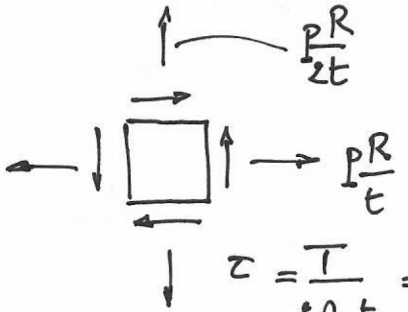
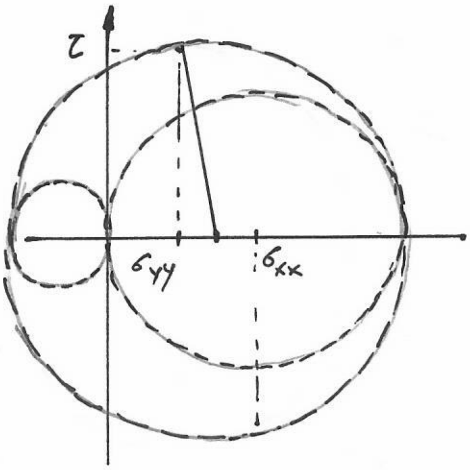
1. (a)



$$\tau = \frac{T}{2A_e t} = \frac{T}{2\pi r^2 t}$$

(b) $\tau = qY$, $\sigma_{xx} = pY$, $\sigma_{yy} = \frac{p}{2}Y$



$$R = \sqrt{\tau^2 + \left(\frac{\sigma_{xx}-\sigma_{yy}}{2}\right)^2}$$

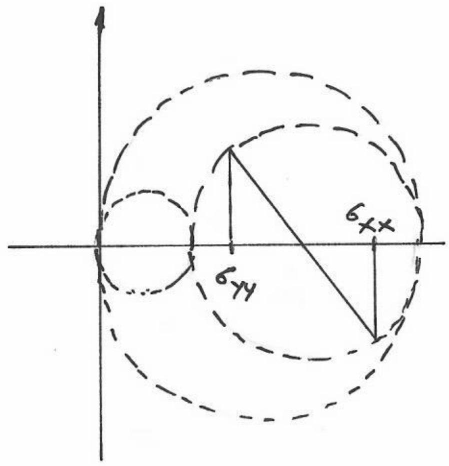
$$= Y\sqrt{q^2 + \frac{(p/2)^2}{4}} = Y\sqrt{q^2 + p^2/16}$$

$$Y/2 = Y\sqrt{q^2 + p^2/16}$$

$$\boxed{16q^2 + p^2 = 4}$$

(c) limiting value : $R = \frac{3}{2}\sigma_{yy}$

$$\rightarrow \sqrt{q^2 + p^2/16} = \frac{3}{2}\frac{p}{2} \rightarrow q^2 + \frac{p^2}{16} = \frac{9}{16}p^2 \rightarrow q = \frac{\sqrt{2}}{2}p$$

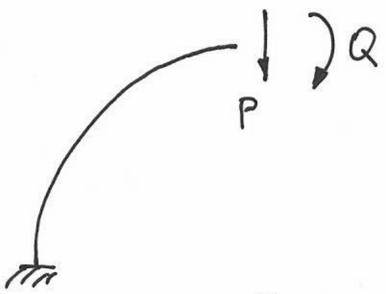


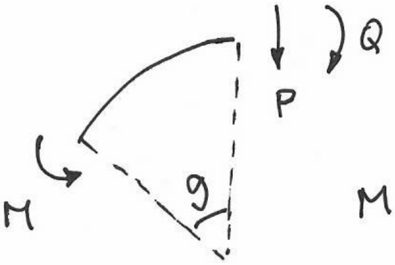
$$\frac{\sigma_{xx}+\sigma_{yy}}{2} + R = Y$$

$$\frac{3p}{4} + \sqrt{q^2 + \frac{p^2}{16}} = 1$$

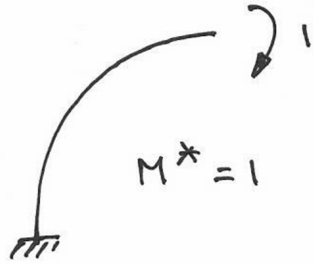
$$\rightarrow \boxed{q^2 - \frac{p^2}{2} + \frac{3p}{2} = 1}$$

