2P4 2021

Section A

1 a) Boundary layer $\frac{\partial p}{\partial y} \approx 0$, Approximately parallel flow

$\left(\frac{\partial p}{\partial y} << \frac{\partial p}{\partial x} \right)$ $(R >> \delta)$

No frictional effects outside boundary layer so by Bernoulli U_e = constant since $\frac{dp}{dx} = 0$ in this case.

$p + \frac{1}{2}\rho U_e^2 = \text{constant}$

$p = \text{constant} \rightarrow U_e = \text{constant}$

[3]

b) $$\tau = \mu \frac{\partial u}{\partial y} = \mu U_e \left[\frac{An}{\delta^n} y^{n-1} + \frac{B}{\delta} + C \right]$$

at $y = 0$, $u_x = 0 \rightarrow \underline{C = 0}$

$$\tau_w = \mu U_e \left[\frac{B}{\delta} \right]$$

$$\rightarrow B = \frac{\tau_w}{U_e}\frac{\delta}{\mu} = \frac{10}{3} \cdot \frac{\frac{1}{2}\rho U_e^2}{U_e Re_\delta} \cdot \frac{\delta}{\mu} = \frac{10}{3} \cdot \frac{\frac{1}{2}\rho U_e^2}{\rho U_e^2 \delta/\mu} \cdot \delta/\mu$$

$$\rightarrow B = \frac{5}{3}$$

at $y = \delta$, $\tau = 0 \rightarrow 0 = \frac{An}{\delta} + \frac{5}{3\delta}$

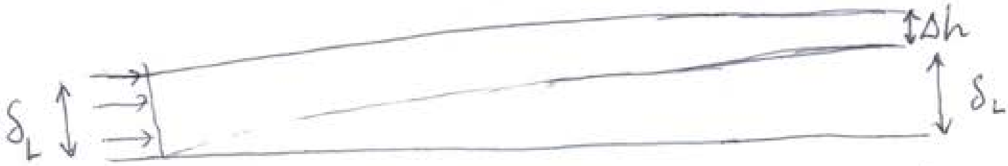
at $y = \delta$, $u_x = U_e \rightarrow 1 = A + \frac{5}{3}$

$$\rightarrow \underline{A = -2/3}$$

$$n = -5/3 / -2/3 = \underline{2.5}$$

[7]

c)



mass conservation: $\rho U_e \delta_L = \rho \int_0^{\delta_L} u \, dy + \rho U_e \Delta h$

$$\delta_L = \int_0^{\delta_L} \frac{u}{U_e} dy + \Delta h$$

$$\Delta h = -\delta_L \int_0^1 \left(-\frac{2}{3}\left(\frac{y}{\delta}\right)^{5/2} + \frac{5}{3}\left(\frac{y}{\delta}\right) \right) d\left(\frac{y}{\delta}\right) + \delta_L$$

$$= \delta_L \left[\left(\frac{2}{3}\right)\left(\frac{2}{7}\right)\left(\frac{y}{\delta}\right)^{7/2} - \left(\frac{5}{6}\right)\left(\frac{y}{\delta}\right)^2 \right]_0^1 + \delta_L$$

$$= \left(\frac{4}{21} - \frac{5}{6} \right) \delta_L + \delta_L$$

$$= \frac{5}{14} \delta_L \quad (0.3571 \, \delta_L)$$ [5]

d) $\underline{\text{S.F.M.E}}$

$$F = -\rho U_e^2 \delta_L + \rho U_e^2 \Delta h + \rho \int_0^{\delta_L} u^2 dy$$

$$\int_0^1 \left(\frac{u}{U_e}\right)^2 d\left(\frac{y}{\delta}\right) = \int_0^1 A^2 \left(\frac{y}{\delta}\right)^{2n} + B^2 \left(\frac{y}{\delta}\right)^2 + 2AB\left(\frac{y}{\delta}\right)^{n+1} d\left(\frac{y}{\delta}\right)$$

$$= \frac{A^2}{2n+1} + \frac{B^2}{3} + \frac{2AB}{n+2}$$

$$= \frac{4/9}{6} + \frac{25/9}{3} - \frac{20/9}{9/2}$$

$$= 287/567 = 0.5062$$

$$F = -\rho U_e^2 \delta_L + \rho U_e^2 \cdot \frac{5}{14} \cdot \delta_L + \frac{287}{567} \rho U_e^2 \cdot \delta_L$$

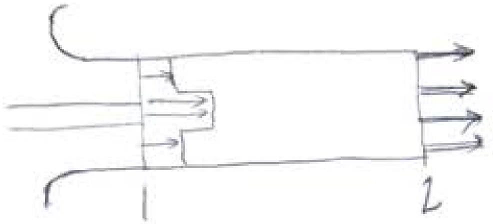
$$F = -\frac{155}{1134} \cdot \rho U_e^2 \delta_L$$ force on fluid

$$\text{Force on plate} = \frac{155}{567} \cdot \frac{1}{2} \rho U_e^2 \delta_L$$

$$K = \frac{155}{567} = 0.2734$$

[10]

