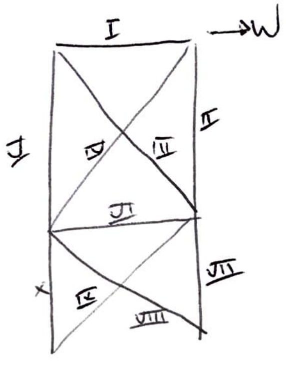
2P2 2021 J0-5

Q1(a) Maxwell's equation

$$s - m = b + r - dj$$
$$s = 10 + 2\times 2 - 2\times 6$$
$$s = 2$$

Q1(b) IV & IX redundant



$$t_0 = \begin{bmatrix} 1 \\ 0 \\ -\sqrt{3} \\ 0 \\ \sqrt{2} \\ 1 \\ -\sqrt{2} \\ -\sqrt{2} \\ 0 \\ 1+\sqrt{2} \end{bmatrix} W$$

(ii) States of self stress

$t_{IV} = 1$ $\quad s_1 = \begin{bmatrix} -\sqrt{3}/3 \\ -0.816 \\ 1 \\ 1 \\ -0.816 \\ -\sqrt{3}/3 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

$t_{IX} = 1$ $\quad s_2 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sqrt{2}/2 \\ -\sqrt{2}/2 \\ 1 \\ 1 \\ -\sqrt{2}/2 \end{bmatrix}$

2P2 JO-5

Q1(b)(iii)

$$t = t_0 + x_1 s_1 + x_2 s_2$$

$$\underset{\sim}{f} = \begin{bmatrix} 1 &&&&&&&&& \\ & \sqrt{2} &&&&&&&& \\ && \sqrt{3} &&&&&&& \\ &&& \sqrt{3} &&&&&& \\ &&&& \sqrt{2} &&&&& \\ &&&&& 1 &&&& \\ &&&&&& 1 &&& \\ &&&&&&& \sqrt{2} && \\ &&&&&&&& \sqrt{2} & \\ &&&&&&&&& 1 \end{bmatrix} \frac{L}{AE}$$

$$\underset{\sim}{e} = \underset{\sim}{f}\,\underset{\sim}{t}$$

$$\Rightarrow \underset{\sim}{e} = \begin{bmatrix} 1 \\ 0 \\ -3 \\ 0 \\ 2 \\ 1 \\ -\sqrt{2} \\ -2 \\ 0 \\ 1+\sqrt{2} \end{bmatrix} \frac{WL}{AE} + \begin{bmatrix} -0.57 \\ -1.15 \\ 1.73 \\ 1.73 \\ -1.15 \\ -0.57 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \frac{x_1 L}{AE} + \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -\sqrt{2}/2 \\ -\sqrt{2}/2 \\ \sqrt{2} \\ \sqrt{2} \\ -\sqrt{2}/2 \end{bmatrix} \frac{x_2 L}{AE}$$

Q1 (b)(iii)

$$\sum f.d = \sum s.e$$

$$s_1 e \Rightarrow \left(-5.788\right)\frac{WL}{AE} + 6.016\frac{x_1 L}{AE} + 0.408\frac{x_2 L}{AE} \qquad (A)$$

$$s_2 . e \Rightarrow -3.414\frac{WL}{AE} + 0.408\frac{x_1 L}{AE} + 4.328\frac{x_2 L}{AE} \qquad (B)$$

$$\cancel{(A) \times \frac{3.414}{5.788} \Rightarrow}$$

$$(B) \times \frac{5.788}{3.414} \Rightarrow -5.788\frac{WL}{AE} + 0.692 x_1 + 7.337 x_2 \qquad (C)$$

$(A-C)$ gives $x_1 = 1.301 x_2$

& solving $x_1 = 0.91W$

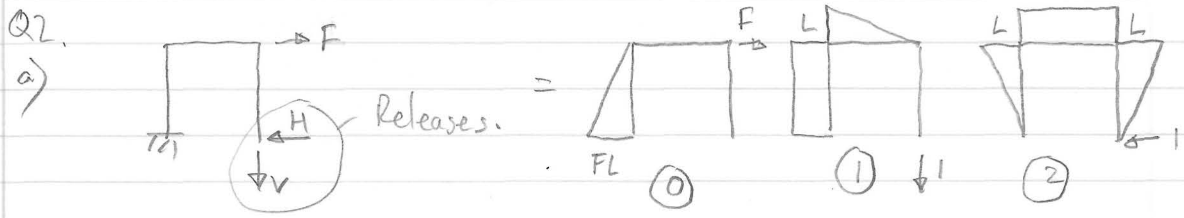
$x_2 = 0.70W$

$$t_0 + x_1 s_1 + x_2 s_2$$

$$\Rightarrow t_0 = \begin{bmatrix} 0.472 \\ -0.747 \\ -0.818 \\ 0.914 \\ 0.668 \\ -0.025 \\ -1.911 \\ -0.712 \\ 0.703 \\ 1.917 \end{bmatrix} W$$

Q2.

a)



Real system = 0 + V 1 + H 2

V.W. $F.\delta = \Sigma \int M \frac{M}{EI} ds$

Real Compatibility

Virtual Equilibrium

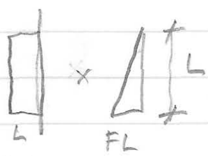
Apply twice, with Virtual Equilibrium as 1 and then 2.

