Q1 Psychometrics

1a)

State 1 1 bar $P_{sat}(10°C) = 0.01228$ bar

10°C $P_{v1} = 0.5 \cdot P_{sat}(10) = 0.00614$ bar

$$\omega = [kg\ H_2O]/[kg\ Air] = \frac{P_{v1}}{P_{A1}} \frac{M_{H_2O}}{M_{Air}} = \frac{P_{v1}}{(P_{Tot} - P_{v1})} \frac{18}{29}$$

$$\omega_1 = \left(\frac{\phi\ P_{sat}}{P_{Tot} - \phi\ P_{sat}}\right) \frac{18}{29} = \frac{0.5 \cdot 0.01228}{(1 - \underbrace{0.5 \cdot 0.01228}_{P_{v1} = 0.00614\ bar})} \frac{18}{29} = \boxed{0.00383 \frac{kg\ H_2O}{kg\ Air}}$$

Conservation of mass

$$\dot{m} = \dot{m}_{Air} + \dot{m}_v$$

$$\frac{\dot{m}_v}{\dot{m} - \dot{m}_v} = \omega \Rightarrow (1+\omega)\dot{m}_v = \dot{m}\omega$$

$$\dot{m}_v = \dot{m} \frac{\omega}{1+\omega} = \frac{0.00383}{1 + 0.00383} = \boxed{0.00382\ kg/s = \dot{m}_{v1}}$$

$$\dot{m}_{Air} = \dot{m}\left(1 - \frac{\omega}{1+\omega}\right) = \dot{m}\left(\frac{1}{1+\omega}\right) = 0.99618\ kg/s = \dot{m}_{A1}$$

1b)

Superheated steam taken at *T* and p_{sat}

$h_{v1} = 2519.2\ \frac{kJ}{kg}$ (10°C) $h_{v2} = 2626.1\ kJ/kg$ (sat curve at T=70°C)

SFEE

$$\dot{Q} = \dot{m}_{Air} C_p (T_2 - T_1) + \dot{m}_{v1}(h_2 - h_1)$$

$$= \underbrace{0.99618 \tfrac{kg}{s}\ 1.1 \tfrac{kJ}{kgK} (70-10)K}_{65.75\ kW} + \underbrace{0.00382 \tfrac{kg}{s} (\overset{h_2}{2626.1} - \overset{h_1}{2519.2}) \tfrac{kJ}{kg}}_{0.41\ kW}$$

$$\boxed{\dot{Q}_{12} = 66.15\ kW}$$

1c) **Given** $\phi_3 = 100\%$, calculate $\dot{m}_3$

$T_3 = 28°C$

$p_{sat} = 0.03783$ bar

Conservation of mass

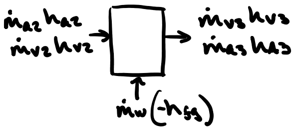
$$\dot{m}_w = \dot{m}_{v3} - \dot{m}_{v1}$$

$$\dot{m}_{v3} = \dot{m}_{a1}\,\omega_3 = \underset{0.99618\ kg/s}{\dot{m}_{a1}} \overbrace{\frac{\overset{1}{\phi}\ \overset{0.03783}{P_{sat}(T_3)}}{\underset{1}{P_{Tot}} - \underset{1}{\phi}\ P_{sat}} \frac{18}{29}}^{\omega_3 = 0.0244}$$

$$\dot{m}_{v3} = 0.0243\ kg/s$$

$$\dot{m}_w = \dot{m}_{v3} - \dot{m}_{v1} = 0.0243\ kg/s - 0.00382\ kg/s = \boxed{0.0205\ kg/s = \dot{m}_w}$$

1d)



solve for $h_f \rightarrow T_w$

$$\Delta \dot{H}_{23} = \dot{m}_a \, c_p (T_3 - T_2) + \dot{m}_{v_3} h_3 - \dot{m}_{v_2} h_2 - \dot{m}_w \boxed{h_f} = 0$$

$$h_f = \left(\dot{m}_a c_p (T_3 - T_2) + \dot{m}_{v_3} h_3 - \dot{m}_{v_2} h_2\right) / \dot{m}_w$$

