Version AJF/6

EGT1
ENGINEERING TRIPOS PART IB

Wednesday 7 June 2017 2 to 4

Paper 5

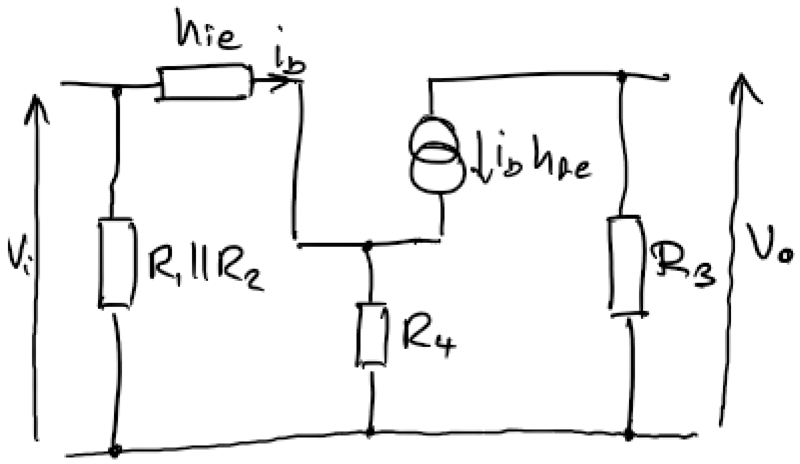
ELECTRICAL ENGINEERING - CRIB

SECTION A

1 (a)

(i) The potential divider formed by R_1 and R_2 sets the base voltage to a well-defined value [1 mark] if I_B is small compared with the current through R_1 and R_2 [1 mark]. If the voltage on the base with respect to earth, V_B, is significantly greater than the base-emitter voltage, V_{BE}, then the voltage across R_4 is also fixed[1 mark]. There will therefore be a stable emitter and collector current, resulting in stable operation of the amplifier circuit [1 mark]. [4]

(ii)



[4]

(iii) By considering the current through R_3,

$$V_o = -i_b h_{fe} R_3$$

Also, performing a mesh current analysis around the loop on the input side of the circuit,

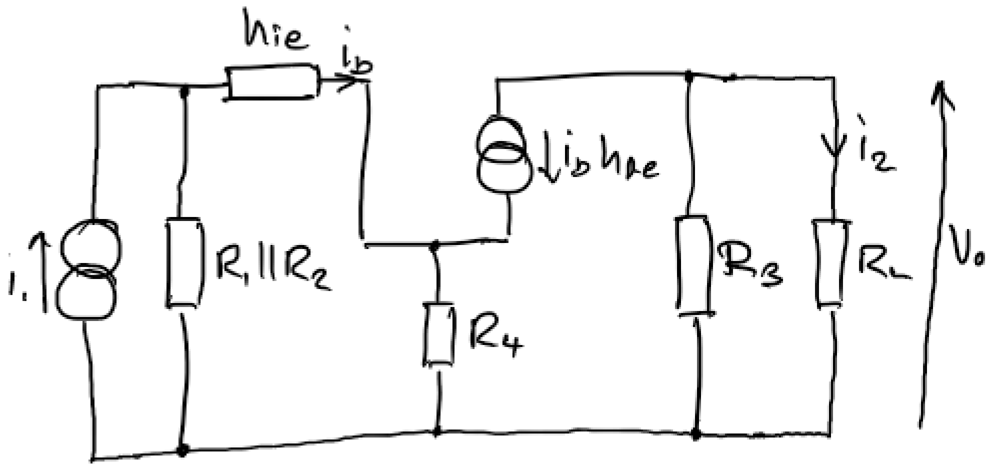
$$V_i = i_b h_{ie} + (1 + h_{fe}) i_b R_4$$

Dividing these two gives the gain

$$\frac{V_o}{V_i} = \frac{-h_{fe} R_3}{h_{ie} + (1 + h_{fe}) R_4}$$ [6]

[Examiner's Note: This was a standard circuit that the students have seen before, and although they were good at drawing the small-signal equivalent circuit, perhaps the familiarity led to some lack of attention in the analysis which resulted in many candidates dropping straightforward marks. Few candidates gave a complete explanation of the circuit construction.]