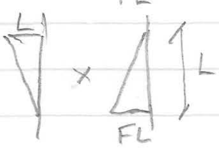
In each case $F.\delta = 0$ as support does not move.

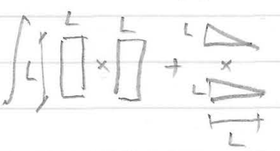
$\therefore \quad 0 = 1.0 + V\,1.1 + H\,1.2$

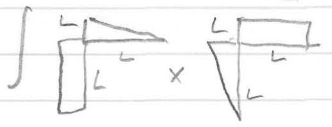
$0 = 2.0 + V\,2.1 + H\,2.2$

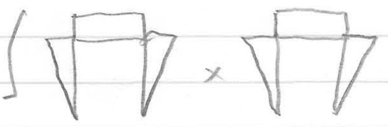
EI is same everywhere, therefore cancels.

$1.0 = \int$  $= \frac{FL^3}{2}$

$2.0 = \int$  $= FL^3$  $\frac{1}{4} = \frac{2}{3}\frac{FL^3}{4} = \frac{FL^3}{6}$

$1.1 = \int$  $= L^3 + L^3$  $=\frac{1}{3} = \frac{4L^3}{3}$

$1.2 = \int$  $= L^3\left(\frac{1}{2}+\frac{1}{2}\right) = L^3$

$2.2 = \int$  $= 2\frac{L^3}{3} + L^3 = \frac{5L^3}{3}$

a) cont'd

$$-\frac{FL^3}{2} = \frac{4L^3}{3}V + L^3 H$$

$$-\frac{FL^3}{6} = L^3 V + \frac{5L^3}{3}H$$

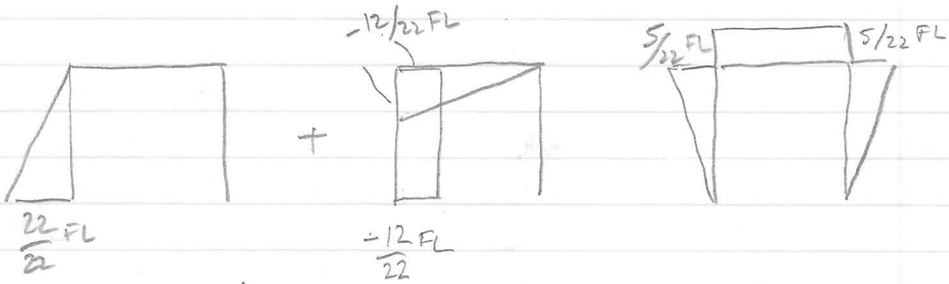
$\times 6 \rightarrow \quad F\begin{bmatrix}-3\\-1\end{bmatrix} = \begin{bmatrix}8 & 6\\6 & 10\end{bmatrix}\begin{bmatrix}V\\H\end{bmatrix} = \underline{\underline{A}}\,\underline{R} = \underline{F}$

$\underline{R} = \underline{\underline{A}}^{-1}\underline{F} \qquad A^{-1} = \begin{bmatrix}10 & -6\\-6 & 8\end{bmatrix}\frac{1}{(80-36)} = \begin{bmatrix}10 & -6\\-6 & 8\end{bmatrix}\frac{1}{44}$

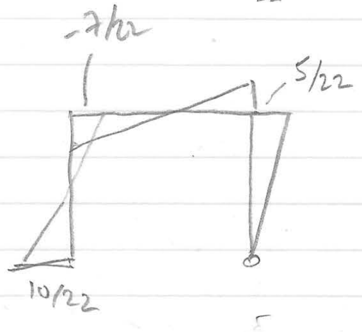
$= \begin{bmatrix}5 & -3\\-3 & 4\end{bmatrix}\frac{1}{22}$

$$\underline{R} = \frac{1}{22}\begin{bmatrix}5 & -3\\-3 & 4\end{bmatrix}\begin{bmatrix}-3\\-1\end{bmatrix}F = \frac{F}{22}\begin{bmatrix}-15+3\\+9-4\end{bmatrix} = \frac{F}{22}\begin{bmatrix}-12\\5\end{bmatrix} = \begin{bmatrix}V\\H\end{bmatrix}$$

b)



=

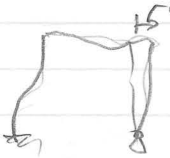


× FL

BMD

c)

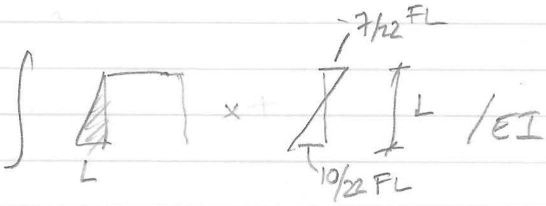
Valid. Equilibrium System.



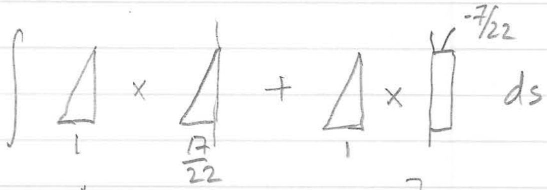
Real compatibility System.