2 (a).





$$M = Q + P(R\sin\theta)$$

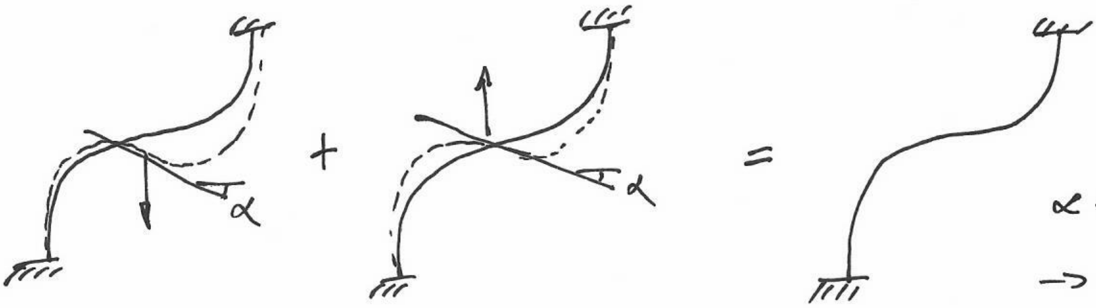


$M^* = 1$

$$\theta = \int_0^{\pi/2} \frac{MM^*}{EI} R\,d\theta = \frac{R}{EI}\int_0^{\pi/2} (Q + PR\sin\theta)\,d\theta$$

$$= \frac{R}{EI}\left[Q\frac{\pi}{2} + PR(-\cos\theta)\Big|_0^{\pi/2}\right] = \frac{R}{EI}\left[Q\frac{\pi}{2} + PR\right]$$

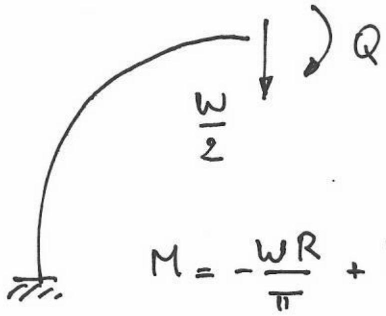
(b). (i)



$\alpha + \alpha = 0$

$\rightarrow \alpha = 0$

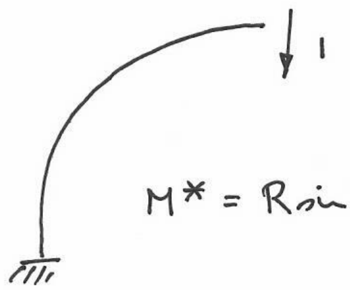
(ii)



$\theta = 0$

(a) $\rightarrow$ $\frac{Q\pi}{2} + \frac{W}{2}R = 0$ $\rightarrow$ $Q = -\frac{WR}{\pi}$

$$M = -\frac{WR}{\pi} + \frac{W}{2}R\sin\theta$$



$M^* = R\sin\theta$

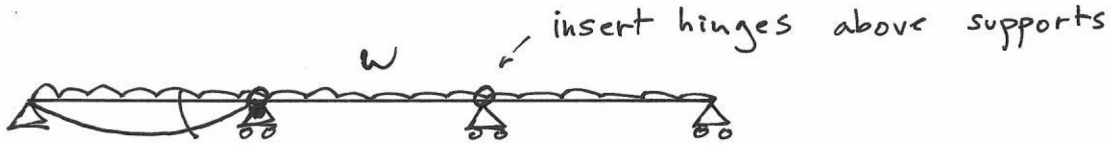
$$v = \int_0^{\pi/2} (R\sin\theta)\left(-\frac{WR}{\pi} + \frac{W}{2}R\sin\theta\right)\frac{R\,d\theta}{EI}$$

$$= -\frac{WR^3}{\pi EI} + \frac{WR^3}{2EI}\int_0^{\pi/2} \frac{1-\cos 2\theta}{2}\,d\theta$$

$$= \frac{WR^3}{EI}\left[\frac{\pi}{8} - \frac{1}{\pi}\right]$$

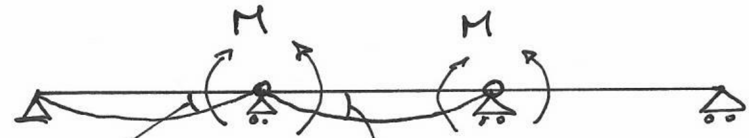
3.