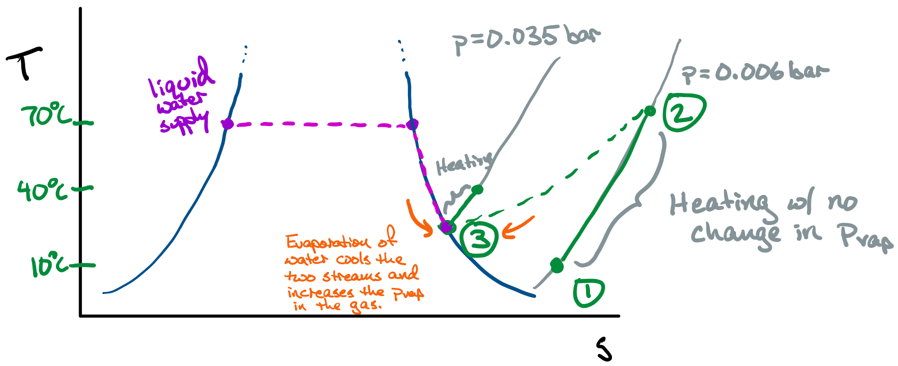
at 28°C   at 70°C

$$= \Big(\underbrace{0.99618 \cdot 1.1 (28-70)}_{-46.0 \text{ kJ/s}} + \underbrace{0.0243 \cdot 2551.9}_{+62.0 \text{ kJ/s}} - \underbrace{0.00382 \cdot 2626.1}_{-10.0 \text{ kJ/s}}\Big) / 0.0205 \text{ kg/s}$$

$$h_f = 292 \text{ kJ/kg} \xrightarrow[\text{From Tables}]{} \boxed{T_w \sim 70°C}$$

Within the evaporating coils no work or heat transfer is done. Rather a mass of liquid is evaporated which when accounting for the water evaporation results in a net zero change in enthalpy within the evaporating coils.

1e)



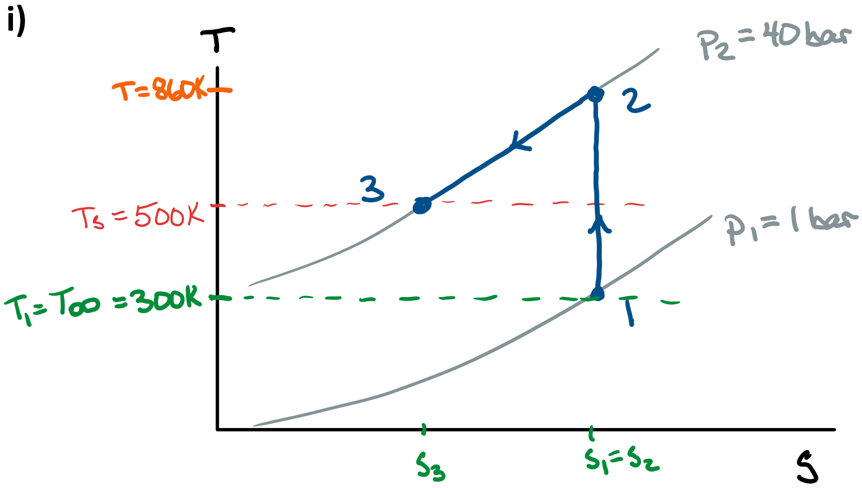
Q1 Psychometrics – 155 attempts, average mark 14.2/25, max. 25 and min. 0.

Psychometrics had not been examined for several years, resulting in relatively few students attempting the question, which was similar to their example paper. Most students who made a valid attempt could define the specific humidity and construct the energy balance to determine the required heating covering parts (a) - (c). Many struggled to solve for the enthalpy of evaporation needed to look up the initial water temperature in the tables in part (d). There were mixed results with the representation of the water vapour on a *T*-*s* diagram, with roughly half of those attempting it constructing a reasonable depiction. The overall low marks were indicative of the bimodal nature where a large fraction of students only attempted parts (a) – (b), leaving the remaining blank. The question was marked with favourable credit given for any partial work.

Q2 Energy Storage

a)

i)



ii)

$$\frac{T_2}{T_1} = \left(\frac{P_2}{P_1}\right)^{\frac{\gamma-1}{\gamma}} = \underbrace{\left(\frac{40}{1}\right)^{0.4/1.4}}_{2.87}$$

$$T_{2a} = 300 \cdot 2.87 = \boxed{860.7\,K}$$

$$\dot{W}_{x12a} = \dot{m}(h_{1a} - h_{2a}) = \dot{m} c_{pa} (T_{1a} - T_{2a})$$

$$= 1\,\text{kg/s}\ 1.1\,\text{kJ/kgK}\ (300 - 860)\,K$$

$$W_{x12a} = \boxed{-616.8\ \text{kW}}$$

iii)

$$\dot{Q}_{23a} = \dot{m} c_p (\overset{=T_S}{T_{3a}} - T_{2a}) = 1\,\frac{\text{kg}}{\text{s}}\ 1.1\,\frac{\text{kJ}}{\text{kgK}} (500 - 860)\,K$$

$$\boxed{\dot{Q}_{23a} = -396.8\,\text{kW}}$$

iv)

$$\Delta \dot{B}_{23a} = (\dot{H}_{3a} - T_0 \dot{S}_{3a}) - (\dot{H}_{2a} - T_0 \dot{S}_{2a})$$