Q2

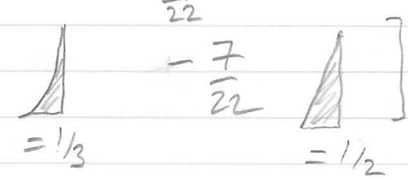
c) V.W. $1.\delta = \int$



$= \frac{[(FL)L]L}{EI} \times \int$



$= \frac{FL^3}{EI}\Big[\frac{17}{22} \times$

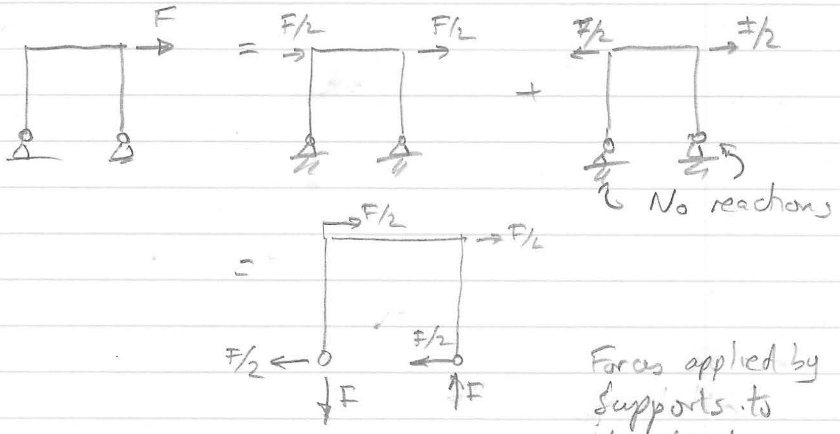


$\Big]$

$= \frac{FL^3}{EI}\left[\frac{17}{66} - \frac{7}{44}\right] \quad \left(= \frac{1}{22}\left[\frac{17}{3} - \frac{7}{2}\right] = \frac{1}{22(6)}(34-21) = \frac{13}{22(6)} \right)$

$$= \frac{13}{132}\frac{FL^3}{EI}$$

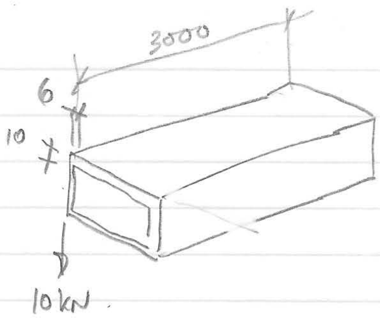
d) Both feet pinned

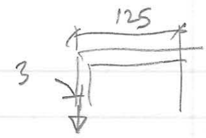


Forces applied by supports to the structure.

Q3

a)



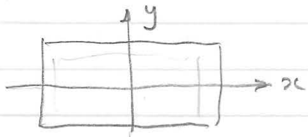


At root,

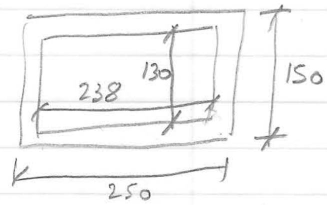
Torque $T = 10\,kN \times 0.122\,m = \underline{1.22\,kNm}$

Moment $M = 10\,kN \times 3\,m = \underline{30\,kNm}$

i)



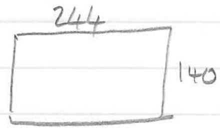
$$I = \left(\frac{bd^3}{12}\right)_{ext} - \left(\frac{bd^3}{12}\right)_{int}$$



I_{xx} (vertical bending) $= \frac{1}{12}\left[(250)(150)^3 - (238)(130)^3\right] = \underline{26.74 \times 10^6\,mm^4}$

I_{yy} (horiz. bending) $= \frac{1}{12}\left[(150)(250)^3 - (130)(238)^3\right] = \underline{49.27 \times 10^6\,mm^4}$

ii) J (Torsion const) $= \dfrac{4A_e^2}{\int ds/t}$ $\quad A_e$ (midwall) $=$



$= 34.16 \times 10^3\,mm^2$

$$\int \frac{ds}{t} = 2\left(\frac{244}{10}\right) + 2\left(\frac{140}{6}\right) = 95.47$$

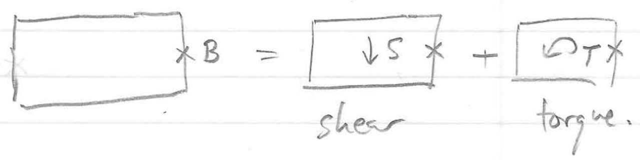
$$J = \frac{4A_e^2}{\int ds/t} = \frac{4(34.16 \times 10^3)^2}{95.47} = \underline{48.89 \times 10^6\,mm^4}$$

iii) Maximum axial stress at A.

$$\sigma = \frac{My}{I} = \frac{[30 \times 10^6\,Nmm][75\,mm]}{[26.74 \times 10^6\,mm^4]}$$

$$= \underline{84.14\,MPa}$$

iv) Max shear stress at B



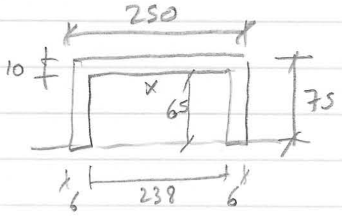
Shear part $\quad q = \dfrac{SA_c\bar{y}}{I}$

$S = 10 \times 10^3\,N$

$$A_c = \frac{\left[(250)(150) - (238)(130)\right]}{2} = \underline{3280\,mm^2}$$

Q3 cont'd.

iv) cont'd.