2.



a) Mass conservation (incompressible)

$$V_2 D^2 = V_p d^2 + V_a (D^2 - d^2)$$

$$V_2 = V_p \left(\frac{d}{D}\right)^2 + V_a \left(1 - \left(\frac{d}{D}\right)^2\right)$$

$$V_2 = V_p \alpha + V_a (1 - \alpha)$$

$$V_2 = \alpha (V_p - V_a) + V_a$$ [3]

b) Bernoulli from atmosphere to ①

$$P_1 + \frac{1}{2}\rho V_a^2 = P_{atm}$$

$$\rightarrow P_1 = P_{atm} - \frac{1}{2}\rho V_a^2$$ [3]

c) S.F.M.E from $1 \rightarrow 2$, no viscous forces on walls

$P_2 = P_{atm}$ straight + parallel flow at 2.

$$\rightarrow P_{atm} D^2 + \rho V_2^2 D^2 = P_1 D^2 + \rho V_p^2 d^2 + \rho V_a^2 (D^2 - d^2)$$

$$V_2^2 = V_p^2 \alpha + V_a^2 (1-\alpha) + (P_1 - P_{atm})/\rho$$

$$\frac{P_1 - P_{atm}}{\rho} = -\frac{1}{2}\rho V_a^2 \text{ (from b)}$$

$$\rightarrow V_2^2 = V_p^2 \alpha + V_a^2 (1-\alpha) - \frac{1}{2}\rho V_a^2$$ [7]

$$V_2^2 = \alpha [V_p^2 - V_a^2] + V_a^2/2$$

d) $\alpha = 0.5$

$$\rightarrow \quad V_2 = (V_p + V_a)/2 \qquad \text{(mass)}$$

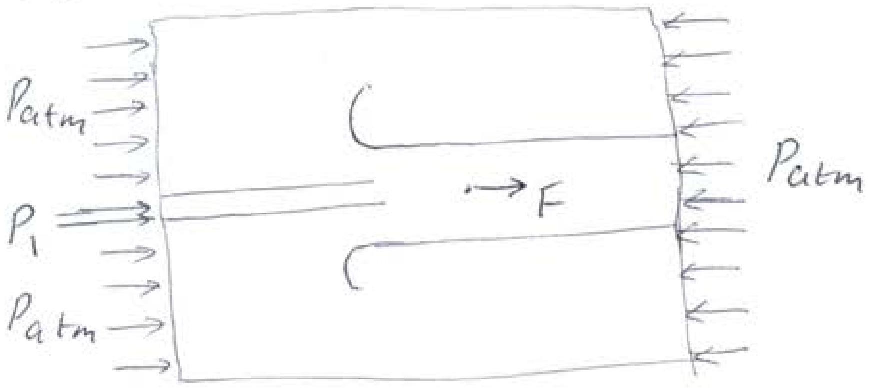
$$V_2^2 = (V_p^2 - V_a^2)/2 + V_a^2/2 = V_p^2/2 \qquad \text{(momentum)}$$

$$\underline{V_2 = V_p/\sqrt{2}}$$

$$\rightarrow \quad V_p/\sqrt{2} = V_p/2 + V_a/2$$

$$\rightarrow \quad V_a = V_p\left(\sqrt{2} - 1\right)$$

Now consider new control volume:



Force on flow

$$F + (P_1 - P_a)\, d^2 \pi/4 = \pi \frac{D^2}{4} \cdot \rho V_2^2 - \frac{\pi d^2}{4} \cdot \rho V_p^2$$

$$F = \rho \frac{V_p^2}{2} D^2 \pi/4 - \rho V_p^2 d^2 \pi/4 - \left(-\tfrac{1}{2}\rho V_a^2\right) d^2 \pi/4$$

$$F = \rho V_p^2 D^2 \pi/4 \left[\frac{1}{2} - \alpha + \frac{1}{2}\left(\frac{V_a}{V_p}\right)^2 \alpha \right]$$

$$= \underline{\rho V_p^2 D^2 \pi/4 \; \frac{\left(\sqrt{2} - 1\right)^2}{4}} = 0.0107\, \pi \rho V_p^2 D^2$$

force on fluid $\rightarrow$

force on duct $\leftarrow$

[10]

e)

The force is applied at the inlet as the flow accelerates into the duct from atmosphere, the pressure drops. The pressure difference on the inlet gives rise to the force pushing the duct to the left. [2]

3.

a)