$$= \underbrace{\dot{m}\left[c_p (T_3 - T_2)\right.}_{\dot{Q}_{23}} \left. - T_0 (s_{3a} - s_{2a})\right]$$

$$= \dot{Q}_{23} - \dot{m} T_0 \left(c_p \ln\left[T_{3a}/T_{2a}\right] - \cancelto{0}{R \ln\left[P_{3a}/P_{2a}\right]}\right)$$

$$= -396.8\,\text{kW} - \underbrace{1\,\text{kg/s}\ 300K \left(1.1\,\frac{\text{kJ}}{\text{kgK}} \ln\left[500K/860K\right]\right)}_{-119.07\ \text{kW}}$$

$$\boxed{\Delta \dot{B}_{23a} = -217.4\ \text{kW}} \qquad \Delta \dot{H}_{23a} = \dot{Q}_{23a} = -396.8\,\text{kW}$$

Discussion: The change in available power represents the maximum amount of work that could have been produced in a Carnot heat engine exchanging heat with a finite temperature, T_{∞}. The change in availability is less than the change in enthalpy as only a fraction (Carnot efficiency) is available for work. The question solely pertains to the changes in enthalpy and availability of the air and not the thermal storage.

b)

i)

$$\dot{Q}_{32b} = \dot{m} c_p (T_{2b} - T_{3b}) = 1\,\text{kg/s} \cdot 1.1\,\frac{\text{kJ}}{\text{kg K}}(500\text{K} - 300\text{K})$$

$$\dot{Q}_{32b} = 220\,\text{kW}$$

$$(\dot{H}_{2b} - T_0 \dot{S}) - (\dot{H}_{2a} - T_0 \dot{S}_{2a})$$

$$\Delta \dot{B}_{32b} = \dot{m}\left[\underbrace{c_p (T_{2b} - T_{3b})}_{\dot{Q}_{32b}} - T_0 (s_{2b} - s_{3b})\right]$$

$$= \dot{Q}_{32b} - \dot{m} T_0 \left(c_p \ln[T_{2b}/T_{3b}] - R \cancelto{0}{\ln[P_{2b}/P_{3b}]}\right)$$

$$= +220\,\text{kW} - \underbrace{1\,\text{kg/s}\; 300\text{K}\left(1.1\,\frac{\text{kJ}}{\text{kg K}} \ln\left[500\text{K}/300\text{K}\right]\right)}_{168.57\,\text{kW}}$$

$$\boxed{\Delta \dot{B}_{32b} = 51.4\,\text{kW}}$$

Discussion: The change in available power upon generation through the heat exchanger is due (1) less heat being added back into the flow stream than upon storage, i.e. $Q_{23b}<Q_{23a}$, and (2) the mean temperature of heat addition is lower upon regeneration relative to storage, i.e. due to the gas being stored at ambient temperature, $T_{3b}=T_{\infty}$ thus leading to lower availability.

ii)

$$T_{1b} = T_{2b}\left(P_{1b}/P_{2b}\right)^{\gamma-1/\gamma} = 500\text{K}\underbrace{\left(1/40\right)^{0.4/1.4}}_{0.35} = 174\,\text{K}$$

$$_{23}x_b = \dot{m} c_p (T_{2b} - T_{1b}) = 1\,\text{kg/s} \cdot 1.1\,\frac{\text{kJ}}{\text{kg K}}(500 - 174)\text{K}$$

$$\boxed{_{23}x_b = 358.3\,\text{kW}}$$ ~ work out is positive

$$\eta_{cycle} = \frac{|\dot{W}_{23b}|}{|\dot{W}_{12a}|} \approx \frac{358.3}{616.7}$$

$$\boxed{\eta_{cycle} = 58\%}$$

The cycle efficiency as a function of T_s

$$\eta = \frac{|\dot{m} c_p (T_s - T_{1b})|}{|\dot{m} c_p (T_{1a} - T_{2a})|}$$

~ where the maximum $T_{2b} = T_s$

~ as storage gets its heat from 2a

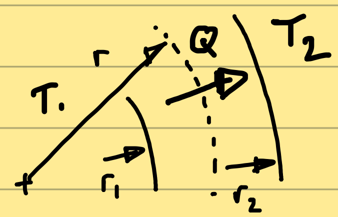
$T_{2a} > T_s$ as $\dot{Q}_{23a} = \dot{m} c_p (T_{2a} - T_{3a})$ where $T_{3a} = T_s$

Discussion: To increase the efficiency we must maximize the regeneration heat addition, such that heat addition equals heat removal, i.e. $Q_{23a} = Q_{23b}$. To maximize availability increasing storage temperature increases efficiency. The balance of heat rejection and addition provides an upper limit for an isotheral T_s for a cyclical process where the air is stored at ambient temperature $T_{3a} = T_{3b} = T_{\infty}$.