$$\bar{y}A = \int y\,dA$$

$$\bar{y} = \frac{(250)(75)\left(\frac{75}{2}\right) - 238(65)\frac{65}{2}}{3280}$$

$$= \underline{\underline{61.08\text{ mm}}}$$

$$\therefore \quad q_{shear} = \frac{SA\bar{y}}{I} = \frac{(10\times10^3\text{ N})(3280\text{ mm}^2)(61.08)\text{ mm}}{26.74\times10^6\text{ mm}^4}$$

$$= \underline{\underline{74.9\text{ N/mm}}}$$

$$q_{torsion} = \frac{T}{2A_e} = \frac{1220\times10^3\text{ Nmm}}{2(34.16\times10^3\text{ mm}^2)} = \underline{\underline{17.86\text{ N/mm}}}$$

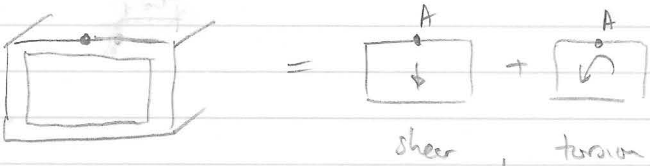
Shear stress $\tau_{shear} = \dfrac{74.9\text{ N/mm}}{12\text{ mm}} = \underline{\underline{6.24\text{ MPa}}}$ ↓

← two webs

$\tau_{torsion} = \dfrac{17.86\text{ N/mm}}{6\text{ mm}} = \underline{\underline{2.98\text{ MPa}}}$ ↑

These are in opposing directions, so $\tau_B = 6.24 - 2.98 = \underline{\underline{3.26\text{ MPa}}}$ ↓

v). Max shear stress at A.



gives zero shear stress at A by symmetry

as before.

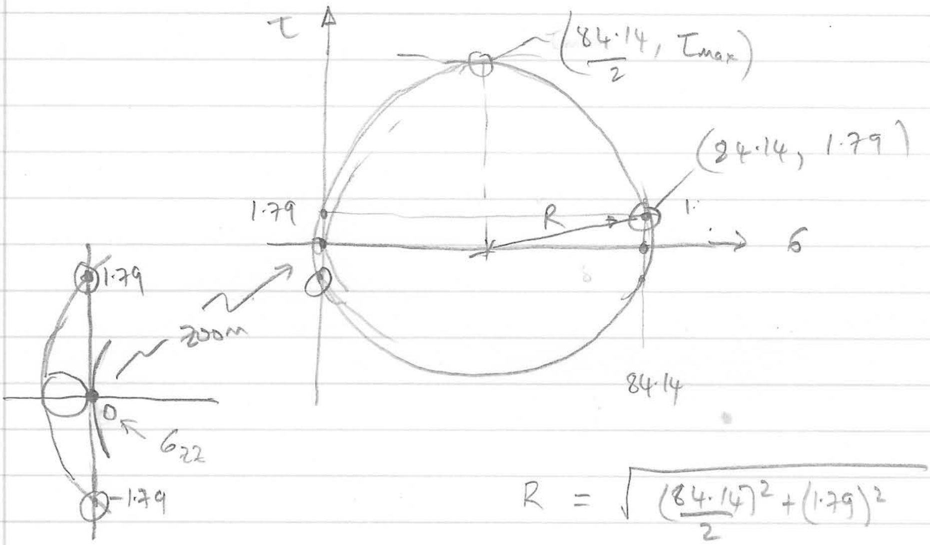
$$\uparrow \; = \frac{17.86\text{ N/mm}}{10\text{ mm}}$$

$$= \underline{\underline{1.79\text{ MPa}}} \leftarrow$$

at A

Q3. v) cont'd.

Axial stress = 84.14 MPa as per part ii)

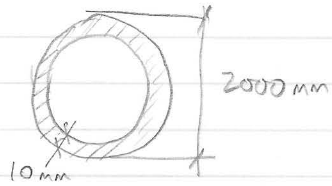


$$R = \sqrt{\left(\frac{84.14}{2}\right)^2 + (1.79)^2} = \underline{\underline{42.11\ MPa}}$$

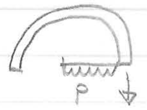
$$\underline{\underline{\tau_{max} = R = 42.11\ MPa}}$$

Section B.

Q4. a)



Hoop stress $\sigma_{\theta\theta}$:



$$\sigma_{\theta\theta}\, t = p\, r_{int}$$

$$\sigma_{\theta\theta} = (2\,\text{MPa})\left(\frac{990}{10}\right)$$

$$= \underline{\underline{198\ \text{MPa}}}$$

Longitudinal stress : σ_{zz} :

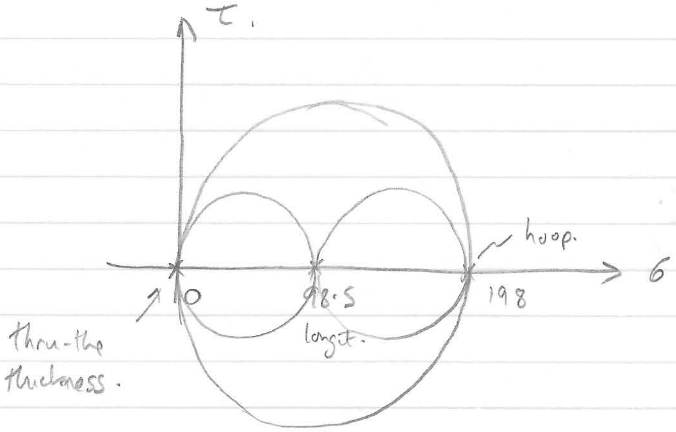
Approximately:

$$(2\pi r t)\sigma_{zz} = (\pi r^2)p$$

$$\sigma_{zz} = \frac{pr}{2t} \approx \frac{2(1000)}{2(10)} \approx \underline{\underline{100\ \text{MPa}}}$$