Momentum (V = constant if fully developed)

$$\tau_w \cdot \pi d \cdot L = (P_A - P_B)\, d^2 \pi/4$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho V^2} = \frac{(P_A - P_B)}{\frac{1}{2}\rho V^2} \cdot \left(\frac{d}{L}\right)\left(\frac{1}{4}\right)$$

$$P_A - P_B = P_{0A} - P_{0B}$$ (fully developed V = const from continuity)

$$\rightarrow K = \frac{P_{0A} - P_{0B}}{\frac{1}{2}\rho V^2} = c_f \cdot 4 \cdot \left(\frac{L}{d}\right) = 1.2$$

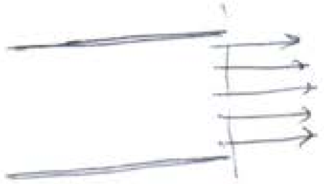
$$\rightarrow \underline{K = 1.2}$$ [5]

b) $V \approx 10\,m/s$ $\quad Re_d \sim \dfrac{1.2 \times 10 \times 0.05}{18.2 \times 10^{-6}}$

$$\sim 33 \times 10^3$$

$Re_d > 10^4 \rightarrow$ expect turbulent flow

$\rightarrow$ c_f is only a function of roughness and independent of Reynolds number [3] c)



If streamlines are straight and parallel at exit $P = P_{atm}$

therefore

$$P_{oC} = P_{atm} + \frac{1}{2}\rho V^2$$

$$\frac{P_{oC} - P_{atm}}{\frac{1}{2}\rho V^2} = 1.0$$

[2]

d)

$P_{oA} = P_{atm}$ (no inlet losses)

$$P_{oB} = P_{atm} - (1.2)\rho\frac{V^2}{2}$$

$$P_{oC} = P_{atm} - (1.2)\rho\frac{V^2}{2} + \left[A + \frac{B}{V} + \frac{C}{V^2}\right]\frac{\rho V^2}{2}$$

$$P_{atm} + \frac{1}{2}\rho V^2 = P_{atm} - (1.2)\rho\frac{V^2}{2} + \left[A + \frac{B}{V} + \frac{C}{V^2}\right]\frac{\rho V^2}{2}$$

$$1 = -1.2 + A + \frac{B}{V} + \frac{C}{V^2}$$

$$\frac{653}{V^2} - \frac{6.53}{V} - 3.33 = 0$$

$$\rightarrow \quad V = 13.06 \text{ m/s (reject -ve)}$$

$$Q = \frac{\pi d^2}{4}.V = \underline{\underline{0.0256 \text{ m}^3/\text{s}}}$$

[10]

e) $(Q\Delta P_0)_{Pipe} = -3.15\ W \qquad (\rho = 1.2\ kg/m^3)$

$(Q\Delta P_0)_{Pump} = 5.76\,W$

2.6 W lost due to mixing of flow downstream of c.

<u>or</u> leaving kinetic energy of flow accounts for 2.6 W additional power from pump.

[5]

94 1

Q4) a) i) $P_1 = P_4 = 1$ bar $T_1 = 288K$ $T_3 = 1400K$

$P_r = 20$ $P_2 = 20$ bar $\eta_c = \eta_t = 0.85$

$\gamma = 1.4$ $c_p = 1005\ J/kgK$.

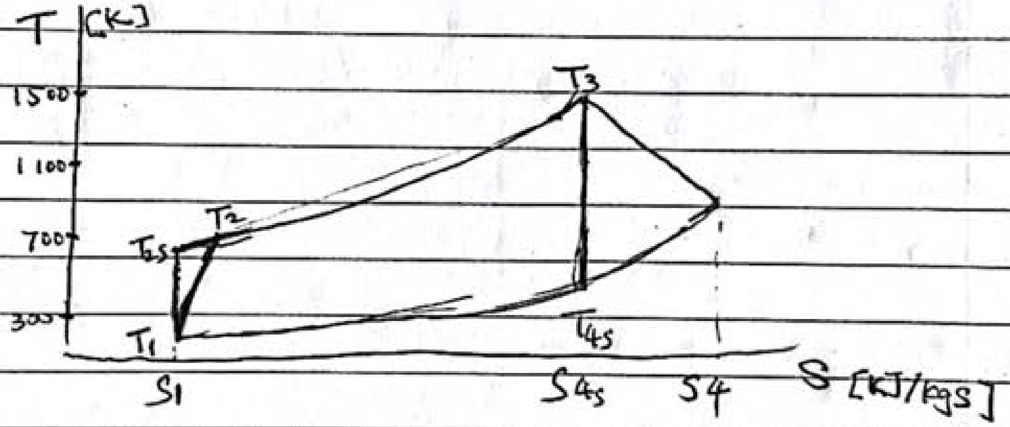
$T_{2s} = T_1 (P_r)^{\frac{\gamma-1}{\gamma}} = 288 \cdot 20^{\frac{0.4}{1.4}} = 288 \cdot 20^{0.2857} = 677.8K$

