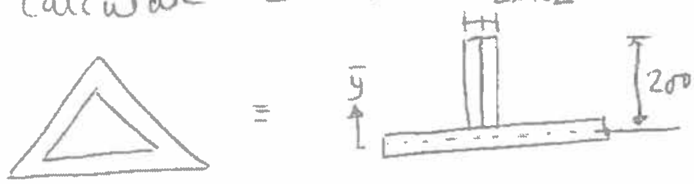
2P2 2019. Jo VI ①

Q1(a)

calculate I value.



$\left(\frac{200-0.5}{2}+0.5\right)$

$A\bar{y} = \sum A_i y_i$

$$\bar{y} = \frac{400(1)(0) + 200(2\sqrt{2})(100.25)}{400 + (199.5)(2\sqrt{2})} = \underline{\underline{58.7\text{mm}}}$$

$(bt^3/12)$ $199.5\sqrt{2}$

$$I = \sum I_{local} + A\bar{y}^2$$

$$= 33.3 + 400(58.7)^2 + 2\left[935756 + 282.1(41.1)^2\right]$$

$$= \underline{\underline{4.20\times10^6}}\ \text{mm}^4.$$

At (a) $\sigma_L = \frac{My}{I}$

$$M = 335\left(\tfrac{1}{19}L\right) + 335\left(\tfrac{2}{19}L\right) \ldots = 335(10)L = \underline{\underline{16.1\text{kNm}}}$$

$$\sigma_{L,A} = \frac{16.1\times58.7}{4.2} = \underline{\underline{225\text{MPa}}}$$

$$\sigma_{L,B} = \frac{16.1\times \cancel{41.1}\ 41.3}{4.2} = \cancel{157\text{MPa}}\ 158\text{MPa}$$

(1)(a)(ii)

$$\tau = \frac{SA\bar{y}}{It}$$

Shear at support = $335 \times 19 = \underline{6365 N}$

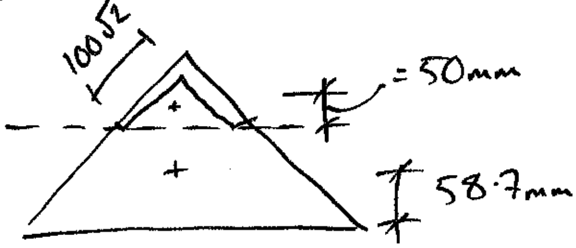
Torsion = $19 \times 335 \times 200 = \underline{1.27 \times 10^6}$ Nmm

at A

$\tau_{shear} = 0$ MPa

$$\tau_{Torsion} = \frac{T}{2A_e t} = \frac{1.27 \times 10^6}{2(200 \times 200)(1)} = \underline{\underline{15.9 \text{ MPa}}}$$

at B



distance $\bar{y} = (100 - 58.7) + 50$
$= \underline{\underline{91 mm}}$

$$\tau_s = \frac{6365 (2 \times 100\sqrt{2} \times 1)(91)}{4.2 \times 10^6 \times 2 \times 1} = \underline{\underline{19.5 \text{ MPa}}}$$

$\tau_T = 15.9$ MPa.

$\therefore$ at A $\tau_a = \underline{15.9 \text{ MPa}}$

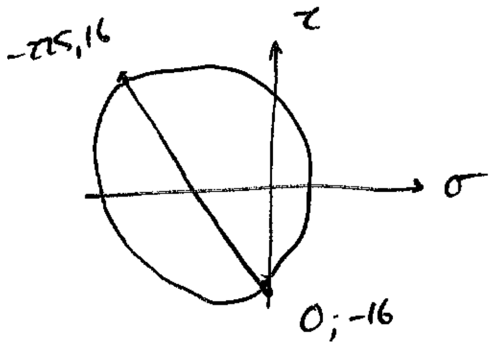
$\tau_b = 19.5 - 15.9 = \underline{3.6 \text{ MPa}}$

& on the other side $\tau_{b,2} = 19.5 + 15.9 = \underline{\underline{35.4 \text{ MPa}}}$

(1)(b)(i)

at A $\sigma_{A,L} = 225\,\text{MPa}$

$\tau_A = 16\,\text{MPa}$

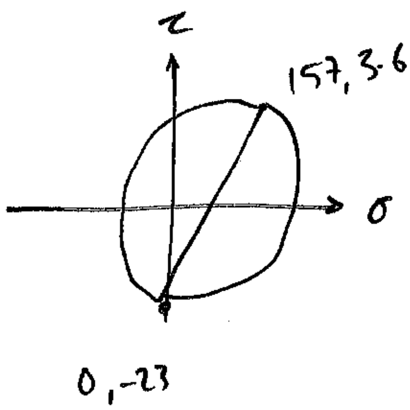


$$\sigma_2 = -\frac{225}{2} - \sqrt{\left(\frac{225}{2}\right)^2 + 16^2} = -226\,\text{MPa}$$

$$\sigma_1 = (\,''\,) + (\,''\,) = +1\,\text{MPa}$$

at B $\sigma_L = +157\,\text{MPa}$

$\tau = 3.6\,\text{MPa}$ [or $\tau = 35\,\text{MPa}$)



$\sigma_1 = 157\,\text{MPa}$

$\sigma_2 = 0\,\text{MPa}$.