(a)



$$\alpha_1 = \frac{wL^3}{24EI}$$

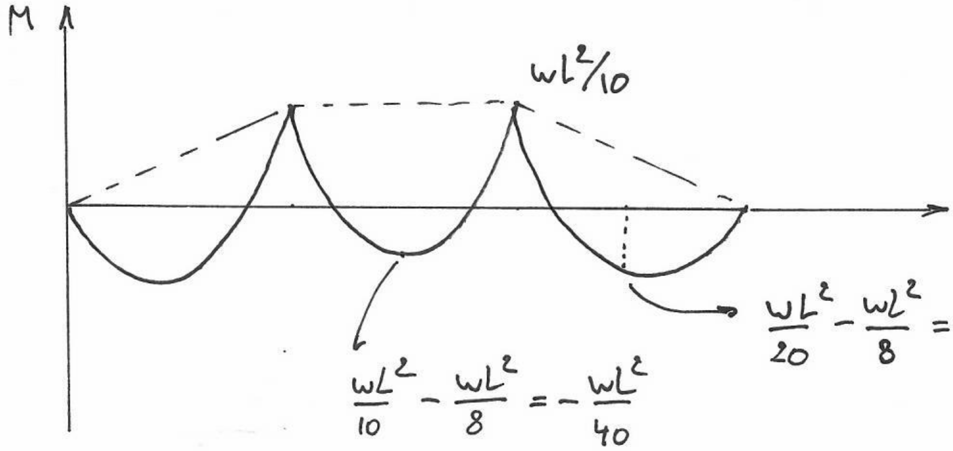
+



$$\alpha_2 = -\frac{ML}{3EI} \qquad \alpha_3 = -\frac{ML}{2EI}$$

$$2\alpha_1 + \alpha_2 + \alpha_3 = 0 \rightarrow \frac{wL^3}{12} - \frac{ML}{3} - \frac{ML}{2} = 0$$

$$\rightarrow M = \frac{wL^2}{10}$$

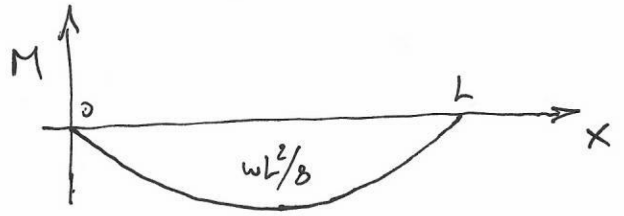


$$\frac{wL^2}{20} - \frac{wL^2}{8} = -\frac{3}{40} wL^2$$

$$\frac{wL^2}{10} - \frac{wL^2}{8} = -\frac{wL^2}{40}$$

$$R = wL + \frac{M}{L} = wL + \frac{wL}{10} = \frac{11}{10} wL$$

3b.