$T_2 = T_1 + (T_{2s} - T_1)/\eta_c = 288 + \frac{677.8-288}{0.85} = \underline{746.6K}$

$T_{4s} = T_3 (P_r)^{-\frac{\gamma-1}{\gamma}} = 1400(20)^{-0.2857} = 594.8K$

$T_4 = T_3 - \eta_t (T_3 - T_{4s}) = 1400 - 0.85(1400 - 594.8) = \underline{715.6K}$

T-s diagram.



ii) $\eta_{th} = \frac{\text{net work output}}{\text{heat input}} = \frac{c_p(T_3-T_4) - c_p(T_2-T_1)}{c_p(T_3-T_2)} = \cancel{\frac{1400-715.6}{1400-746.6}}$

$= \frac{(1400-715.6)-(746.6-288)}{1400-746.6} = 0.3455 \simeq \underline{\underline{34.6\%}}$

available Power. $b_3 - b_2 = (h_3 - h_2) - T_0(S_3 - S_2)$

$= c_p(T_3 - T_2) - T_0 c_p \ln(T_3/T_2)$

$= c_p[(T_3 - T_2) - T_0 \ln(T_3/T_2)]$

$= 1.005[653.4 - 288 \cdot 0.6287] = 474.7$

$\eta_{ex} = \frac{W_x}{\Delta b_{32}} = \frac{1.005(684.4 - 458.6)}{474.7} = \frac{226.93}{474.7} = \cancel{0.478 = 47.8\%}$

iii) available power lost in turbine exhaust:

$b_4 - b_1 = (h_4 - h_1) - T_0(S_4 - S_1) = c_p[(\Delta T_{4-1}) - T_0 \ln[\frac{T_4}{T_1}]]$

$= 1.005 \cdot (427.6 - 288 \cdot 0.909) = \underline{\underline{165.9\ kJ/kgK}}$.

b)i) $P_r = 40$ $P_2 = 40\text{ba}$ $T_{2s}' = T_1 \cdot P_r^{\frac{\gamma-1}{\gamma}} = 288 \cdot 40^{0.2857} = 826.3\text{ K}$

$T_2' = T_1 + \frac{T_{2s}' - T_1}{\eta_c} = 921.3\text{K}$

$T_3' - T_2' = T_3 - T_2$ as heat input constant.

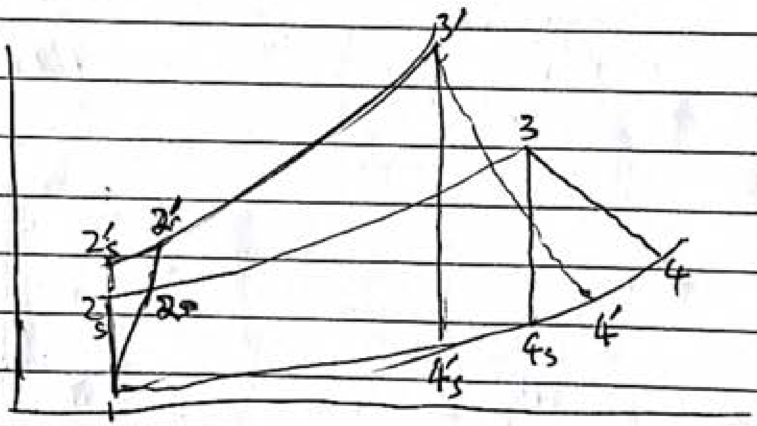
$T_3' = T_2' + \Delta T_{32} = 921.3 + (1400 - 746.6) = 1574.7\text{ K}$

$T_{4s}' = T_3 \cdot P_r^{-\frac{\gamma-1}{\gamma}} = 1574.7 \cdot 40^{-0.2857} = 548.8\text{ K}$

$\cancel{T_4' = \eta_t \cdot (T_3' - T_{4s}') = 0.85(1574.7 - 548.8) = .85 \cdot 1025.8 =}$

$T_3' - T_4' = \eta_t (T_3' - T_{4s}')$ $T_4' = T_3' - \eta_t (T_3' - T_{4s}')$

$T_4' = 1574.7 - 0.85 \cdot (1574.7 - 548.8) = \cancel{702.9}\ 702.7\text{K}$



b)ii) $b_4' - b_1 = \Delta h_{4-1} - T_0(\Delta s_{4-1}) = c_p\left[(T_4' - T_1) - T_0 \ln \frac{T_4'}{T_1}\right]$

$= 1.005\left[(702.7 - 288) - 288 \ln\left(\frac{702.7}{288}\right)\right] = 158.6\text{ kJ/kg·K}$

Slightly lower than aiii due to slightly lower T_4', thus reduced available power loss.