[Exactly. $\sigma_{zz} = p\dfrac{\pi r_i^2}{\pi(r_o^2 - r_i^2)} = \dfrac{p\, r_i^2}{(r_o - r_i)(r_o + r_i)} = \dfrac{p\, r_i^2}{t\,(r_o + r_i)}$

$$= \frac{p\, r_i}{t\left(r_o/r_i + 1\right)} = \frac{2(990)}{10\left(1 + \frac{1000}{990}\right)} = \underline{\underline{98.5\ \text{MPa}}}$$]



hoop = xx say

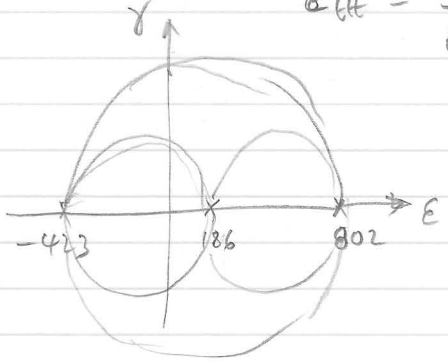
longit = yy say.

b). Principal strains:

$$\varepsilon_{hoop} = \frac{1}{E}\left(198 - 0.3(98.5)\right) = \underline{\underline{802 \times 10^{-6}}}$$

$$\varepsilon_{longit} = \frac{1}{E}\left(98.5 - 0.3(198)\right) = \underline{\underline{186 \times 10^{-6}}}$$

$$\varepsilon_{ttt} = \frac{1}{E}\left(0 - 0.3(198 + 98.5)\right) = \underline{\underline{-423 \times 10^{-6}}}$$



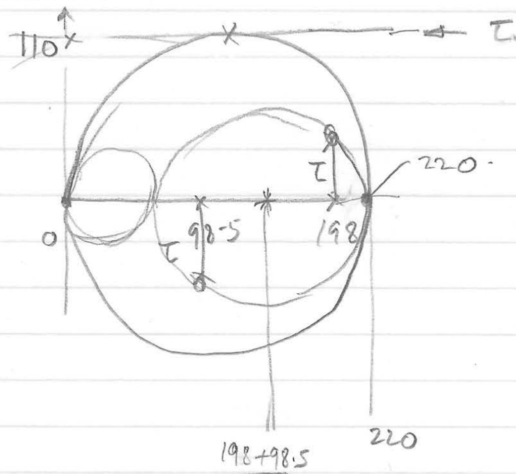
Q4 c) i Torque $\tau = \dfrac{Tr}{J}$

$J = 2\pi r^3 t$
$= 2\pi(10^9)(10)$
$= \underline{62.8 \times 10^9 \text{ mm}^4}$

$\tau = \dfrac{T(10^3 \text{ mm})}{62.8 \times 10^9 \text{ mm}^4}$

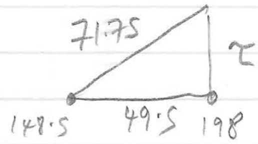
$\Rightarrow \quad \underline{T_{max} = (62.8 \times 10^6)\, \tau_{max}} \quad (\text{mm}^3)$

i) Tresca:



$\dfrac{198+98.5}{2}$
$= 148.5$

$R = 220 - 148.5 = 71.75.$

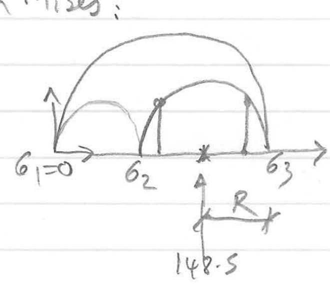


$\tau = \sqrt{(71.75)^2 - (49.5)^2}$

$= \underline{51.94 \text{ MPa}}$

$\therefore \quad T_{max} = (62.8 \times 10^6 \text{ mm}^3)(51.94 \text{ N/mm}^2)$
$= 3261 \times 10^6 \text{ Nmm}$
$= \underline{3261 \text{ kNm}}$

ii) Von Mises:



$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2Y^2$

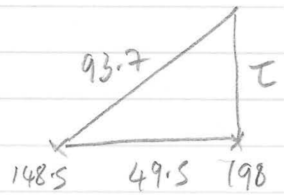
$(148.5 - R)^2 + (2R)^2 + (148.5 + R)^2 = 2Y^2$

$148.5^2 - \cancel{297}R + R^2 + 4R^2 + 148.5^2 + \cancel{297}R + R^2 = 2Y^2$

$2(148.5)^2 + 6R^2 = 2Y^2$
$(148.5)^2 + 3R^2 = Y^2$
$R^2 = \dfrac{Y^2 - (148.5)^2}{3}$

$R = \sqrt{\dfrac{(220)^2 - (148.5)^2}{3}} = \underline{93.7 \text{ MPa}}$

Q4 c) cont'd.



$$\tau = \sqrt{(93.7)^2 - (49.5)^2}$$

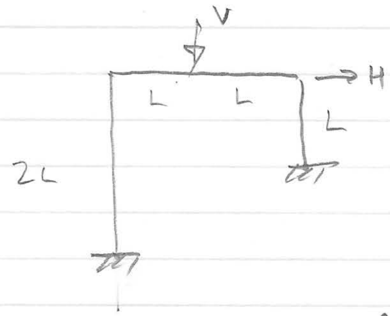
$$= \underline{\underline{79.58 \text{ MPa}}}$$

$T = 62.8 \times 79.58 = \underline{\underline{4997 \text{ kNm}}}$ Von Mises

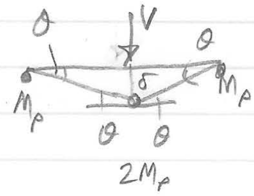
(cf 3261 kNm Tresca).