(b) (i) The new small-signal circuit including the current source and load resistance is



We can immediately analyse the output side of the circuit as the current $i_b h_{fe}$ passes through the parallel combination R_{3L},

$$Vo = -i_b h_{fe} R_{3L}$$

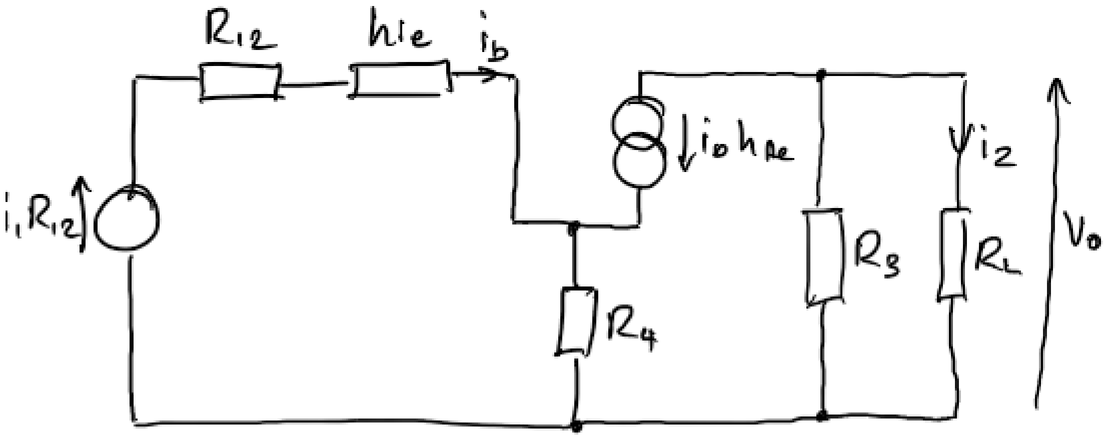
However, we also know that i_2 passes through R_L only, giving

$$Vo = i_2 R_L$$

So equating these two gives

$$i_2 = \frac{-i_b h_{fe} R_{3L}}{R_L}$$

We can convert the input side of the circuit to a Thevenin equivalent



Therefore, a mesh current analysis around the input loop gives

$$i_1 R_{12} = i_b \left(R_{12} + h_{ie}\right) + \left(1 + h_{fe}\right) i_b R_4$$

Therefore, dividing the two expressions for the currents gives

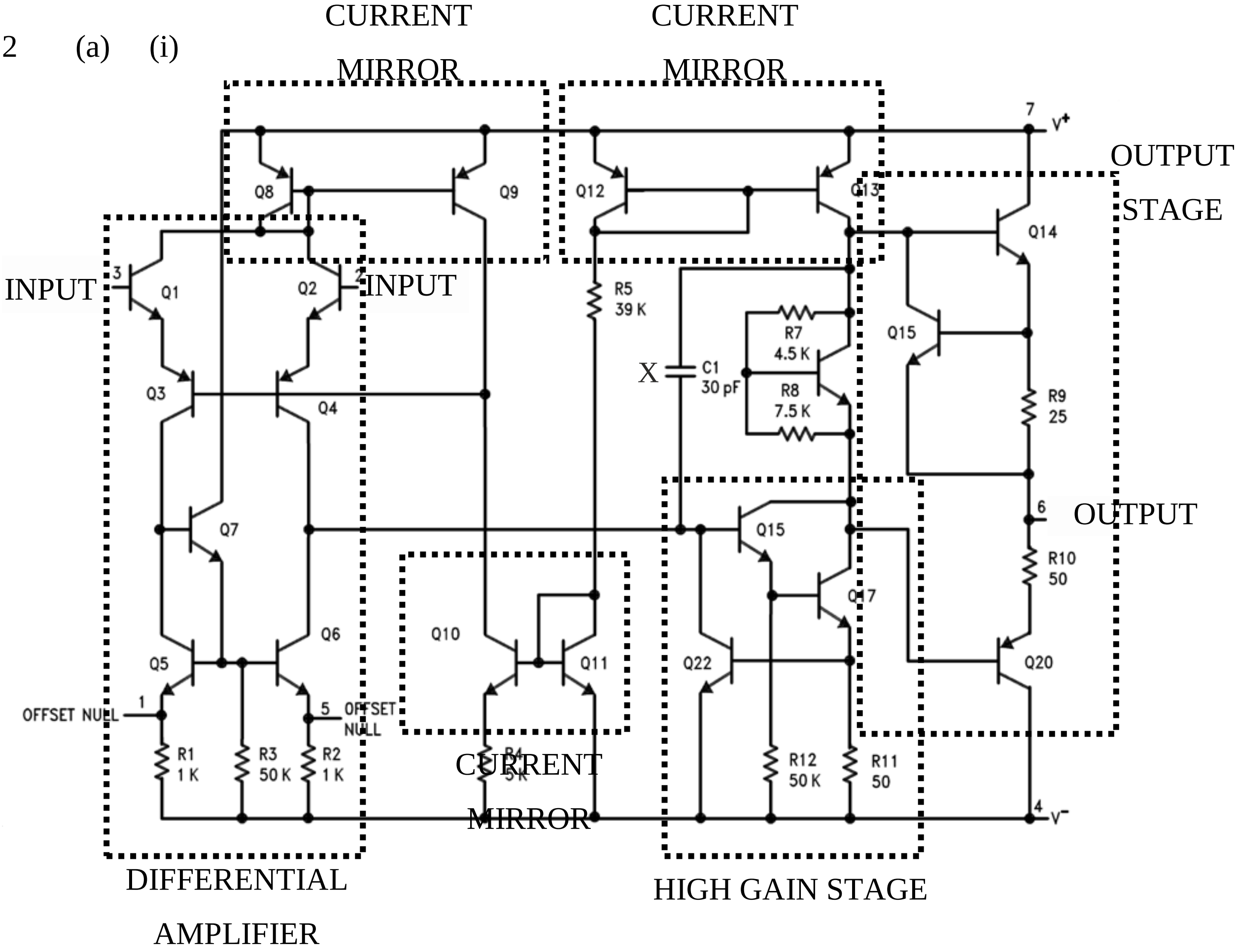
$$\frac{i_2}{i_1} = \frac{-h_{fe} R_{12} R_{3L}}{R_L \left[R_{12} + h_{ie} + R_4 \left(1 + h_{fe}\right)\right]}$$ [7]