$$M = \frac{w}{2}x(L-x) = \frac{wL}{2}x - \frac{w}{2}x^2$$

$$EI\alpha = \frac{wL}{4}x^2 - \frac{w}{6}x^3 + C$$

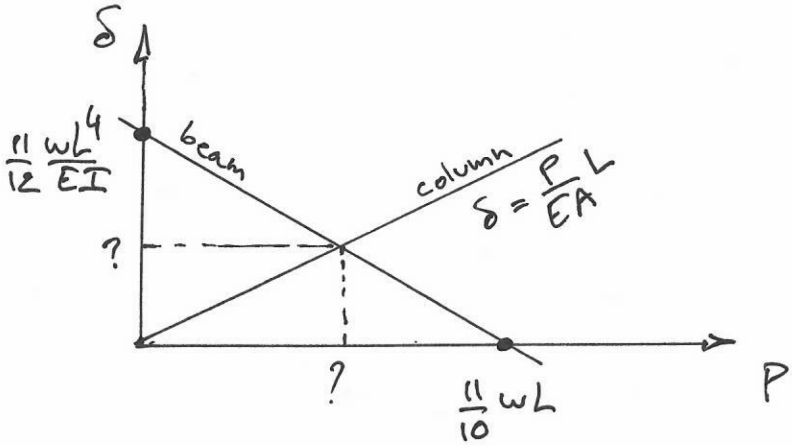
$$\alpha = 0 \quad \text{for } x = \frac{L}{2} \quad \rightarrow \quad C = -\frac{wL^3}{24}$$

$$EIv = \frac{wL}{12}x^3 - \frac{w}{24}x^4 - \frac{wL^3}{24}x$$

$$x = \frac{L}{3} \rightarrow \quad EIv = \frac{wL^4}{(12)(27)} - \frac{wL^4}{(27)(81)} - \frac{wL^4}{(3)(24)}$$

$$L \curvearrowright 3L \qquad EIv = \frac{wL^4}{4} - \frac{wL^4}{24} - \frac{9wL^4}{8} \Rightarrow v = \frac{11}{12}\frac{wL^4}{EI}$$

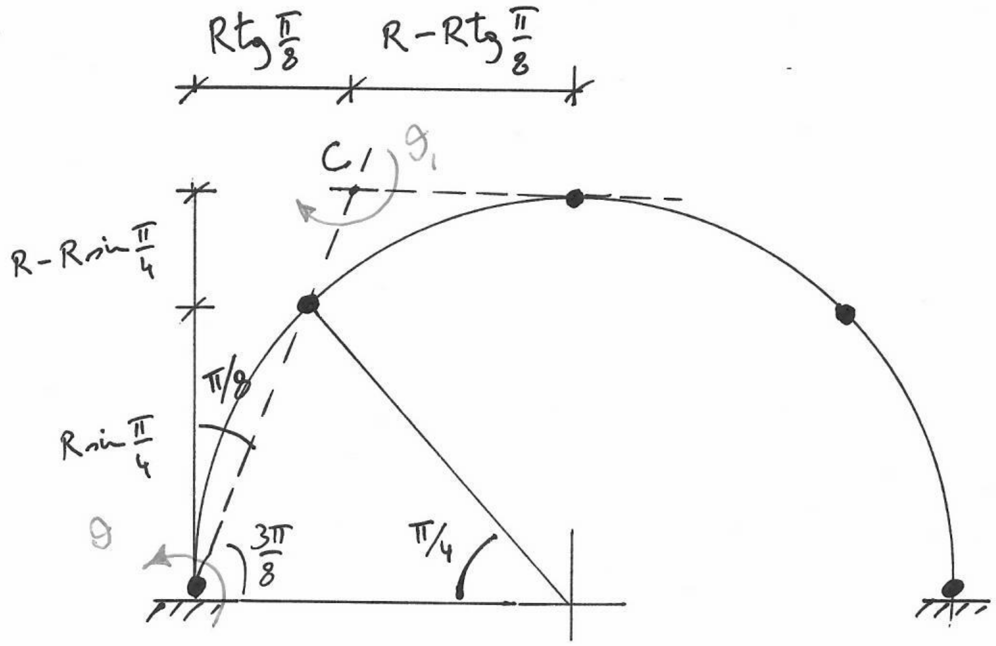
3c.

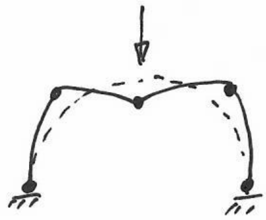


$$\delta = -\frac{\frac{\cancel{11}}{12}\frac{\cancel{w}L^4}{EI}}{\frac{\cancel{11}}{10}\cancel{w}L}P + \frac{11}{12}\frac{wL^4}{EI} = \frac{PL}{EA} \qquad \Rightarrow \qquad P = \frac{11}{2}w\frac{AL^3}{6I + 5AL^2}$$

$$\delta = \frac{PL}{EA}$$

4.





a.