Q5.

a)



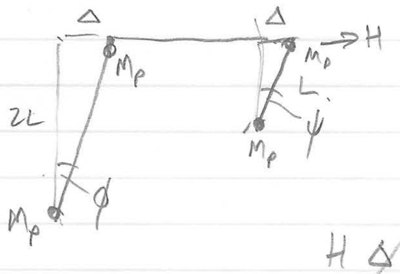
i) Beam:



$$V.\delta = 2M_p\theta + 2M_p(2\theta) = 6M_p\theta$$

$$\theta = \frac{\delta}{L} \Rightarrow V = \frac{6M_p}{L} \Rightarrow \boxed{\frac{VL}{M_p} = 6}$$

ii) Sway.



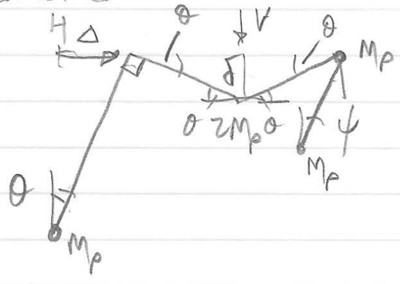
$$H\Delta = 2M_p\phi + 2M_p\psi$$

$$\phi = \Delta/2L \qquad \psi = \frac{\Delta}{L}$$

$$H \cancel{\Delta} = \cancel{2} M_p \frac{\cancel{\Delta}}{\cancel{2}L} + 2M_p \frac{\cancel{\Delta}}{L}$$

$$H = \frac{M_p}{L}\left(1+2\right) = \frac{3M_p}{L} \Rightarrow \boxed{\frac{HL}{M_p} = 3}$$

iii) Combined



$$H\Delta + V\delta = M_p\theta + 2M_p(2\theta) + M_p(\theta+\psi) + M_p\psi$$

$$\theta = \frac{\Delta}{2L} \qquad \theta = \frac{\delta}{L} \Rightarrow \underline{\underline{\Delta = 2\delta}}$$

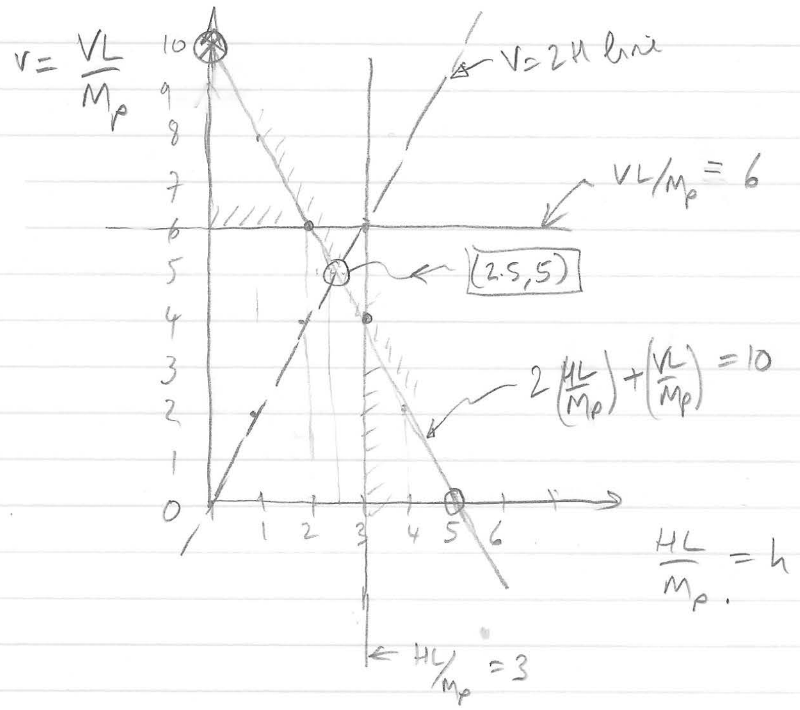
$$[\psi = \Delta/L = 2\delta/L]$$

$$H(2\delta) + V\delta = M_p\frac{\delta}{L} + 4M_p\frac{\delta}{L} + M_p\left(\frac{\delta}{L} + \frac{2\delta}{L} + \frac{2\delta}{L}\right)$$