Q2 Thermodynamics of Energy Storage – 285 attempts, mean 18.0/25, max. 25 and min. 0.

The thermodynamics of energy storage examined the availability of flow within a perfect gas undergoing compression/expansion and heat exchange. The students did very well in depicting the cycle on a *T-s* diagram and determining the work and heat required for the processes in parts (a)(i)-(iii). The availability of the flow was numerically determined by most students correctly, but few identified the correct reason that the change in availability had a lower absolute change than the enthalpy part (a)(iv). Most students determined the correct heat required and availability change in regeneration, as well as the work produced and cycle efficiency in part (b). The only challenge students had was correctly explaining the results and discussing the impact of storage temperature. The overall high marks were indicative of the unimodal distribution, with most students answering nearly all numerical questions correctly and missing minor marks in their written discussion of results.

Q3)

a) 

$Q = -\lambda 2\pi r \dfrac{dT}{dr}$ $\qquad Q = \Delta T / R \Rightarrow R = \dfrac{T_1 - T_2}{Q}$ 1

1

$$\int_{r_1}^{r_2} \frac{dr}{r} = \int_{T_1}^{T_2} \frac{-\lambda 2\pi}{Q}\, dT \Rightarrow \ln(r_2/r_1) = \frac{2\pi\lambda}{Q}(T_1 - T_2) \Rightarrow R = \frac{1}{2\pi\lambda}\ln\left(r_1/r_2\right) \;//$$ 1

FOR UNIT LENGTH

b) $\lambda_f = 0.05\ W m^{-1} K^{-1}$ $\qquad Pr = \dfrac{c_p \mu}{\lambda} = \dfrac{4.23\times10^3 \times 0.3\times10^{-3}}{0.05}$

$\mu = 0.3\times10^{-3}\ Pa s$

$c_p = 4.23\times10^3\ J kg^{-1} K^{-1}$ $\qquad = 25.38\,[-]$ 1

1

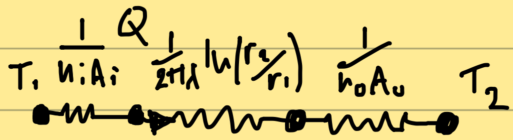
$r_i = 4\,mm \Rightarrow Re_D = \dfrac{\rho \bar{u} D}{\mu} = \dfrac{4\dot{m}}{D\mu} = \dfrac{4\times0.01}{0.008\times0.3\times10^{-3}} = 5305.2\ [-]$

$D = 0.008\,m$

1

$$Nu_D = 0.023\, Re_D^{0.8}\, Pr^{0.4} = 0.023\times5.305\times10^3 \times 25.4^{0.4} = 80.0\ [-]$$

i) $Nu_D = \dfrac{hD}{\lambda_f} \Rightarrow h = \dfrac{Nu_D \lambda_f}{D} = \dfrac{80\times0.05}{0.008} = \boxed{500.2\ W m^{-2} K^{-1}}$ 1

ii) 

$\lambda_s = 0.1\ W m^{-1} K^{-1}$, 1m tube

$R = R_i + R_{wall} + R_o$ 1

$= 1/h_i 2\pi r_i + \dfrac{1}{2\pi\lambda_s}\ln(r_o/r_i) + 1/h_o 2\pi r_o$

$= \dfrac{1}{500\times2\pi\times0.004}$ 1 $+ \dfrac{1}{2\pi\times0.1}\ln\{7/4\}$ 1 $+ \dfrac{1}{7.5\times2\pi\times0.007}$ 1

$= 0.0795 + 0.891 + 3.03 = \boxed{4.0\ KW^{-1}}$

c) Increased external surface area can drop resistance of external convection quicker than the increase in resistance

i) of the wall. 2

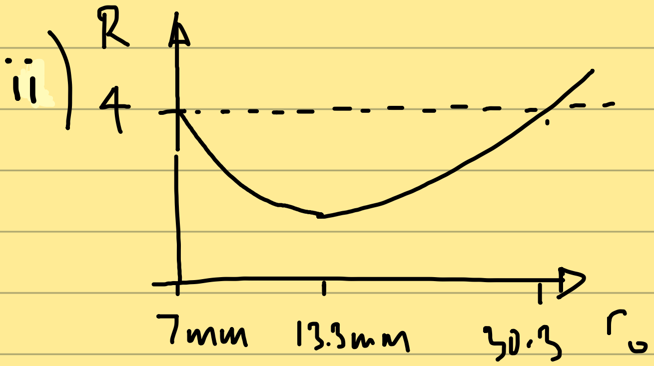
$$R = \frac{1}{2\pi}\left\{ \frac{1}{h_i r_i} + \frac{1}{\lambda_s}\ln\left(\frac{r_o}{r_i}\right) + \frac{1}{h_o r_o} \right\}$$

