 1 6input NA

OP \ MN	00	01	11	10
00	0	0	1	0
01	0	1	1	1
11	1	1	1	1
10	0	1	1	1

Product of sums

$$\bar{Z} = \bar{M}\bar{N}\bar{O} + \bar{O}\bar{P}\bar{M} + \bar{O}\bar{P}\bar{N} + \bar{M}\bar{N}\bar{P}$$

$$\bar{Z} = \overline{(M+N+O)} + \overline{(O+P+M)} + \overline{(O+P+N)} + \overline{(M+N}$$

$$Z = \overline{\overline{(M+N+O)} + \overline{(O+P+M)} + \overline{(O+P+N)} + \overline{(M+N}}$$

Total number of gates required: 4 3input NOR & 1 4 input NOR.

7 (a) LSB = M1 , MSB = P1

Input impedance at N1 is $4R = 4k\Omega$

Input impedance at P1 is $R = 1k\Omega$.

When all inputs are connected to 5V (HIGH), the total current is

$$I = \frac{5}{8R} + \frac{5}{4R} + \frac{5}{2R} + \frac{5}{R}$$

$$V_{out} = -\frac{R}{2} \cdot I = -\frac{R}{2} \times 5\left(\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + 1\right)$$

$$= -\frac{5}{2} \times \frac{15}{8} = -\frac{75}{16} = -4.6875 \text{ V}$$

(b) Drawbacks of this design:

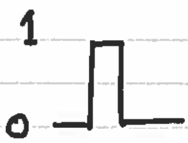
i) Resistance values are difficult to implement due to physical dimensions for DACs with higher bits. The range of resistor values is wide.

ii) Accuracy of the resistors is difficult to achieve, esp. for DACs with higher bits

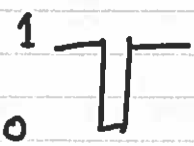
iii) Input impedance of each bit is different.

An R-2R ladder DAC can be used to avoid the abovementioned pro
In this design the values of resistors required are R & 2R only.

8(a) A static hazard in a combinational digital circuit is when a signal undergoes a momentary transition when it is supposed to remain unchanged. A static hazard can
of 0 or 1.

Static 0 hazard 

The signal is expected to stay at 0.

Static 1 hazard 

The signal is expected to stay at 1.

CD \ AB	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	0	0

$Z = A\bar{C} + \bar{A}D$

To remove static 1 hazard, an additional term is needed to cover the adjacent but non-overlapping groups. This is shown in the dotted group in the k-map.

Therefore the function required to avoid static 1 hazard is:

$$Z = A\bar{C} + \bar{A}D + \bar{C}D.$$

CD \ AB	00	01	11	10
00	0	0	1	1
01	1	1	1	1
11	1	1	0	0
10	0	0	0	0

$\bar{Z} = AC + \bar{A}\bar{D}.$

To remove static 0 hazard, an additional term is needed to cover the adjacent but non-overlapping groups. This is shown in the dotted group in the k-map.

Therefore the function required to avoid static 0 hazard is:

$$Z = AC + \bar{A}\bar{D} + C\bar{D}.$$

9 (a) Truth table for 4 bit synchronous up counter:

Present				Next				J_3	K_3	J_2	K_2	J_1	K_1	J_0	K_0
0	0	0	0	0	0	0	1	0	X	0	X	0	X	1	X
0	0	0	1	0	0	1	0	0	X	0	X	1	X	X	1
0	0	1	0	0	0	1	1	0	X	0	X	X	0	1	X
0	0	1	1	0	1	0	0	0	X	1	X	X	1	X	1
0	1	0	0	0	1	0	1	0	X	X	0	0	X	1	X
0	1	0	1	0	1	1	0	0	X	X	0	1	X	X	1
0	1	1	0	0	1	1	1	0	X	X	0	X	0	1	X
0	1	1	1	1	0	0	0	0	X	X	1	X	1	X	1
1	0	0	0	1	0	0	1	X	0	0	X	0	X	1	X
1	0	0	1	1	0	1	0	X	0	0	X	1	X	X	1
1	0	1	0	1	0	1	1	X	0	0	X	X	0	1	X
1	0	1	1	1	1	0	0	X	0	1	X	X	1	X	1
1	1	0	0	1	1	0	1	X	0	X	0	0	X	1	X
1	1	0	1	1	1	1	0	X	0	X	0	1	X	X	1
1	1	1	0	1	1	1	1	X	0	X	0	X	0	1	X
1	1	1	1	0	0	0	0	X	1	X	1	X	1	X	1