Tresca (at A) $\lambda = \dfrac{275}{\sigma_1 - \sigma_2} = \underline{\underline{1.21}}$

Von Mises $(\sigma_1-\sigma_2)^2 + (\sigma_2-\sigma_3)^2 + (\sigma_1-\sigma_3)^2 = 27^2$

$\gamma = \underline{\underline{1.22}}$

By inspection no need to check B.

Question 1

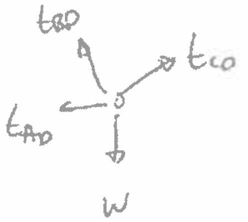
Answered by 115 candidates. The question was moderately well answered. Most students spent too long creating long winded calculations of the second moment of area. The majority were able to calculate the moment caused at the base of the cantilever. Shear stresses arise from both the force in the cable and its eccentricity (torsion) and spotting this was essential. Section (b) - yield criteria – was answered quite well, although the need to comment on the values flummoxed some candidates. Part (d) was well answered.

Q2 (a) (i)

3 bars, 2 nodal freedoms ⇒ 1 state of self stress

(b)(i) ignore BC

take AD as redundant bar.

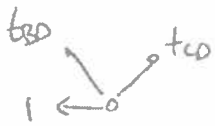


$t_{AD} = 0$

$t_{BC} = t_{CD} = \dfrac{W}{\sqrt{2}}$

$$\mathbf{t}_0 = \begin{bmatrix} t_{AD} \\ t_{BD} \\ t_{CD} \end{bmatrix} = W \begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$ particular eqm solution

(b)(ii) $t_{AD} = 1$



$t_{CD} = 1/\sqrt{2}$ $\quad t_{BD} = -1/\sqrt{2}$

$$\mathbf{s} = \begin{bmatrix} 1 \\ -1/\sqrt{2} \\ +1/\sqrt{2} \end{bmatrix}$$

(iii) general solution

$$\mathbf{t} = \mathbf{t}_0 + x\,\mathbf{s}$$

$$\mathbf{e} = \mathbf{F}\,\mathbf{t} \qquad F = \frac{L}{AE}\begin{bmatrix} 1 & & \\ & \sqrt{2} & \\ & & \sqrt{2} \end{bmatrix}$$

hence $$\mathbf{e} = \frac{LW}{AE}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + \frac{Lx}{AE}\begin{bmatrix} 1 \\ -1 \\ +1 \end{bmatrix}$$

(2)(b)(iii) contd.

By virtual work

$$\underset{\sim}{s} \cdot \underset{\sim}{e} = 0$$

$$\frac{L}{AE}\left(W\left(0 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right)\right) + x\left(1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}\right) = 0$$

$$\frac{L}{AE}\left(0 + x(1+\sqrt{2})\right) = 0$$

$$x = 0$$

thus elastic soln equal to our choice of particular soln

$$\underset{\sim}{t}_e = W\begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix}$$

(2)(c)(i)

Now add term due to heating

$$\underset{\sim}{e} = \underset{\sim}{f}\,\underset{\sim}{t} + \begin{bmatrix} \alpha L T \\ 0 \\ 0 \end{bmatrix}$$

Compatibility equation $S.e = 0$

$$\Rightarrow \frac{L}{AE}\left(x(1+\sqrt{2})\right) + \alpha L T = 0$$

hence $x = \dfrac{-AE\alpha T}{(1+\sqrt{2})}$

therefore $t = W\begin{bmatrix} 0 \\ 1/\sqrt{2} \\ 1/\sqrt{2} \end{bmatrix} - \dfrac{AE\alpha T}{(1+\sqrt{2})}\begin{bmatrix} 1 \\ -1/\sqrt{2} \\ +1/\sqrt{2} \end{bmatrix}$

2(c)(ii)

Total extensions are given by:

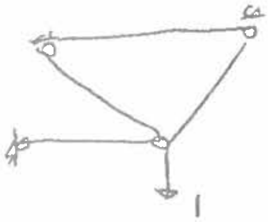
$$\underset{\sim}{e} = Ft + \alpha L T$$

$$= \frac{WL}{AE}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} - \frac{\alpha L T}{1+\sqrt{2}}\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} + \alpha L T\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

(2)(c)(ii)

Use virtual work to find deflections:

→ unit load of 1, down

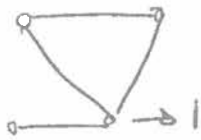


$$t_v^* = \frac{1}{\sqrt{2}}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$\delta_v \cdot 1 = t_v^* \cdot e$$

$$= \frac{2}{\sqrt{2}}\frac{WL}{AE} + 0 + 0 = \underline{\underline{\sqrt{2}\frac{WL}{AE}}}$$

Horizontal deflection, apply unit load to the right



$$t_h^* = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\delta_h \cdot 1 = t_h^* e$$

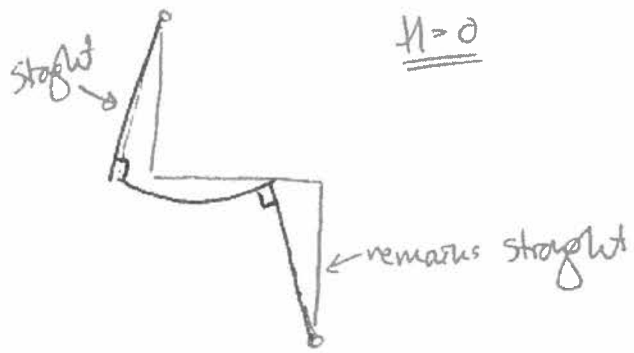
$$= \frac{-\alpha LT}{(1+\sqrt{2})} + \alpha LT$$

$$= \underline{\underline{\alpha LT\left(\frac{-1+(1+\sqrt{2})}{\sqrt{2}+1}\right) = \left(\frac{\sqrt{2}}{1+\sqrt{2}}\right)\alpha LT}}$$

Question 2

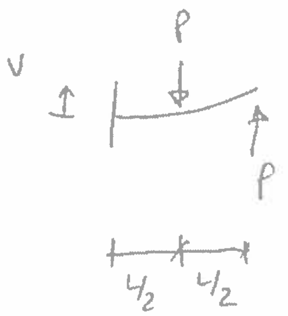
This question was answered by 274 candidates. The question was generally well answered. The number of redundancies was identified well, although most students made their lives more difficult by retaining Bar IV throughout the analysis. It could be removed as it makes no difference to the final solution. Part (b) follows the process given in the lecture notes very closely and most students were able to apply this. Part (c) led to a few more hiccups, particularly when considering which bars are affected by the temperature change. Part (d) was poorly answered, and many students were not able to apply virtual work to the problem.

Q3 (a)(i) 8



$\underline{\underline{H = 0}}$

(ii) consider mid-span BC



$$v = \frac{PL^3}{3} - \frac{P\left(L/2\right)^3}{3} - \frac{P\left(L/2\right)^2}{2}\left(\frac{L}{2}\right.$$

$$\underline{\underline{v = \frac{11PL^3}{48EI}}}$$ 1

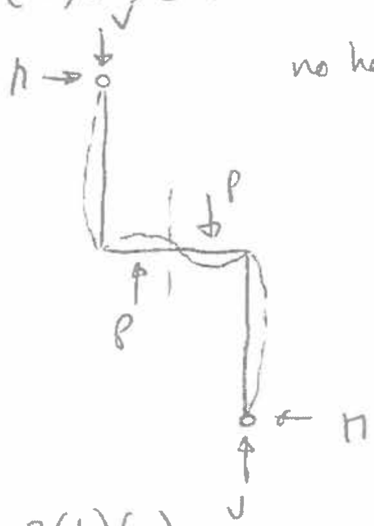
(iii) $h = \theta_B \cdot 2L$

$$\theta_B = -\frac{P\left(L/2\right)^2}{2} + \frac{P\left(L^2\right)}{2}$$

$$= \frac{3}{8}\frac{PL^2}{EI}$$

$$h = \frac{6}{8}\frac{PL^3}{EI} = \underline{\underline{\frac{3}{4}\frac{PL^3}{EI}}}$$ 2

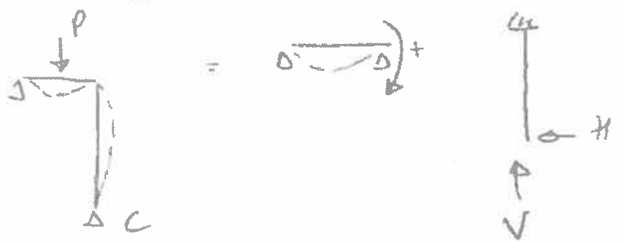
(3)(b)(ii)



no horizontal movement.