$$\frac{\partial R}{\partial r_o} = \frac{1}{2\pi}\left\{ \frac{1}{\lambda_s r_o} - \frac{1}{h_o r_o^2} \right\} = 0 \qquad 2$$

$$\frac{1}{\lambda_s \cancel{r_o}} - \frac{1}{h_o r_o \cancel{r_o}} = 0 \qquad 1$$

$$\frac{1}{\lambda_s} = \frac{1}{h_o r_o} \Rightarrow r_o = \lambda_s / h_o = 0.1/7.5 = \boxed{13.3\ \text{mm}}$$

ii)



$$\lambda_s\left(2\pi R_7 - \frac{1}{h_i r_i} - \frac{1}{h_o r_o}\right) = \ln\left\{\frac{r_o}{r_i}\right\} \qquad 2$$

$$r_o = r_i \exp\left\{\lambda_s\left(2\pi R_7 - \frac{1}{h_i r_i} - \frac{1}{h_o r_o}\right)\right\} \qquad 1$$

$$r_o^{n+1} = 4\times10^{-3} \exp\left\{0.1\left(2\pi\times4 - \frac{1}{500\times4\times10^{-3}} - \frac{1}{7.5\, r_o^n}\right)\right\} \qquad 1$$

1st guess in 14mm, Gauss iteration until:

$$\boxed{r_o = 30.3\text{mm} \;//} \qquad 2$$

N.B.

$$r_0^{n+1} = \frac{1}{h_0}\left[2\pi R_? - \frac{1}{h_i r_i} - \frac{1}{\lambda_s}\ln\left\{\frac{r_0^n}{r_i}\right\}\right]^{-1}$$ 4

This converges on $r_0 = 7\,mm$ regardless of starting value.

d) i) $Nu = \frac{hd}{\lambda_f} \sim \left\{ \frac{g d^3 \beta \Delta T}{\nu^2} \right\}^{1/3} \sim d$ 1

$\Rightarrow$ $\boxed{h \sim \text{const.} \quad \text{no error}}$ 1

ii) $Nu = \frac{hd}{\lambda_f} \sim \left(\frac{\rho u d}{\mu}\right)^{4/5}$ 1

$$h \sim d^{-1/5}$$

$$\frac{h'}{h} = \left(\frac{2\cancel{d}}{\cancel{d}}\right)^{-1/5}$$

$$h' = h\, 2^{-1/5} = 0.87\, h$$ 1

$\Rightarrow \frac{h - h'}{h} = \frac{1 - 0.87}{1} =$ $\boxed{13\% \quad \text{reduction} //}$

Q3 Conduction – 167 attempts, mean 15.5/25, max. 25 and min. 0.

This quesiton was a relatively popular heat transfer question compared to previous years. It started with an easy lead-in, asking the students to derive the conduction resistance for a cylindrical pipe. This part was done correctly by almost all candidates. Part (b)(i) asked the students to estimate the heat transfer coefficient on the inside of the tube from the information given. This required them to find the Reynolds No. and the Prandtl No., then pick the turbulent correlation from the Databook. Approximately 70% of the students did this perfectly, and another 20% decided to use the laminar correlation, often justifying this with a comment that the mass flow rate "*looked low*". Several candidates attempted to use the Biot No. Part (b)(ii) asked the candidates to find the overall thermal resistance of the pipe. Most candidates did this perfectly (full marks were given for the correct method, regardless of the heat transfer coefficient found in the previous section). Approximately 10% of students incorrectly used the cross-sectional area of the pipe, rather than the surface area. Next, in part (c), the candidates had to explain why the resistance of the pipe might decrease and then find the worst case by differentiation of the resistance expression. The differentiation conveniently removed carried errors in the internal (but constant) heat transfer coefficient, whilst full marks were given where the correct method was used, but the resistance expression was wrong. The next part asked the students to use an iterative solution to find the radius at which the resistance would first increase with increasing radius. Well over half the candidates used the solver in their calculator to derive the correct result, and a small minority made swift progress by hand. Finally, in part (d), the candidates were asked to consider scaling the external heat transfer coefficient under natural and forced convection conditions with changing radius. This was done well by approximately half the candidates, with the other half considering the scaling of the Nusselt No. alone.