(b)

J_3

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	0	0	0	0
01	0	0	1	0
11	X	X	X	X
10	X	X	X	X

K_3

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	X	X	X
01	X	X	X	X
11	0	0	1	0
10	0	0	0	0

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	0	0	1	0
01	X	X	X	X
11	X	X	X	X
10	0	0	1	0

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	X	X	X
01	0	0	1	0
11	0	0	1	0
10	X	X	X	X

J_1

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	0	1	X	X
01	0	1	X	X
11	0	1	X	X
10	0	1	X	X

K_1

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	X	1	0
01	X	X	1	0
11	X	X	1	0
10	X	X	1	0

J_0

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	1	X	X	1
01	1	X	X	1
11	1	X	X	1
10	1	X	X	1

K_0

Q_3Q_2 \ Q_1Q_0	00	01	11	10
00	X	1	1	X
01	X	1	1	X
11	X	1	1	X
10	X	1	1	X

$J_3 = Q_0 Q_1 Q_2$

$K_3 = Q_0 Q_1 Q_2$

$J_2 = Q_1 Q_0$

$K_2 = Q_1 Q_0$

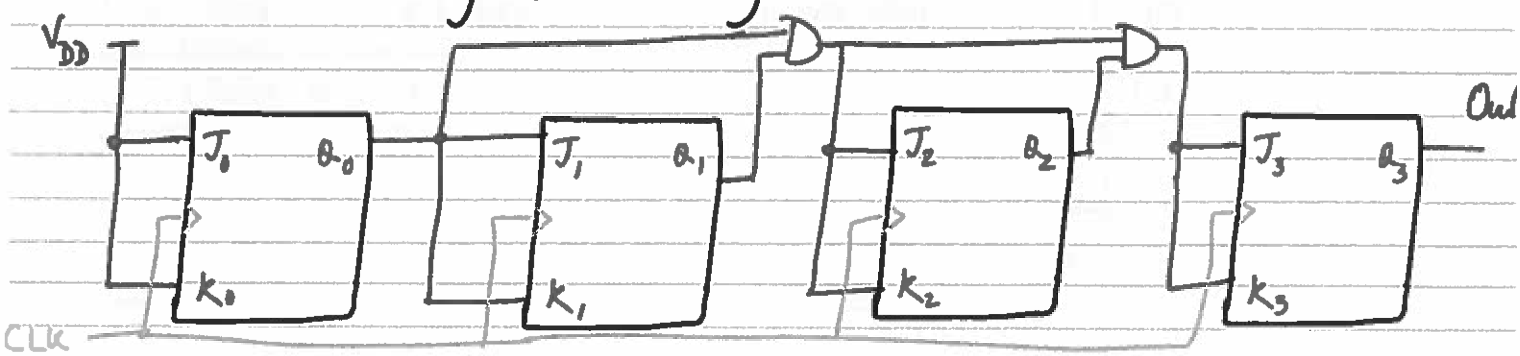
$J_1 = Q_0$

$K_1 = Q_0$

$J_0 = 1$

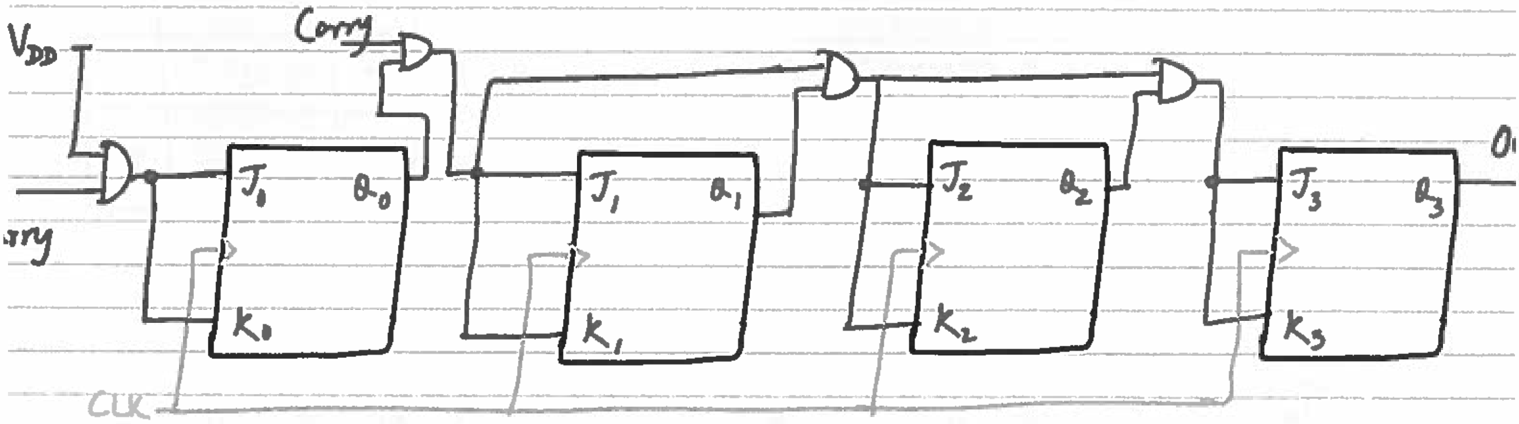
$K_0 = 1$

(c) Without carry in & carry out

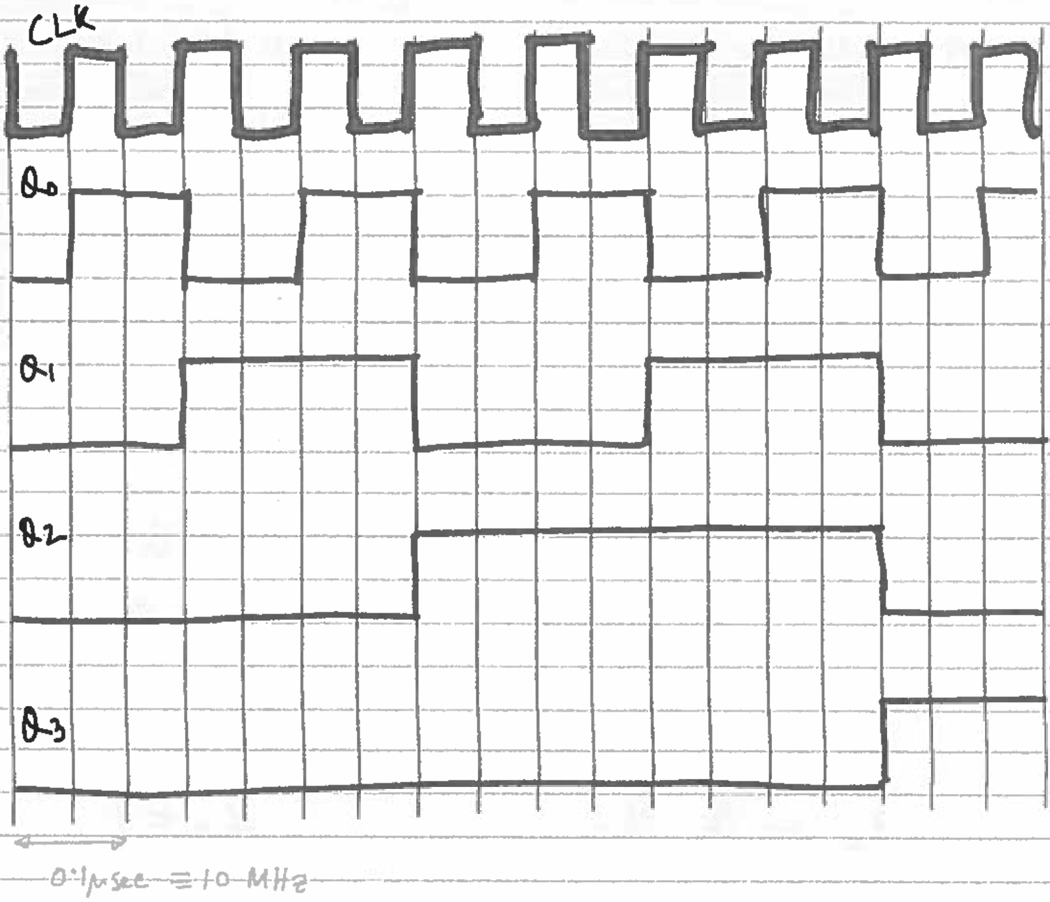


With carryin & Carry out :