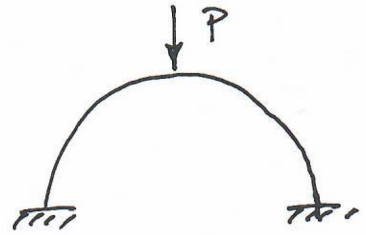
$$\left(R\sin\frac{\pi}{4}\right)\vartheta = R\left(1-\sin\frac{\pi}{4}\right)\vartheta_1$$

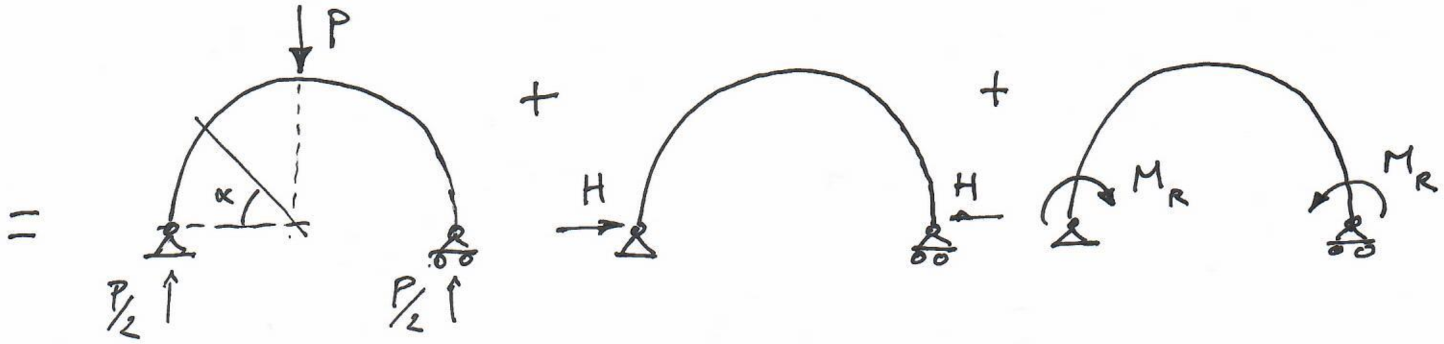
$$\Rightarrow \vartheta_1 = \frac{1}{\sqrt{2}-1}\,\vartheta$$

$$P\vartheta_1 R\left(1-tg\frac{\pi}{8}\right) = 2M_p\left(2\vartheta+2\vartheta_1\right)$$

$$\Rightarrow P = \frac{4M_p\left(1+\sqrt{2}-1\right)}{R\left(1-tg\frac{\pi}{8}\right)} \approx 9.66\,\frac{M_p}{R}$$

4b





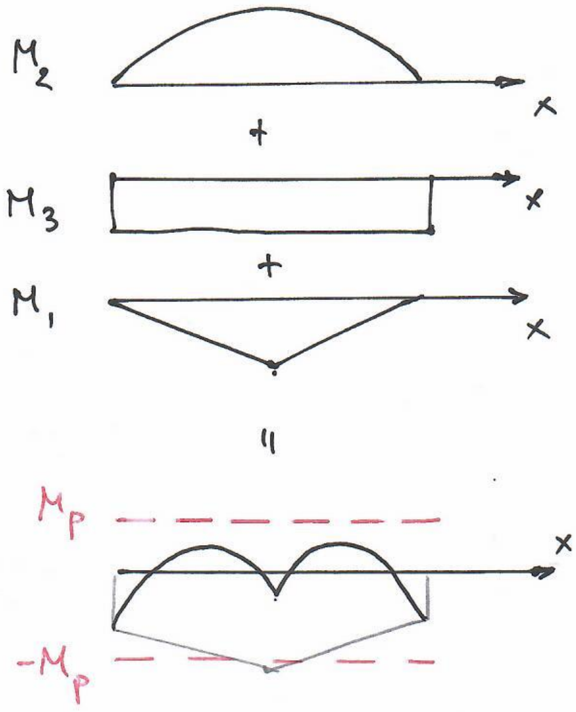
$$M_1 = \frac{-P}{2}(R - R\cos\alpha) \qquad M_2 = HR\sin\alpha \qquad M_3 = -M_R$$