[redacted]

4

a) $\partial V/\partial t = 0 \quad \nabla \rho = 0$ steady, incompressible

$$(\nabla \times V) \times V + \frac{1}{2}\nabla(V \cdot V) = -\frac{\nabla P}{\rho}$$

$$\omega \times V + \frac{1}{2}\nabla(V \cdot V) + \nabla P/\rho = 0$$

$$\nabla\left[P + \rho \frac{V \cdot V}{2}\right] = \rho V \times \omega \rightarrow \underline{\nabla P_0 = \rho \underline{V} \times \underline{\omega}}$$

[5]

b) $\partial u/\partial x = A \cos(2\pi x/L)\cos(2\pi y/L) \times 2\pi/L$

$\partial v/\partial y = -A \cos(2\pi x/L)\cos(2\pi y/L) \times 2\pi/L$

i)

$\nabla \cdot V = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$

[5]

ii) $\partial u/\partial y = -A \sin(2\pi x/L)\sin(2\pi y/L) \times 2\pi/L$

$\partial v/\partial x = A \sin(2\pi x/L)\sin(2\pi y/L) \times 2\pi/L$

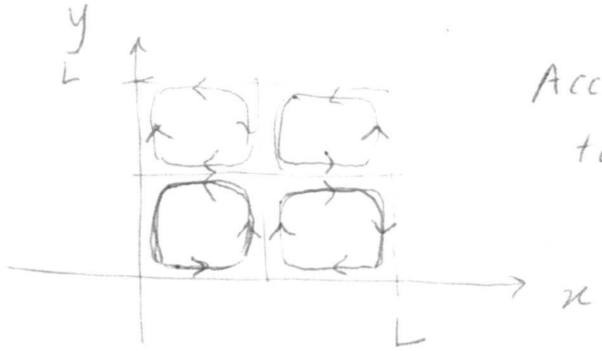
$$\omega = \partial v/\partial x - \partial u/\partial y = A \sin(2\pi x/L)\left(\frac{2\pi}{L}\right) 2 \sin(2\pi y/L)$$

$$= A\left(\frac{4\pi}{L}\right)\sin\left(\frac{2\pi x}{L}\right)\sin\left(\frac{2\pi y}{L}\right)$$

[5]

Not irrotational because $\omega \neq 0$ unless $A = 0$

iii)



According to part (a) total pressure gradient is orthogonal to V thus can only apply Bernoulli along streamlines (not across streamlines)

[3]

c i) A units of speed (m/s)

B units of pressure (Pa)

C units of specific energy (J/kg , m^2/s^2) [2]

ii) $$\left(\begin{pmatrix} u \\ v \end{pmatrix} \cdot \begin{pmatrix} \partial/\partial x \\ \partial/\partial y \end{pmatrix} \right) \begin{pmatrix} u \\ v \end{pmatrix} = -\frac{1}{\rho} \begin{pmatrix} \partial P/\partial x \\ \partial P/\partial y \end{pmatrix}$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x}$$

$$s(x) = \sin(2\pi x/L) \quad c(x) = \cos(2\pi x/L)$$

$$A\, s(x) c(y) \left(A\, c(x) c(y) \right) \frac{2\pi}{L} + \left(-A\, c(x) s(y) \right) \left(-A\, s(x) s(y) \right) \frac{2\pi}{L}$$

$$= -\frac{1}{\rho} (\rho C) \left(-s(2x) \right) 4\pi/L$$

$$\frac{2\pi}{L} A^2 \left[s(x)\, c(x)\, c^2(y) + s(x)\, c(x)\, s^2(y) \right]$$

$$= C\, s(2x)\, 4\pi/L$$

$$A^2 s(x)\, c(x) = C \underbrace{s(2x)\,(2)}_{4\, s(x)\, c(x)}$$

$$\rightarrow \quad C = A^2/4$$

[5]

Q4 Euler Equations – 189 attempts, mean 16.1/25, max. 24 and min. 2.

The problem involved the use of the vector form of the Euler equations. It was generally done well. Most students were able to derive the relationship between the gradient in total pressure and vorticity. Students also showed a good understanding of the use of divergence of the velocity field for mass conservation. Most students were not able to sketch the streamlines for the given flow field, and there was a mix of responses as to whether Bernoulli could be applied for the given flow – most recognized that the flow was not irrotational but failed to spot that total pressure was still constant along streamlines. Only a very small number of students were able to expand the Euler equations to find the equation relating to the x-momentum.

5.

a) $V_b = V_c = Q_0/A$

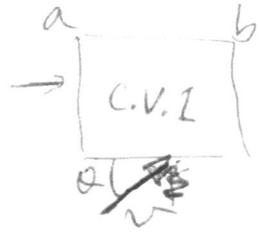
$V_a = (1-\alpha)\, Q_0/A$

$V_d = V_a = (1-\alpha)\, Q_0/A$ [3]

b) MOMENTUM

$\left(P_a A + \rho V_a^2 A\right) = \left(P_b A + \rho V_b^2 A\right)$

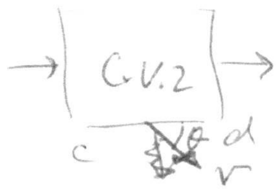
$+ \, v \cos\theta \, \dot{m}_{bleed}$



$\left(P_d A + \rho V_d^2 A\right) = \left(P_c A + \rho V_c^2 A\right)$

$+ \, v \cos\theta \, \dot{m}_{bleed}$

$V_a = V_d, \quad V_b = V_c$



$\rightarrow \quad P_d - P_a = P_c - P_b$

[5]

c)

i) $K = \dfrac{-\Delta P}{\frac{1}{2}\rho V^2} = \dfrac{P_d - P_a}{\frac{1}{2}\rho V_d^2}$