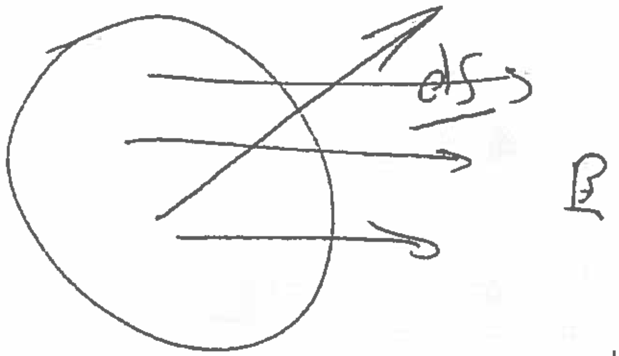
Inspecting $J_0, k_0, J_1, k_1, J_2, k_2$ & $J_3 . k_3$, previous stages (carry in) should be incorporated with 'AND' gates.



(d)



II (i) (i)

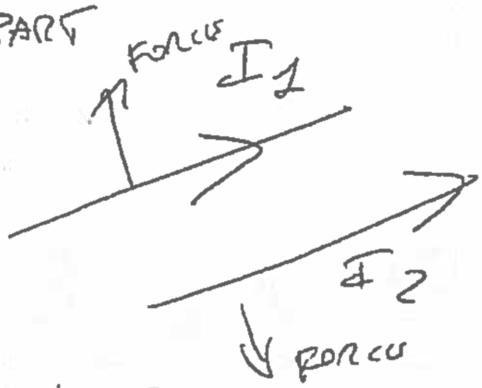


THE TOTAL MAGNETIC FLUX ϕ THROUGH A LOOP IS GIVEN BY

$$\phi = \iint_S \underline{B} \cdot d\underline{S}$$

WHERE $\underline{B}$ IS THE MAGNETIC FLUX DENSITY

(B) $\underline{B}$ IS DEFINED IN TERMS OF THE FORCE BETWEEN WIRE CARRYING CURRENTS I_1, I_2 A DISTANCE r APART



$$F = \mu_0 \frac{I_1 I_2 L}{2\pi r} = B I_2 L$$

WITH $B = \dfrac{\mu_0 I_1}{2\pi r}$ DUE TO CURRENT I_1

IN THE ELECTROMAGNETS IN FIG 1 THE MAGNETIC FLUX IS CONFINED ALMOST ENTIRELY TO THE MAGNETIC MATERIAL. SINCE THE CROSS SECTIONS OF THE MAGNETIC MATERIAL AND THE AIR GAP ARE THE SAME, THE MAGNETIC FLUX DENSITY B IS ALSO THE SAME

FROM AMPERE'S LAW

$$\oint \underline{H} \cdot \underline{dl} = NI$$

$$H_g l_g + H_c l_c = NI$$

FROM CONSERVATION OF MAGNETIC FLUX

$$B_g = B_c$$ ASSUMING NO SATURATION AND NO FRINGING FIELD

AND

$$B_c = \mu_r \mu_0 H_c$$

$$B_g = \mu_0 H_g$$

HENCE

$$I = \frac{1}{N}\left(\frac{B_g}{\mu_0} l_g + \frac{B_c}{\mu_r \mu_0} l_c\right)$$

$$= \frac{1}{\mu_0 N}\left(l_g + \frac{l_c}{\mu_r}\right) B_g$$

$$= \frac{1}{4\pi 10^{-7}\, 200}\left(2 \times 10^{-3} + \frac{40 \times 10^{-3}}{250}\right) 0.2 = 1.72A$$

$$\iint_S \bar{D}\cdot d\bar{S} = \varphi$$

$$\varphi = \epsilon_0 \bar{E}_r \cdot \bar{A}_r$$

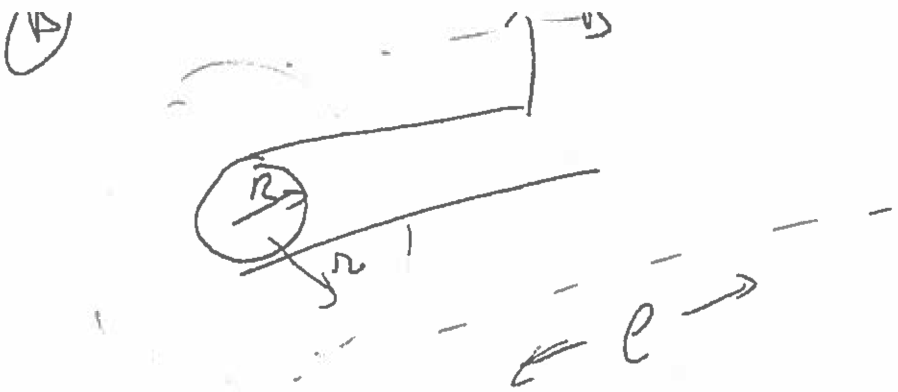
$$E_r = \frac{\varphi}{\epsilon_0 4\pi r^2}$$