$$2H + V = \frac{M_p}{L}\left[1+4+1+4\right] = 10\frac{M_p}{L}$$

$$\boxed{2\left(\frac{HL}{M_p}\right) + \left(\frac{VL}{M_p}\right) = 10}$$

Q. 5 cont'd

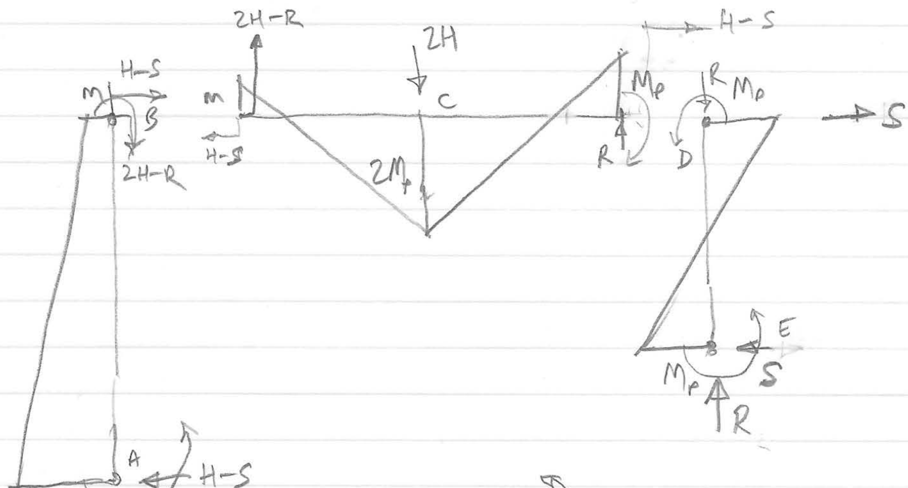


b) ii) $V = 2H \rightarrow$ Combined mechanism governs

$2h + v = 10$

$2h + 2h = 10$

$h = 2.5 \quad , \quad v = 5.$



E ↺+ $\quad 2M_p = SL$

$$\underline{\underline{S = \frac{2M_p}{L}}}$$

A ↺+ $\quad M_p - m = (H-S)\,2L$

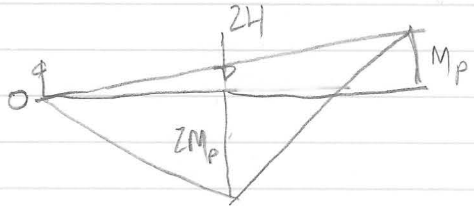
$M_p - m = 2LH - \frac{2M_p}{L}(2L) = 2LH - 4M_p.$

$m = 5M_p - 2LH \qquad$ but $\frac{HL}{M_p} = 2.5$ so $HL = 2.5M_p$

$$\underline{\underline{m = 5M_p - 5M_p = 0}}$$ ← This is less than yield (obviously).

5b ii)

Check beam:



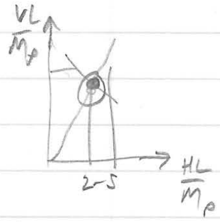
Free BM = "$\frac{WL}{4}$" = $\frac{(2H)(2L)}{4}$

= HL

But from diagram Free BM = $2.5\,M_p$

$\therefore$ $2.5 M_p = HL$

$2.5 = \frac{HL}{M_p}$ ✓



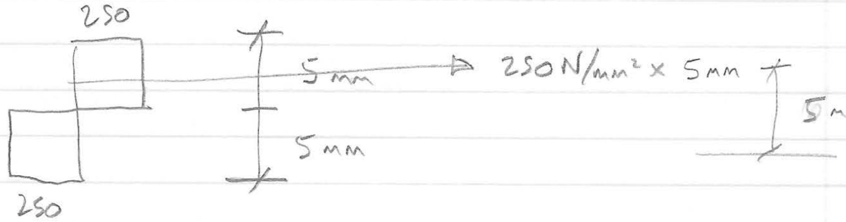
Yes.

$\therefore$ BMD is in equilibrium with applied loads

and nowhere exceeds yield

$\therefore$ Lower Bound Solution also.

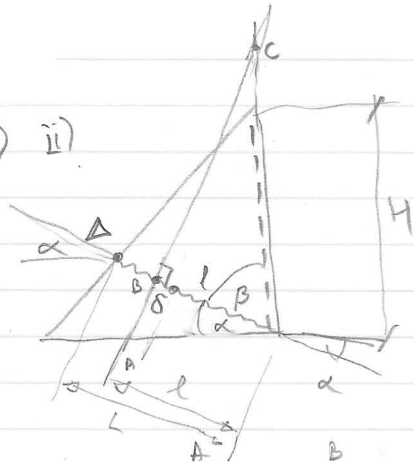
Q6 a) i).



$$M_p = \sigma_y \frac{d^2}{4} = 250\ N/mm^2 \times (25\ mm^2)$$

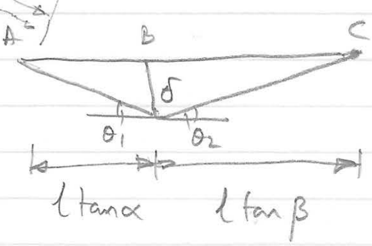
$$= \underline{\underline{6250\ Nm/m}}$$

Q6 a) ii)



Let geared point B on Yield Line move down by δ

Geometry



$$d(WD) = m_p \cdot (\theta_1 + \theta_2) \cdot dl \qquad \theta_1 = \frac{\delta}{l\tan\alpha} \qquad \theta_2 = \frac{\delta}{l\tan\beta}.$$

$$= m_p \left[\frac{\delta}{l\tan\alpha} + \frac{\delta}{l\tan\beta} \right] dl.$$

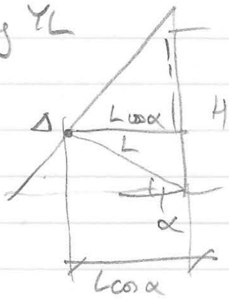
Now $\frac{\delta}{l} = \frac{\Delta}{L}$