biii) $\eta_{th} = \frac{\dot{W}_{out} - \dot{W}_{in}}{\text{heat in}} = \frac{c_p[(T_3' - T_4') - (T_2' - T_1)]}{c_p(T_3' - T_2')} = \frac{\overbrace{872 - 633.3}^{=238.7}}{653.4}$

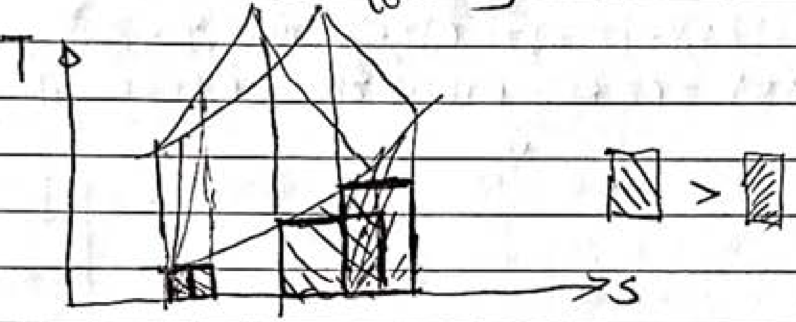
$= 0.3653 = 36.53\%$ (~2% ↑) [$\dot{w}$ ~3.5% ↑]

$\eta_{2nd} = \frac{w_x}{b_3' - b_2'} = \frac{238.7}{499.02} = 0.4783 = 47.83\%$

$b_3' - b_2' = \Delta h_{3'-2'} - c_p T_0 \ln \frac{T_3'}{T_2'} = c_p\left[(1574.7 - 921.3) - 288 \cdot \ln \frac{1574.7}{921.3}\right]$

$= c_p(653.4 - 154.38) = 501.5\text{ kJ/kg·K}$.

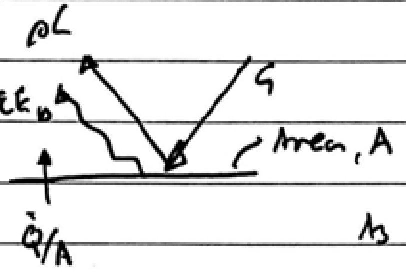
C). The loss of available Power in compressor & turbine in due ~~to~~ viscous forces and turbulence mixing which are dissipative and create irreversibility. Increasing ~~pressure~~ pressure ratio means more wetted surfaces and higher loss. The efficiency can only be improved by improving turbomachinery efficiency to reduce such irreversibilities.



1/4

Q5)

a) i)



$J = \rho G + \varepsilon E_b \Rightarrow G = \frac{J - \varepsilon E_b}{(1-\varepsilon)}$

$\frac{\dot{Q}}{A} = J - G$

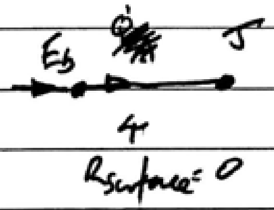
As grey, opaque $\varepsilon = \rho \Rightarrow \frac{\dot{Q}}{A} = J - \left(\frac{J - \varepsilon E_b}{(1-\varepsilon)}\right)$ loss

$$\frac{\dot{Q}}{A} = \frac{E_b - J}{(1-\varepsilon)/\varepsilon}$$

$$\Rightarrow \dot{Q} = \frac{E_b - J}{\left[(1-\varepsilon)/A\varepsilon\right]} = \frac{E_b - J}{R_{surface}}$$

where $R_{surface} = \frac{1-\varepsilon}{A\varepsilon}$

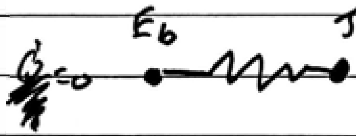
ii) Black body $\varepsilon \to 1$ so $R_{surface} = 0$ i.e. short circuit



$\Rightarrow E_b = J$

Perfect insulator $R_{surface} = ?$

but now $\dot{Q} = 0$ so open circuit



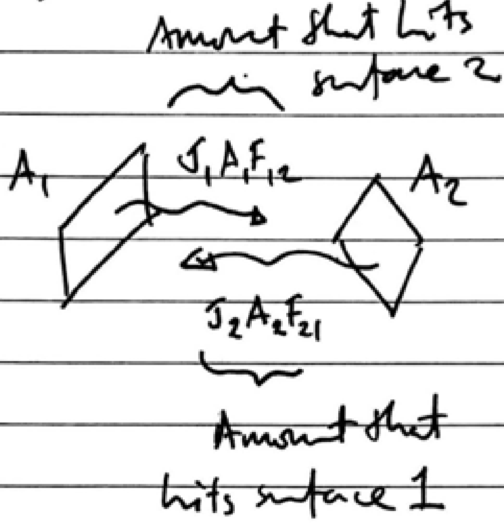
so $E_b = J$

but not the same!