$$V = \int_{d/2}^{\infty} \frac{\varphi}{4\pi \varepsilon_0} \cdot \frac{dr}{r^2} = \frac{\varphi}{4\pi \epsilon_0 d/2}$$

$$C = \frac{\varphi}{V} \Rightarrow C = 4\pi \epsilon_0 \frac{d}{2} = 2\pi \epsilon_0 d$$

GAUSSIAN SURFACE. AREA $A_r = 4\pi r^2$

(b)



NOW SYMMETRY D IS ALWAYS IN RADIAL DIRECTION

ρ = CHARGE DENSITY

FROM GAUSS' LAW : FLUX OF D = CHARGE ENCLOSED

CHARGE ENCLOSED = ρl

FLUX OF D = $2\pi r l D$

$$\Rightarrow 2\pi r l D = \rho l$$

$$\Rightarrow D = \frac{\rho}{2\pi r} \qquad E = \frac{\rho}{2\pi r \varepsilon_0}$$

(c) IF SEVERAL CHARGES ARE INTRODUCED INTO A REGION, THEIR EFFECT IS THE SUM OF THEIR INDIVIDUAL EFFECTS.

FOR $+\rho$ CHARGE DENSITY OF WIRE 1, THE VOLTAGE BETWEEN THE WIRES IS (NEGLECTING THE CHARGE ON THE SECOND WIRE)

$$V = \int_{R}^{d-R} \frac{\rho}{2\pi\varepsilon_0} \frac{dr}{r} = \frac{\rho}{2\pi\varepsilon_0} \ln\left(\frac{d-R}{R}\right)$$

— ρ is on wire 2 and results in similar partial [illegible]

thus $$V = 2\frac{\rho}{2\pi\varepsilon_0}\ln\frac{d-R}{R}$$

Capacitance per unit length

$$C_\ell = \frac{\rho}{|V|} = \frac{\pi\varepsilon_0}{\ln\frac{d-R}{R}}$$

d)

$$C_\ell = \frac{\pi\varepsilon_0}{\ln\left(\frac{2-10^{-2}}{10^{-2}}\right)} = 5.25\ pF$$

Max E-field is at the surface of either wire

$$E_{max} = \frac{\rho}{2\pi\varepsilon_0}\left[\frac{1}{R}+\frac{1}{d-R}\right]$$ → neglect

$d >> R$

$$E_{max} = \frac{1}{2\pi\varepsilon_0}\cdot\frac{1}{R}\cdot\frac{\pi\varepsilon V}{\ln\frac{d-R}{R}}$$

$$\Rightarrow V = 2R\ln\left(\frac{d-R}{R}\right)E = 2\times 10\,10^{-3}\ln\left(\frac{2-10^{-2}}{10^{-2}}\right)\cdot 10^{6} \approx 10k$$

$$H = \frac{I}{2\pi r}$$

DISTANCE FROM THE WIRE THE MAGNETIC FIELD CAN BE TAKEN AS CONSTANT OVER THE COIL

$$B = \mu_0 H$$

$$\Phi_0 = B \times AREA = \frac{\mu_0 I}{2\pi r} \times A =$$

$$= \frac{\cancel{4}^{2}\cancel{\pi} 10^{-7} \cdot 2}{\cancel{2\pi}\, 0.15} \times 10^{-4} = 2.66\, 10^{-10}\ \text{WB}$$

$$V = N \frac{d\left[\Phi_0 \sin(2\pi f t)\right]}{dt}$$

$$V = N \Phi_0 2\pi f \cos(2\pi f t)$$

WITH N = NUMBER OF TURNS AND $f = 50$ HZ

PEAK VOLTAGE V_P WHEN $\cos(2\pi f t) = 1$

$$N = \frac{V_P}{\Phi_0 2\pi f} = \frac{2\,10^{-6}}{2.66\, 10^{-10}\ 2\pi\, 50} = 24$$