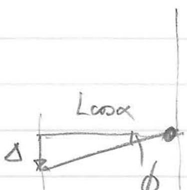
$$WD = m_p \frac{\Delta}{L} \left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta} \right) \int_0^L dl$$

$$= m_p \Delta \left(\frac{1}{\tan\alpha} + \frac{1}{\tan\beta} \right)$$

along sagging YL

Along hogging YL

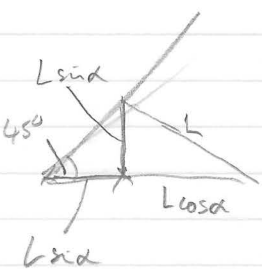




$$\phi = \frac{\Delta}{L\cos\alpha}.$$

$$WD = H m_p \phi$$

$$= \frac{H\Delta}{L\cos\alpha} m_p$$



$$H = L\cos\alpha + L\sin\alpha.$$

$$\frac{H}{L} = \cos\alpha + \sin\alpha.$$

$$WD = \left[\frac{\cos\alpha + \sin\alpha}{\cos\alpha} \right] \Delta m_p.$$

$$= \left[1 + \tan\alpha \right] \Delta m_p$$

Q6. d) ii)

$$\text{Int. WD} = m_p \Delta \left[1 + \tan\alpha + \frac{1}{\tan\alpha} + \frac{1}{\tan\beta} \right]$$

$$= m_p \Delta \left[1 + 2\tan\alpha + \frac{1}{\tan\alpha} \right] \qquad \tan\beta = 1/\tan\alpha$$

$$\text{Ext WD} = \text{Vol of pyramid} = \frac{W\Delta}{3}$$

$$\therefore \quad \frac{W\Delta}{3} = m_p \Delta \left[1 + 2\tan\alpha + \frac{1}{\tan\alpha} \right]$$

$$W = 3m_p \left[1 + 2\tan\alpha + \frac{1}{\tan\alpha} \right]$$

Let $\tan\alpha = t$

$$W = 3m_p \left[1 + 2t + \frac{1}{t} \right]$$

$\therefore$ Max when

$$f = 2t + \frac{1}{t} \text{ is max}$$

$$\frac{df}{dt} = 2 - \frac{1}{t^2} = 0$$

$$t^2 = \frac{1}{2} \qquad t = 1/\sqrt{2} = 0.7071$$

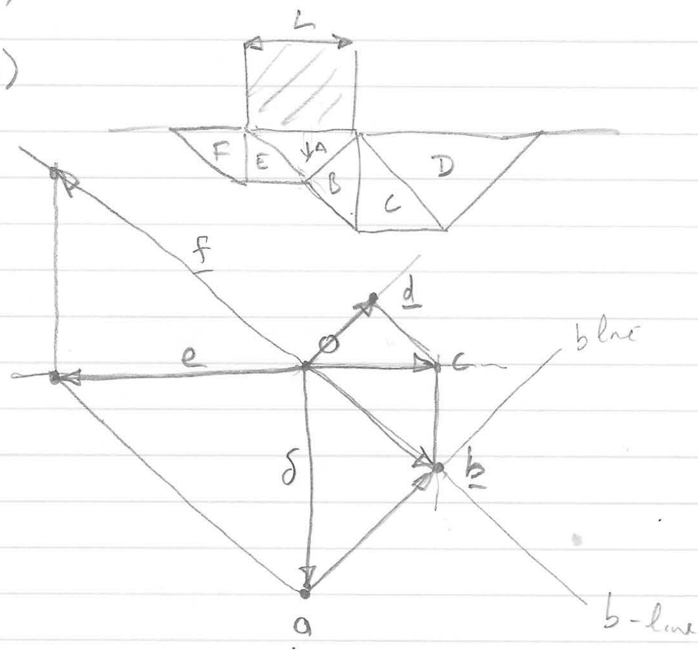
$$\theta = 35.26 \text{ degrees}$$

$$W = 3\,(6250\ \text{Nm/m}) \Big[\underbrace{1 + 2\left(\tfrac{1}{\sqrt{2}}\right) + \sqrt{2}}_{1+2\sqrt{2}} \Big]$$

$$= 71.78\ \text{kN}$$

Q6. b)

i)



		Length.	
AB	$\delta/\sqrt{2}$	$L/\sqrt{2}$	$\frac{\delta L}{2}$
BC	$\delta/2$	L	$\frac{\delta L}{2}$
CD	$\frac{\delta}{2\sqrt{2}}$	$\sqrt{2}L$	$\frac{\delta L}{2}$
OB	$\frac{\delta}{\sqrt{2}}$	$\frac{L}{\sqrt{2}}$	$\frac{\delta L}{2}$
OC	$\delta/2$	L	$\frac{\delta L}{2}$
OD	$\delta/2\sqrt{2}$	$\sqrt{2}L$	$\frac{\delta L}{2}$
			$\sum_R = 3\delta L$
AE	$\sqrt{2}\delta$	$L/\sqrt{2}$	δL
EF	δ	$L/2$	$\delta L/2$
OE	δ	$L/2$	$\delta L/2$
OF	$\sqrt{2}\delta$	$\frac{L}{\sqrt{2}}$	δL
			$\sum_L = 3\delta L$

also

Total WD $= 6\delta L k$ internally.

externally WD $= (\sigma L)\delta$

$\therefore$ $\underline{\underline{\sigma = 6k}}$

upper bound.