Q5.) cont.

2/4

a) iii)



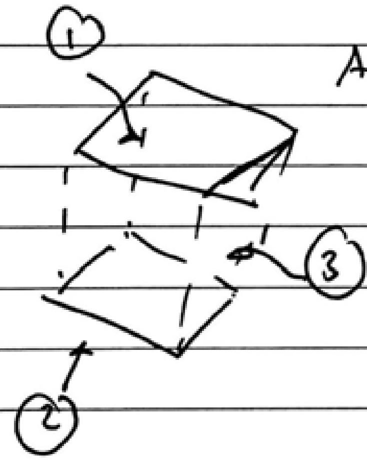
$$\boxed{A_1 F_{12} = A_2 F_{21} \quad \text{reciprocity rule}}$$

$$\dot{Q}_{12} = J_1 A_1 F_{12} - J_2 A_2 F_{21}$$

$$\Rightarrow \dot{Q}_{12} = \frac{J_1 - J_2}{\left[\frac{1}{A_1 F_{12}}\right]}$$

$$\Rightarrow \dot{Q}_{12} = \frac{J_1 - J_2}{R_{space}} \quad \text{where } R_{space} = \frac{1}{A_1 F_{12}} = \frac{1}{A_2 F_{21}}$$

b) i)



$A_1 = A_2$

$F_{12} = 0.65 = F_{21}$

$\cancel{F_{11}}^{0} + F_{12} + F_{13} = 1$

$\Rightarrow F_{13} = 1 - F_{12} = 0.35$

~~$A_1 F_{13} = A_3 F_{31}$~~

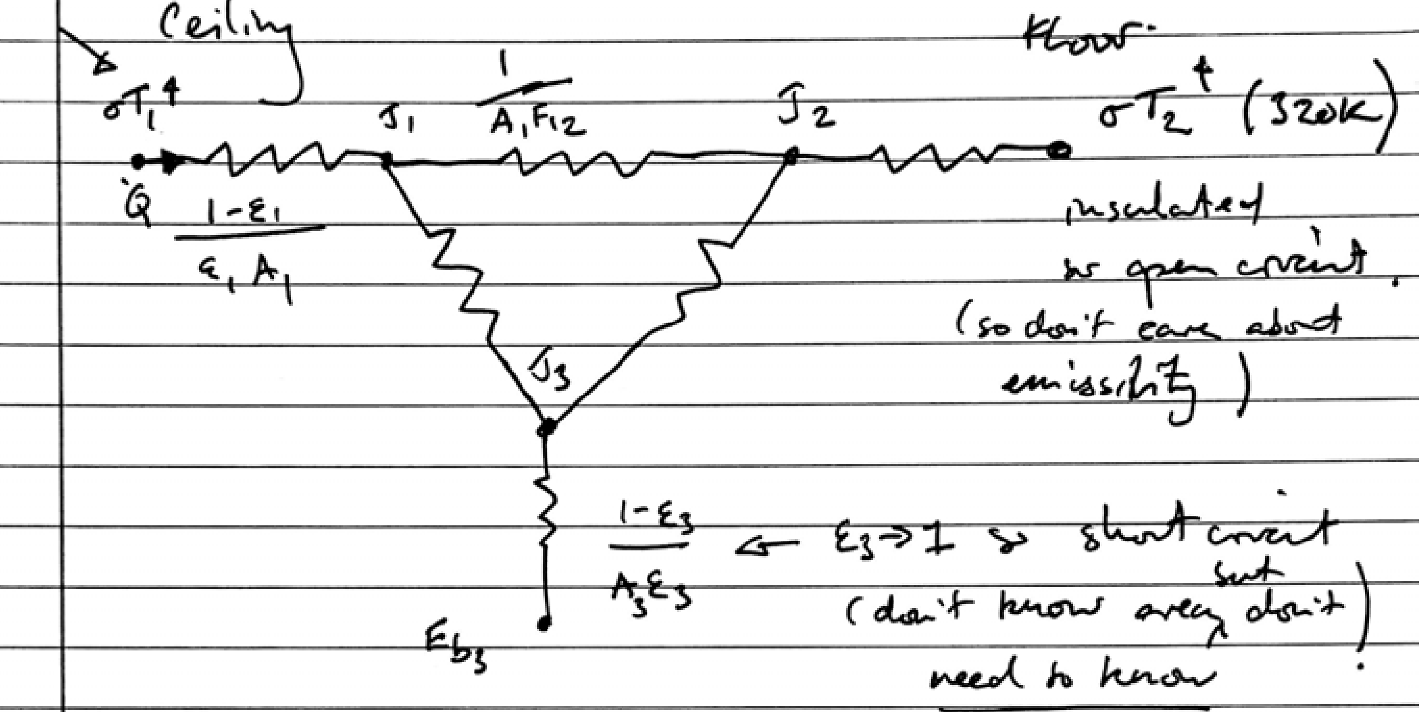
N.B. $F_{13} = F_{23}$ by symmetry.