$\Delta P_T = P_b - P_c = P_d - P_a = K \times \frac{1}{2}\rho V_d^2$

$\Delta P_T = \dfrac{\rho K}{2}(1-\alpha)^2 Q_0^2/A^2$

[5]

ii) Power from pump $= Q_0 \Delta P_T$

$= \dfrac{\rho K}{2}(1-\alpha)^2 Q_0^3/A^2$

Loss across screen $= (1-\alpha) Q_0 \Delta P_T$

$= \dfrac{\rho K}{2}(1-\alpha)^3 Q_0^3/A^2$ [5]

iii) loss of bleed

$$= Q_0 \Delta P_T - (1-\alpha)\, Q_0 \Delta P_T$$

$$= \alpha Q_0 \Delta P_T$$

$$= \rho \frac{K}{2} \frac{Q_0^3}{A^2} \alpha (1-\alpha)^2$$

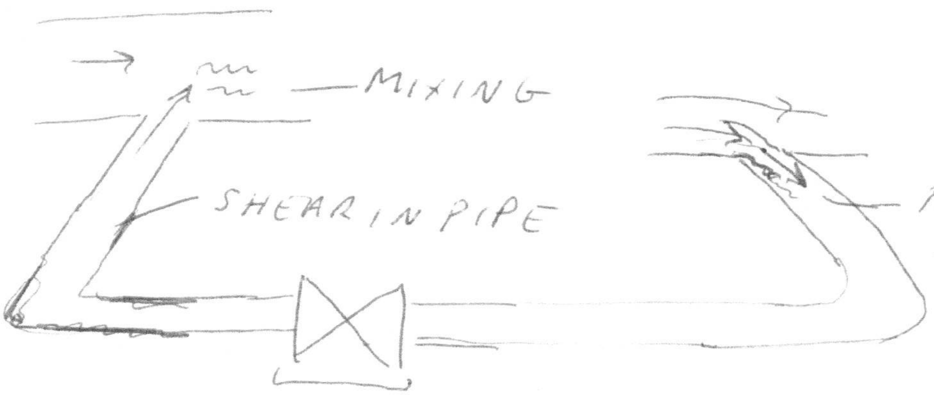
$$\frac{\partial \text{loss}}{\partial \alpha} = \rho \frac{K}{2} \frac{Q_0^3}{A^2} \left[(1-\alpha)^2 - 2\alpha(1-\alpha) \right]$$

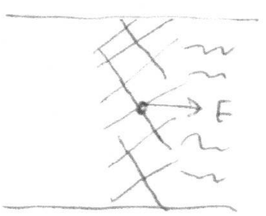
At max $\quad (1-\alpha)^2 = 2\alpha(1-\alpha)$

$$1-\alpha = 2\alpha$$

$\boxed{\alpha = 1/3}$ for max bleed loss

LOSS SOURCES





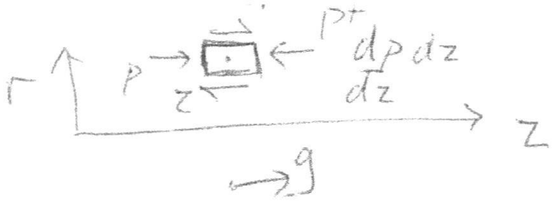
PRESSURE LOSS DUE DRAG ON SCREEN

(also pump losses if efficiency < 100% due to viscous dissipation in pump) [4]

Q5 Control Volume – 169 attempts, mean 15.7/25, max. 25 and min. 0.

The problem involved a bleed flow to and from a duct. Virtually all students were capable of applying continuity to find expressions for velocity up and downstream of the bleed – although a small minority were worryingly incapable of demonstrating this very basic concept. The application of momentum to the two control volumes was poorly made – students seemed to be very confused about how to deal with the bleed momentum. Students knew how to relate total pressure change to mechanical work done, and most spotted how to derive the loss due to bleed – demonstrating a better understanding of energy than momentum.