3(b)(ii)

extract half frame



$$h_{BC} = \frac{H(2L)^3}{3EI} = \frac{8}{3}\frac{HL^3}{EI}$$

$$\theta_B = \frac{2HL^2}{3EI} - \frac{PL^2}{16EI}$$

$$h = \theta_B \cdot 2L + h_{BC}$$

$$= \frac{4}{3}HL^3 - \frac{PL^3}{8} + \frac{8}{3}HL^3 \quad \underline{\underline{= 0}}$$

$$\therefore \quad 4HL^3 = \frac{PL^3}{8}$$

$$\underline{\underline{H = \frac{P}{32}}}$$

$$\theta_B = \frac{2PL^2}{3(32)} - \frac{PL^2}{16} = \frac{-PL^2}{24EI}$$

3(b)(ii) contd.

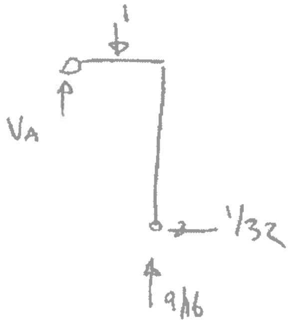
Moments about centre

$$P\left(\frac{L}{2}\right) + \frac{P}{32}(2L) = V(L)$$

$$\frac{18}{32}P = V \quad \text{or} \quad \frac{9}{16}P$$

Case 1	Case 2
$H = 0$	$H = P/32$
$V = P$	$V = 9/16\,P$

note for completeness for (b)(ii)

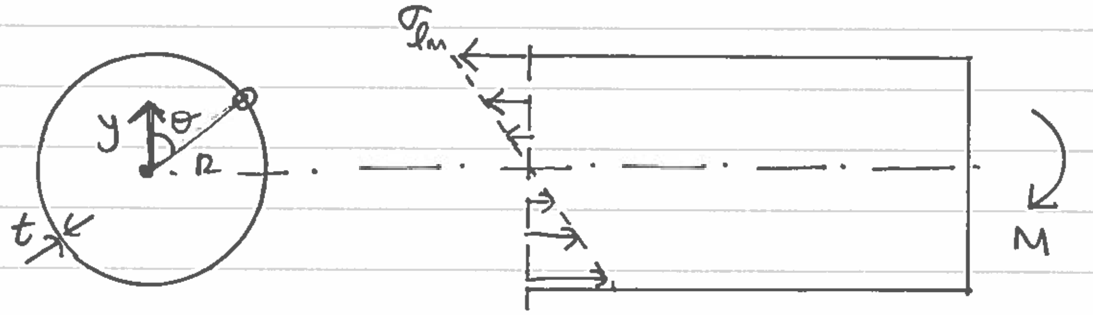


$$V_A = \frac{7}{16}P$$

Question 3

This question was answered by 80 candidates. The question concerns symmetry and anti-symmetry. Most candidates were unable to draw a structurally convincing deflected shape and thus not knowing how the frame behaved were unable to identify the boundary conditions that allow the problem to be solved. Note here that deflected shape drawing is a test of communication, not art, skills. Only a handful of students correctly identified the expressions for vertical and horizontal deflections, mainly because the boundary conditions were not considered carefully enough. Thinking about the deflected shape would help with this part of the question. In section (b)(i) most students drew a different deflected shape, and indeed this shape is the easier of the two to visualise. Part (b)(ii) was not well answered, again stemming from not seeing how the frame in Part (a) was working.

4a)



FROM EULER-BERNOULLI BEAM THEORY:

$$\sigma_{lm} = My/I \quad , \text{ WHERE } \quad I = \int y^2 \, dA = \pi R^3 t$$

FROM MECH. DATA BOOK, INTEGRATION OR.

$$I = \frac{\pi}{4}\left(R_o^4 - R_i^4\right)$$

$$R_o = R_i + t$$

$$R \simeq R_o \simeq R_i$$

$$\therefore I = \pi R^3 t$$

FROM GEOMETRY:

$$y = R\cos\vartheta$$

6 $$\therefore \sigma_{lm} = \frac{M\cos\vartheta}{\pi R^2 t}$$

b) $p = 1200\,\text{kPa}$; $E = 210\,\text{GPa}$; $\nu = 0.3$; $R = 1.5\,\text{m}$; $t = 10\times10^{-3}\,\text{m}$

STRESS:

$$\sigma_{hp} = \frac{pR}{t} = \frac{1200\times10^3 \times 1.5}{10\times10^{-3}} = 180\,\text{MPa}$$

$$\sigma_{lp} = \frac{pR}{2t} = 90\,\text{MPa}$$

STRAINS:

$$\varepsilon_{hp} = \frac{1}{E}\left(\sigma_h - \nu\sigma_{lp} - \nu\sigma_t\right) = \frac{1}{210\times10^9}\left(180\times10^6 - 0.3\times 90\times10^6\right) = 7.29\times10^{-4}$$