$= 0.35$

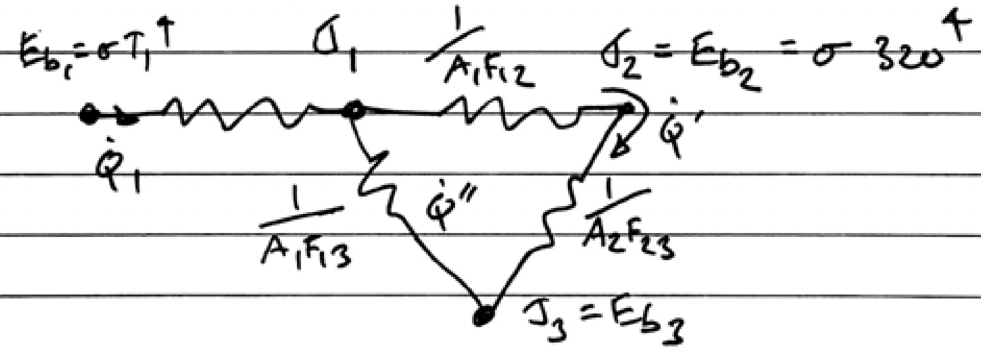
Q5) cont.

3/4

what we want.



so



$$\dot{Q}' = \frac{J_1 - E_{b_2}}{\frac{1}{\cancel{A_1}F_{12}}} = \frac{E_{b_2} - J_3}{\frac{1}{\cancel{A_2}F_{23}}} \Rightarrow J_1 = \frac{E_{b_2} - J_3}{\frac{1}{F_{23}}} \cdot \frac{1}{F_{12}} + E_{b_2}$$

$$\Rightarrow J_1 = \frac{\sigma 320^4 - \sigma 300^4}{\frac{1}{0.35}} \cdot \frac{1}{0.65} + \sigma 320^4$$

$$= 667.38 \ \mathrm{W/m^2}$$

Q5) cont.

$$\frac{E_{b_1} - J_1}{\frac{1-\varepsilon_1}{\varepsilon_1 A_1}} = \frac{J_1 - J_2}{\frac{1}{A_1 F_{12}}} + \frac{J_1 - J_3}{\frac{1}{A_1 F_{13}}}$$

$$E_{b_1} = \left(\frac{1-\varepsilon_1}{\varepsilon_1}\right)\left((J_1 - J_2)F_{12} + (J_1 - T_3)F_{13}\right) + J_1$$

$$= 697.43 \, W/m^{-2}$$

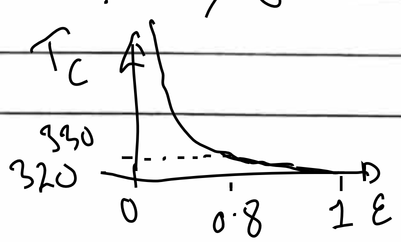
$$T_1 = 333.0 \, K$$

$$\dot{Q} = \frac{E_{b_1} - J_1}{\frac{1-0.8}{0.8 \times 5}} = \frac{697.43 - 667.38 \times 4}{0.2}$$

$$Q = 600.9 \, W$$

iii) Put it in the ceiling instead, no effect on floor as insulated.

N.B. No effect on heat flow, just the required ceiling temperature.



Q6 a i) $P_3 = 40 bar$ $T_3 = 500°C$ $h_3 = 3446 kJ/kg$ (superheated steam).

$P_4 = P_1 = 0.05 bar$. $T_4 = T_1 = \frac{28.96 + 36.16}{2} = 32.56°C$

$S_{4s} = 7.0922$ $(= S_3)$ $\quad x_{4s} = \frac{S_{4s} - S_{4f}}{S_{4g} - S_{4f}} = 0.83494$

$h_3 = 3446 kJ/kg$, $h_{4s} = h_{4f} + x_{4s} \cdot h_{fg} = 2160 kJ/kg$ $\quad$ ($h_{4f} = 1365 kJ/kg$)

$h_4 = h_3 - \eta_t (h_3 - h_{4s}) = 2417.3$

$x_4 = \frac{2417.3 - 136.5}{2424} = \underline{0.941}$

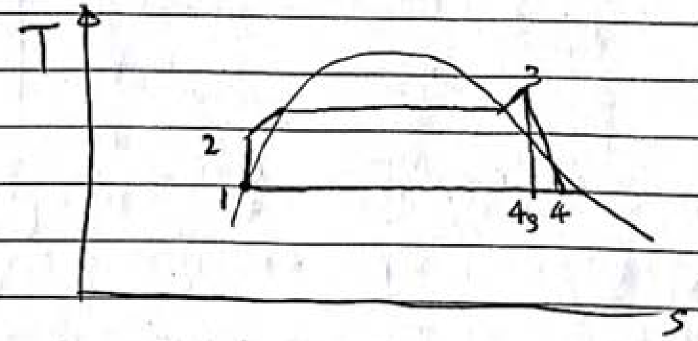
$w = h_3 - h_4 = 3446 - 2417 = 1029 kJ/kg$

$\dot{W}_x = 10 kg/s \cdot (1029 kJ/kg) = \underline{10.29 MW}$.