6 a)



Equilibrium

$$p\,2\pi r\,dr - \left(p + \frac{dp}{dz}dz\right)2\pi r\,dr$$

$$+\left[\tau + \frac{\partial \tau}{\partial r}dr\right]2\pi\left[r + dr\right]$$

$$-\tau\,2\pi r\,dz + \rho g\,dr\,dz\,2\pi r = 0$$

$$\rightarrow \quad -\frac{dp}{dz}\left(2\pi r\,dr\,dz\right)$$

$$+\left[\cancel{\tau\,2\pi r} + \tau\,2\pi\,dr - \cancel{\tau\,2\pi r}\right]dz$$

$$+\left[\frac{\partial \tau}{\partial r}dr\,2\pi r + \cancel{\frac{\partial \tau}{\partial r}dr\,2\pi\,dr}\right]dz$$

$$+\rho g\,dr\,dz\,2\pi r = 0$$

$$-\frac{dp}{dz} + \frac{\tau}{r} + \frac{\partial \tau}{\partial r} + \rho g = 0$$

$$\frac{1}{r}\frac{\partial(\tau r)}{\partial r} = \frac{\partial(\tau)}{\partial r} + \frac{\tau}{r}$$

$$\rightarrow \quad -dp/dz + \frac{1}{r}\frac{\partial}{\partial r}(\tau r) + \rho g = 0$$

$$\frac{d}{dr}(\tau r) = r\left[\frac{dp}{dz} - \rho g\right]$$

[5]

~~3~~. b)

No streamline curvature, therefore
$p = p_{atm}$ at inlet and exit

and p is uniform across inlet + exit [2]

hence $\partial p / \partial r = 0$

c)

$$\tau_w \pi (2R) L + \rho g \pi R^2 L = 0$$

$$\rightarrow \quad \tau_w = -\frac{\rho g R}{2}$$

FROM INTEGRATION

$$R \tau_w = -\rho g R^2 / 2$$ [3]

d)

$$\frac{d(\tau r)}{dr} = r(-\rho g)$$

$$\tau r = -\frac{r^2}{2}(\rho g)$$

$$[\tau = \mu \partial u / \partial r]$$

$$\rightarrow \quad \frac{du}{dr} = \frac{-\rho g}{2\mu} r$$

$$u - u_0 = \frac{-\rho g r^2}{4\mu}$$

$$u = u_0 - \frac{\rho g}{4\mu} r^2$$

But we know $-u_0 = -\frac{\rho g R^2}{4\mu}$

$$\rightarrow u_0 = \rho g R^2 / 4\mu$$

$$\frac{u}{U_0} = 1 - \left(\frac{r}{R}\right)^2$$

$$\rightarrow u = U_0 \left[1 - (r/R)^2\right]$$

[5]

e)

$R = 1\,mm$

$\rho = 1000\,kg/m^3$

$\mu = 1.14 \times 10^{-3}$

$g = 9.81\,m/s^2$

$\pi R^2 U_b = \int u\, 2\pi r\, dr$

$= 2\pi U_0 \int_0^R \left[1 - (r/R)^2\right] r\, dr$

$\pi R^2 U_b = 2\pi U_0 \left[\frac{r^2}{2} - \frac{r^4}{4R^2}\right]_0^R = 2\pi U_0 \left[\frac{R^2}{4}\right]$

$U_b = U_0/2$

$$U_0 = \frac{\rho g R^2}{4\mu} = 2.15\,m/s$$

$$U_b = U_0/2 = 1.075\,m/s$$

$$\tau_w = -4.905\,Pa$$

$$\rightarrow \quad c_f = \frac{|\tau_w|}{\frac{1}{2}\rho V_b^2} = 0.0085$$

[4]

f) $\text{Force} = \rho g \pi R^2 L$ (weight of water) ④

$= 0.302\,N$ UPWARDS [2]

g) $Re_d = \dfrac{\rho U_b d}{\mu} = 1886$

→ Yes in laminar regime (< 2000)

Eddy viscosity μ_T approximates momentum
→ exchange of turbulence as an additional viscosity (added to the molecular μ)

The first part is the same

$$\frac{d(r\tau)}{dr} = -r\rho g$$

but we can no longer use $\tau = \mu \partial u / \partial r$

need to include μ_T

$$\rightarrow \quad \tau = (\mu + \mu_T)\frac{du}{dr}$$

This is an approximation → μ_T depends on flow conditions (not a fluid property). [4]

Q6 Viscous Flow – 256 attempts, mean 15.4/25, max. 25 and min. 1.

By far the most popular question, attempted by around 80 percent of the cohort. This was also the most challenging question and involved gravity-driven viscous flow in a pipe. A lot of students were not able to get the correct force balance for a fluid element in cylindrical polar coordinates – clearly more familiar with the cartesian form. Deriving the velocity profile was also challenging for many. This was made more difficult by the direction of the wall shear stress, which tripped up a few students. A good number of students understood the importance of the Reynolds' number and the expected range for laminar flow.