6 $$\varepsilon_{lp} = \frac{1}{E}\left(\sigma_{lp} - \nu\sigma_h - \nu\sigma_t\right) = \frac{1}{210\times10^9}\left(90\times10^6 - 0.3\times180\times10^6\right) = 1.71\times10^{-4}$$

$$\varepsilon_{tp} = \frac{1}{E}\left(\sigma_t - \nu\sigma_{lp} - \nu\sigma_h\right) = \frac{1}{210\times10^9}\left(-0.3\times180\times10^6 - 0.3\times90\times10^6\right) = -3.86\times10^{-4}$$

4c) i) LOADS CAUSED BY FLUID AND SELF-WEIGHT (kN/m)

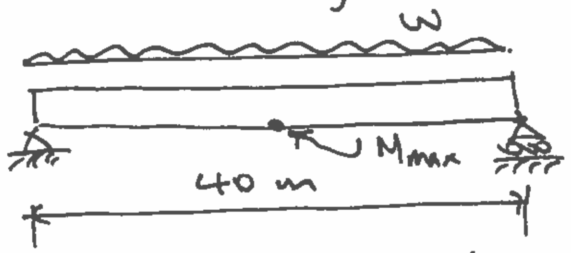
$$W = \underbrace{\left(8\,\text{kN/m}^3 \times \pi \times 1.5^2\right)}_{\text{FLUID}} + \underbrace{\left(76.9\,\text{kN/m}^3 \times \pi \times 3 \times 10 \times 10^{-3}\right)}_{\text{SELF WEIGHT CYLINDER}}$$

$$= 56.56 + 7.25$$

2

$$= 63.81\ \text{kN/m}$$

TREAT CYLINDER AS SIMPLY-SUPPORTED BEAM WITH UDL:



$$M_{max} = wL^2/8 = 63.81 \times 40^2/8 = 12{,}762\ \text{kNm}$$

$$\sigma_{lm} = \frac{M \cos \phi}{\pi R^2 t} = \frac{12762 \times 10^6}{\pi \times 1500^2 \times 10} = 180.5\ \text{MPa}$$

$\therefore$ TOTAL LONGITUDINAL STRESS $\sigma_l = \sigma_{lm} + \sigma_{lp}$

$$= 180.5 + 90 = \underline{\underline{270.5\ \text{MPa}}}$$

TOTAL LONGITUDINAL STRAIN $\varepsilon_l = \varepsilon_{lm} + \varepsilon_{lp}$

$$= \frac{1}{210 \times 10^9}\left(180.5 \times 10^6\right) + 1.71 \times 10^{-4}$$

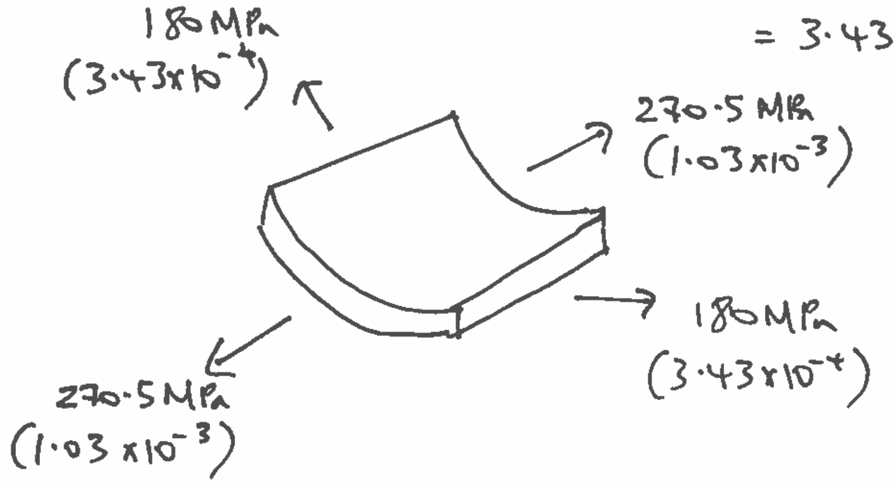
5

$$= 1.03 \times 10^{-3}$$

TOTAL HOOP STRESS $\sigma_h = \sigma_{hp} = \underline{\underline{180\ \text{MPa}}}$

TOTAL HOOP STRAIN $\varepsilon_h = \varepsilon_{hp} - \nu \dfrac{\sigma_l}{E} = 7.29 \times 10^{-4} - \dfrac{1}{210 \times 10^9}\left(0.3 \times 270.5 \times 10^6\right)$

$$= 3.43 \times 10^{-4}$$



4c)ii) AT POINT OF MAXIMUM MOMENT :
- NO SHEAR FORCE
- NO TORQUE
- σ_ℓ AND σ_h ARE PRINCIPAL STRESSES ∴ NO MOHR CIRCLE REQUIRED.