Bottom : Set $M_3 = -M_R = -M_p$

Top : Set $-\frac{PR}{2} + HR \cancel{-M_p} = \cancel{-M_p}$

$\alpha = 90°$

$$\Rightarrow H = \frac{P}{2}$$



$$\Rightarrow M = \frac{P}{2}(R\sin\alpha + R\cos\alpha - R) - M_p$$

M_{max} ?

$$\frac{P}{2}R(\cos\alpha - \sin\alpha) = 0$$

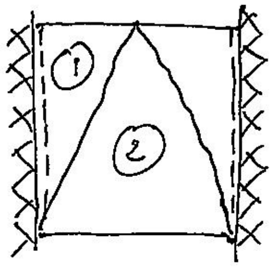
$$\Rightarrow \alpha = 45°$$

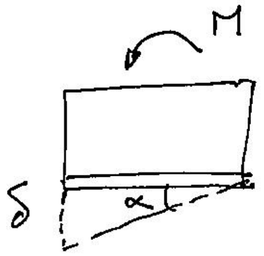
$$M_{max} = \frac{P}{2}R(\sqrt{2} - 1) - M_p$$

Set $M_{max} = M_p$

$$\Rightarrow P = \frac{4M_p}{(\sqrt{2}-1)R} = 9.66\,\frac{M_p}{R}$$

5a

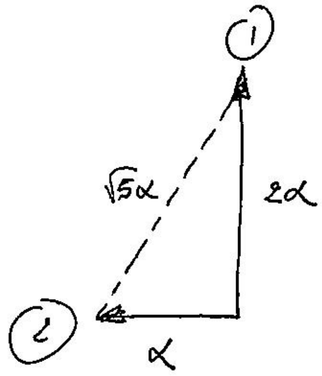




$$\delta = \alpha b$$

$$W = M\alpha \quad \Rightarrow \quad W = \frac{M\delta}{b}$$

b.

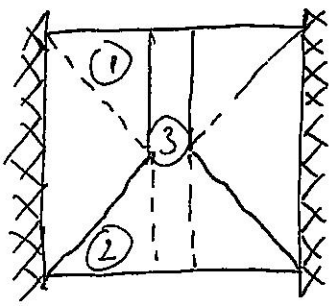


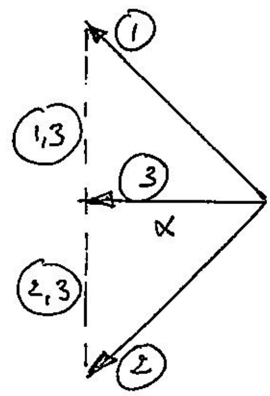
$$2m\left(2\alpha b + \sqrt{5}\alpha\frac{\sqrt{5}}{2}b\right) = M\alpha$$

$$2m\left(2b + 2.5b\right) = M$$

$$M = 9mb$$

c.