$w_f = 0.001003(40 - 0.05) \doteq 4.0 kJ/kg$.

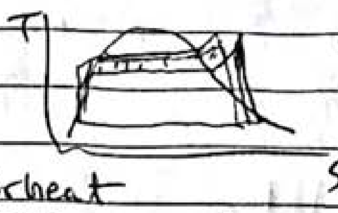
$\eta_{th} = \frac{W_{net}}{Q_{in}} = \frac{\Delta h_{3-4} - w_f}{\Delta h_{3-2}} = \frac{1029 - 4}{3446 - (1365 + 4)} = 0.31$
$= \underline{31\%}$

a. ii)



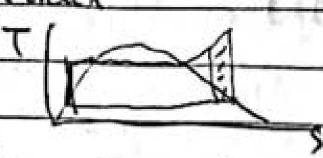
a. iii) need to increase the area enclosed by the cycle in T-s diagram.

Increase P_2.



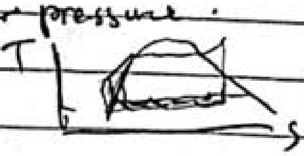
- effective.
- higher boiler P. material limit.
- wetter exhaust if T_3 the same.

Increase superheat



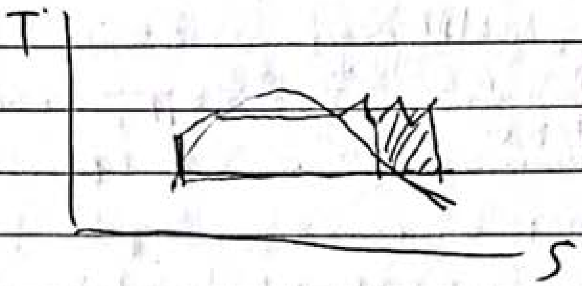
- limited by materials.

lower condenser pressure.



- wetter exhaust.
- requires lower T or better cooling technology.

reheat

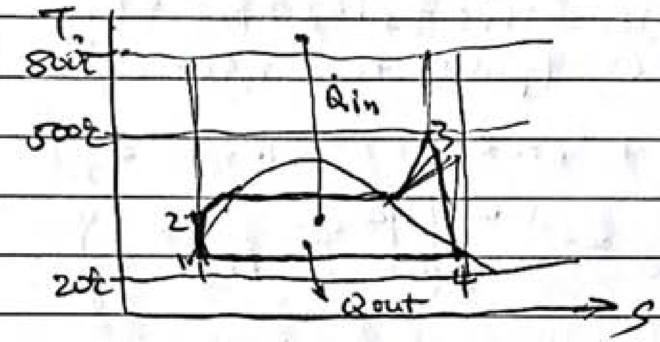


— V. Good.

— many stages results in diminished returns ~~cost~~ + increased costs.

best is combination of those to achieve an optimum.

b.i



$S_3 = 7.1$ kJ/kgK

$S_4 = 7.9$ kJ/kgK

$S_1 = S_2 = S_{4f} = 0.47$ kJ/kgK.

$$\dot{q}_{in} = h_3 - h_2 = 3305 \text{ kJ/kg}$$

$$q_{out} = h_4 - h_1 = 2281 \text{ kJ/kg}$$

Power lost in boiler $= T_0 \Delta S_{2-3} = T_0\left(\frac{-q_{in}}{T_{in}} + \Delta S_{3-2}\right)$

$$= 293 \cdot \left[\frac{-3305}{1073} + \overset{6.63}{(7.1 - 0.47)}\right] = \underline{\underline{104.0 \text{ kJ/kg}}}$$

Power lost in condenser $= T_0 \Delta S_{4-1} = (h_4 - h_1)\left(1 - \frac{T_0}{T_4}\right)$

$$= (2417.3 - 136.5)\left(1 - \frac{293}{306}\right)$$

$$= \underline{\underline{96.9 \text{ kJ/kg}}}$$

much larger ~~power lost~~ available power lost in boiler.

ii. could place another cycle between the heat source & the boiler to reduce the temperature of heat into boiler. e.g. ~~to~~ have a combined cycle of gas turbine topped steam turbine.