∴ FROM VON MISES:

$$2Y^2 = (\sigma_\ell - \sigma_h)^2 + (\sigma_h - \sigma_t)^2 + (\sigma_t - \sigma_\ell)^2$$

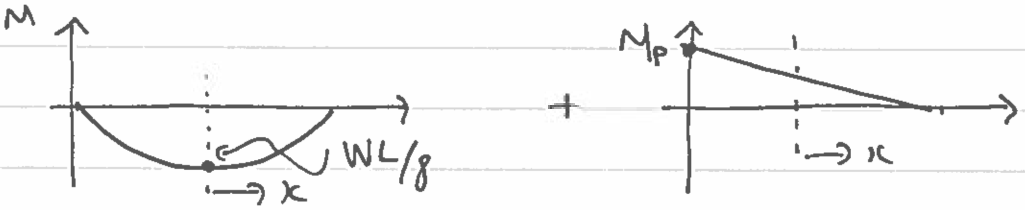
$$Y = \left[\frac{(270.5 - 180)^2 + (180)^2 + (270.5)^2}{2}\right]^{1/2}$$

$$= \underline{\underline{238.5\,\text{MPa}}}$$ ← MINIMUM YIELD STRENGTH REQUIRED.

5 a) <u>LOWER BOUND SOLUTION:</u>

BENDING MOMENTS ALONG SPAN L:

SUPERIMPOSE 2 STANDARD CASES:

2 

$$M = \frac{WL}{8}\left(1 - \frac{4x^2}{L^2}\right) \qquad M = -M_p\left(\frac{1}{2} - \frac{x}{2}\right)$$

1 $$\therefore \quad M = \frac{WL}{8}\left(1 - \frac{4x^2}{L^2}\right) - M_p\left(\frac{1}{2} - \frac{x}{L}\right)$$

MAXIMUM $\frac{dM}{dx} = 0 = \frac{WL}{8} \cdot \frac{8x}{L^2} + \frac{M_p}{L} = 0$

$\therefore x = M_p/W \leftarrow$ SET MOMENT HERE TO M_p TO FIND MAX SAFE VALUE.

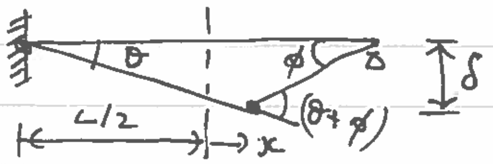
2 $$M_p = \frac{WL}{8}\left[1 - \frac{4}{L^2}\left(\frac{M_p}{W}\right)^2\right] - M_p\left[\frac{1}{2} - \frac{1}{L}\left(\frac{M_p}{W}\right)\right]$$

RE-ARRANGE TO GIVE QUADRATIC IN $\left(M_p/WL\right)$

$$-\frac{1}{2}\left(\frac{M_p}{WL}\right)^2 + \frac{3}{2}\left(\frac{M_p}{WL}\right) - \frac{1}{8} = 0$$

2 $$\frac{M_p}{WL} = 0.08579 \Rightarrow \underline{\underline{W = \frac{11.657 M_p}{L}}}$$

5 b) <u>UPPER BOUND SOLUTION:</u>

3 

COMPATIBILITY:

$$\theta = \delta/\left(\tfrac{L}{2} + x\right)$$
$$\phi = \delta/\left(L/2 - x\right)$$

WORK DONE $= W\delta/2$

$$\text{ENERGY DISSIPATED} = M_p\theta + M_p(\theta + \phi)$$
$$= \frac{2M_p\delta}{(L/2 + x)} + \frac{M_p\delta}{(L/2 - x)}$$

$$\therefore \quad \frac{1}{2} = \left(\frac{M_p}{W}\right)\left(\frac{3L/2 - x}{L^2/4 - x^2}\right)$$

CORRECT VALUE $x = 0.08579L$

$$\frac{1}{2} = \left(\frac{M_p}{W}\right)\frac{5.8248}{L}$$

4 $\Rightarrow W = \underline{\underline{\dfrac{11.657M_p}{L}}}$ IDENTICAL TO LOWER BOUND $\therefore$ THIS IS THE COLLAPSE LOAD.