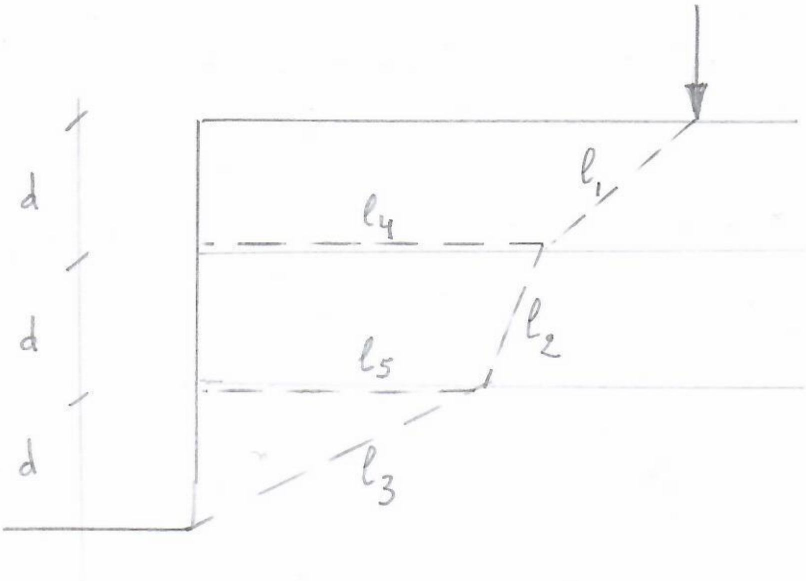


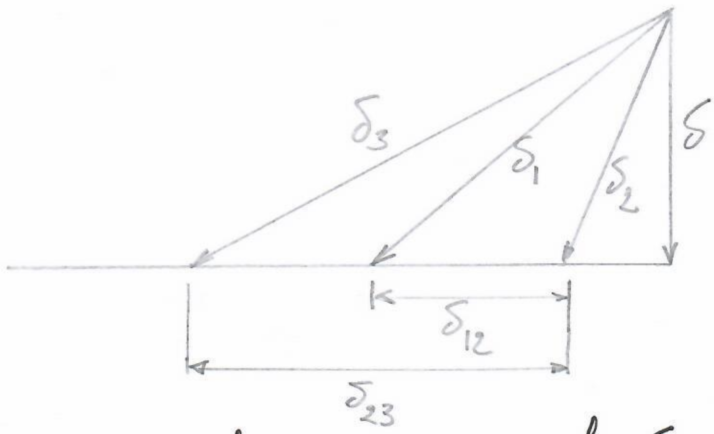


$$2m\left(\frac{\sqrt{2}}{2}b\sqrt{2}\alpha 2 + \alpha b\right) = M\alpha$$

$$M = 6mb$$

3.





$$\frac{\delta_2}{\delta} = \frac{l_2}{d} \Rightarrow \delta_2 = \frac{l_2}{d}\delta \qquad \delta_3 = \frac{l_3}{d}\delta \qquad \delta_1 = \frac{l_1}{d}\delta$$

$$P\delta = k_1 l_1 \delta_1 + k_2 l_2 \delta_2 + k_1 l_3 \delta_3 + k_1 l_4 \delta_{12} + k_1 l_5 \delta_{23}$$

$$= k_1 l_1^2 \frac{\delta}{d} + k_2 l_2^2 \frac{\delta}{d} + k_1 l_3^2 \frac{\delta}{d} + k_1 L_1 \delta_{12} + k_1 L_2 \delta_{23}$$

$$= k_1 \left[(L-L_1)^2 + d^2 \right] \frac{\delta}{d} + k_2 \left[(L_1 - L_2)^2 + d^2 \right] \frac{\delta}{d} + k_1 (L_2^2 + d^2) \frac{\delta}{d}$$

$$+ k_1 L_1 \| (L-L_1) - (L_1 - L_2) \| \frac{\delta}{d} + k_1 L_2 \| 2L_2 - L_1 \| \frac{\delta}{d}$$

$$\Rightarrow Pd = k_1 \left[L^2 - L_1^2 + 3L_2^2 - LL_1 + 2d^2 \right] + k_2 \left[(L_1 - L_2)^2 + d^2 \right]$$

$$\frac{\partial P}{\partial L_1} = 0 \Rightarrow k_1 \left[-2L_1 - L \right] + k_2 \left[2(L_1 - L_2) \right] = 0$$

$$10 L_1 - 12 L_2 = L$$

$$\frac{\partial P}{\partial L_2} = 0 \Rightarrow k_1 \left[6 L_2 \right] - 2k_2 (L_1 - L_2) = 0$$

$$12 L_1 - 18 L_2 = 0$$

$$L_1 = L/2$$
$$L_2 = L/3$$

$$\Rightarrow P = k_1 \left[\frac{3}{4} \frac{L^2}{d} + 8d \right]$$