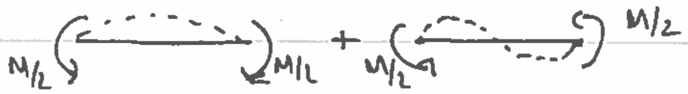
5 c) SUPERIMPOSE THE FOLLOWING DATA BOOK CASES

① 

$\Rightarrow \delta_1 = 5WL^3/384EI$

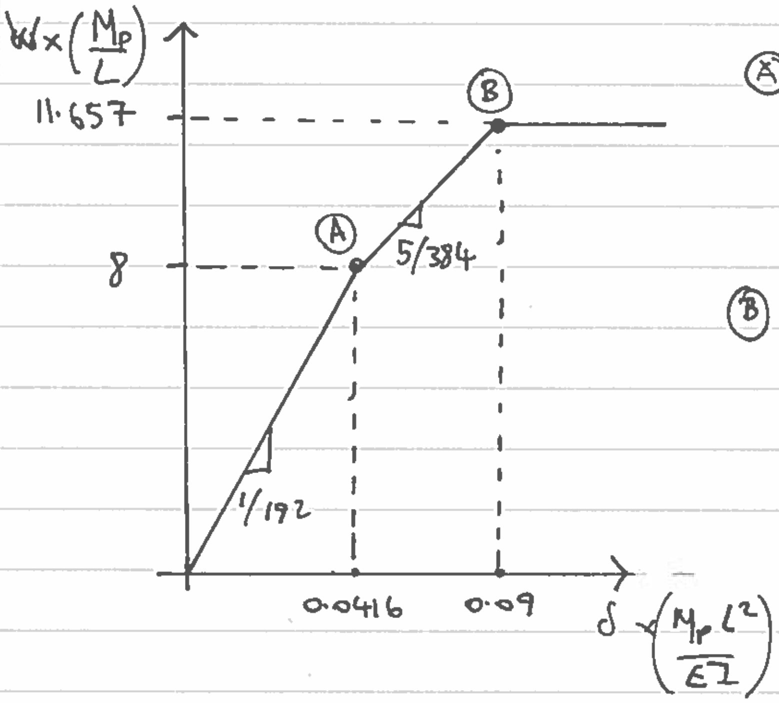
$+$

② 
$M = WL/8$

$\Rightarrow$ 

$$\delta_2 = -ML^2/16EI = WL^3/128EI$$

5 $\therefore \delta$ AT CENTRE $= \delta_1 + \delta_2 = \dfrac{5WL^3}{384EI} - \dfrac{WL^3}{128EI} = \underline{\underline{\dfrac{WL^3}{192EI}}}$

5 d) 

Ⓐ: $\delta = WL^3/192EI$

WHEN $M_p = WL/8 \rightarrow W = 8M_p/L$

$\therefore \delta = \dfrac{LM_p}{24EI} = 0.0416\,\underline{\dfrac{L^2M_p}{24EI}}$

2 Ⓑ ADDITIONAL LOAD AFTER 1ST PLASTIC HINGE FORMS AT ROOT:

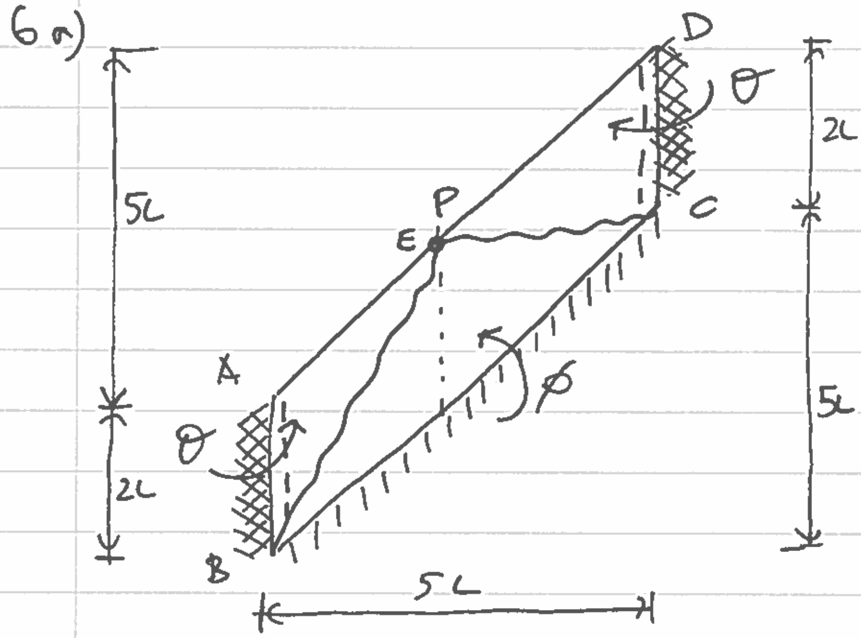
$$\frac{11.657M_p}{L} - \frac{8M_p}{L} = \frac{3.657M_p}{L}$$

$$\therefore \delta_B - \delta_A = \frac{5L^3}{384EI} \times \frac{3.657M_p}{L}$$

4 $= 0.048L^2M_p/EI$

$$\therefore \delta_B = (0.0416 + 0.048)LM_p/EI = \underline{0.09L^2M_p/EI}$$

6 a)



$\theta = 2\delta/5L$

$\phi = \delta/\sqrt{2}L$

WORK DONE : WD = $P\delta$

INTERNAL ENERGY DISSIPATED: (BY PROJECTION METHOD)

$$ED = \underbrace{m_p\sqrt{50}L\phi + m\,5L\theta}_{\text{SAGGING}} + \underbrace{m_p 2L\theta . 2}_{\text{HOGGING}}$$

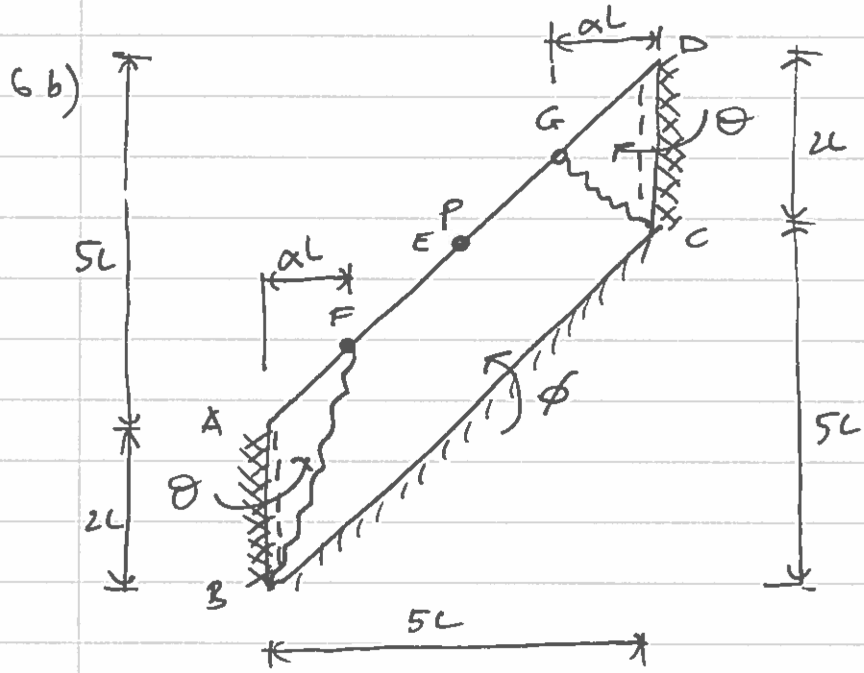
$$= m_pL\left(\sqrt{50}\,\phi + 9\theta\right)$$

$$= m_pL\left(5\delta/L + 18\delta/5L\right) = 43m_p\delta/5$$

WD = ED

$P\delta = 43m_p\delta/5 \Rightarrow P = 43m_p/5 = \underline{\underline{8.6m_p}}$

6 b)



$\theta = \delta/\alpha L$

$\phi = \delta/\sqrt{2}L$

$WD = P\delta$

$$ED = \underbrace{m_p\sqrt{8}\,\alpha L\phi + m_p 4L\theta}_{\text{SAGGING}} + \underbrace{m_p 2L\theta . 2}_{\text{HOGGING}}$$

$$= m_p L(\sqrt{8}\alpha\phi + 8\theta)$$

$$= m_p L(2\alpha\delta/L + 8\delta/\alpha L) = 2m_p\delta(\alpha + 4/\alpha)$$

$WD = ED$

$$P\delta = 2m_p\delta(\alpha + 4/\alpha)$$

$$\therefore P = 2m_p(\alpha + 4/\alpha)$$

$$\frac{dP}{d\alpha} \Rightarrow 2m_p(1 - 4/\alpha^2) = 0 \Rightarrow \underline{\alpha = 2}.$$

$$\therefore P = 2m_p(2+2)$$

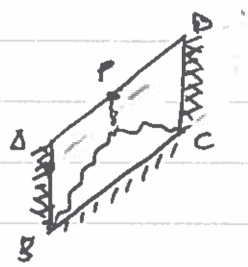
$$\underline{\underline{P = 8m_p}}$$

3 6ci) BEST ESTIMATE OF THE COLLAPSE LOAD IS LOWEST UPPER BOUND i.e. $\underline{\underline{P = 8m_p}}$ ($\Rightarrow$ MECHANISM (b) WITH $\alpha = 2$.)

6d) NO EFFECT, BECAUSE NO CHANGE IN WORK DONE AND ENERGY DISSIPATED (PROJECTED LENGTHS). THIS IS BECAUSE

3 THE ADDITIONAL LENGTH WOULD BE IN THE SIMPLY-SUPPORTED EDGE BC, AND THERE IS NO ENERGY DISSIPATED IN ROTATION ABOUT A SIMPLY-SUPPORTED EDGE.

1 [NOTE THAT REDUCING THE LENGTH BELOW $4L$ ($\alpha = 2$) WOULD TRIGGER A DIFFERENT COLLAPSE MECHANISM $\Rightarrow$ REQUIRES A FRESH CALCULATION]



6cii) $t = 15mm$; $\sigma_y = 350 N/mm^2$

4 $$\therefore m_p = \sigma_y t^2/4 = 350\times10^6 \times 0.015^2/4 = 19.69 kN/m$$

$$P = 8m_p = 8 \times 19.64 = \underline{\underline{157.5 kN}}$$