(ii) To improve the gain of the circuit, we have to look for terms that only appear in either the numerator of denominator of the current gain expression *only*. We could try finding a different transistor with a lower h_{ie}, but this is not going to give a significant improvement. R_4 is an obvious target. Some students suggested removing it completely, but this would change the operating point of the circuit. A better approach is to put a capacitor in parallel with R_4 which will appear as a short circuit at the desired operating frequencies. Therefore, the small-signalk analysis for the current gain will now give

$$\frac{i_2}{i_1} = \frac{-h_{fe} R_{12} R_{3L}}{R_L \left[R_{12} + h_{ie}\right]}$$

which would be a significant improvement, but without the d.c. operating point being affected. [4]

2 (a) (i)



[6]

(ii) The differential amplifier gives a high common mode rejection ratio, so that common mode signals, such as noise, are suppressed relative to the desired differential mode signal. The differential amplifier also gives a high input impedance [2 marks]. The high gain stage applies a high gain amplification to the output of the differential amplifier [2 marks]. The output stage is a buffer which has as low output impedance [2 marks]. [6]

(iii) The 30 pF capacitor marked at position X in Fig. 3 reduces the gain at high frequencies by effectively short circuiting the collector and base of the Darlington Pair which makes up the high gain stage [1 mark]. This ensures stable operation of the operational amplifier circuit [2 marks]. [3]

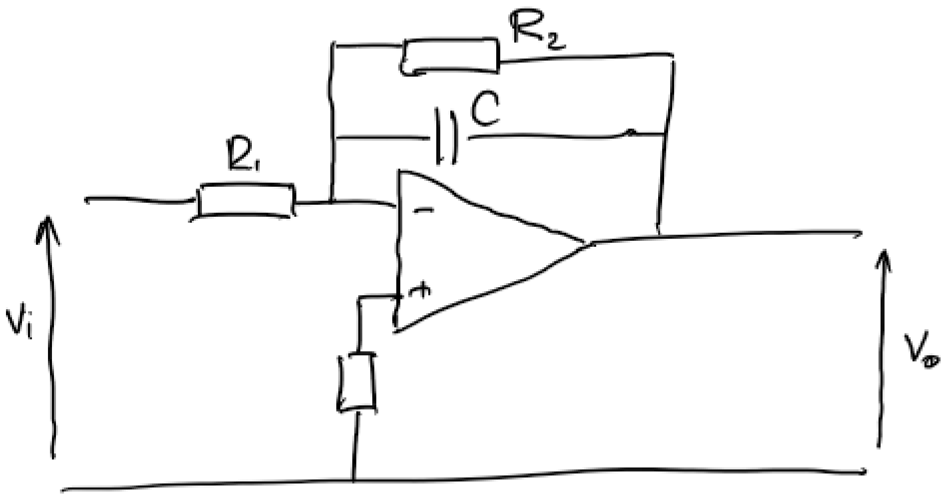
(b)

(i) The operational amplifier is ideal, and so, as the non-inverting input is connected to earth, the inverting input behaves like a virtual earth. Also, the unput impedance is infinite so no current flows into the inverting input. This allows us to simply sum the other currents at the inverting input node

$$\sum I = 0 = C\frac{d}{dt}\left(-v_i\right) - \frac{v_o}{R}$$

$$v_o = -RC\frac{dv_i}{dt}$$ [6]

(ii) The circuit diagram is



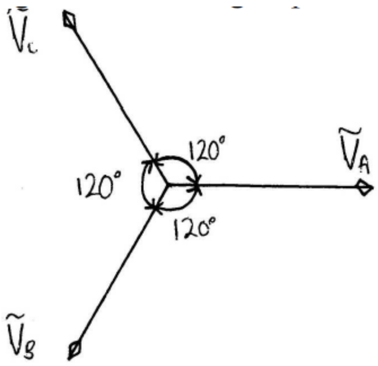
It is the resistor R_2 which gives the well-defined d.c. voltage gain. Many students left this off for which they were penalised 2 marks. [4]

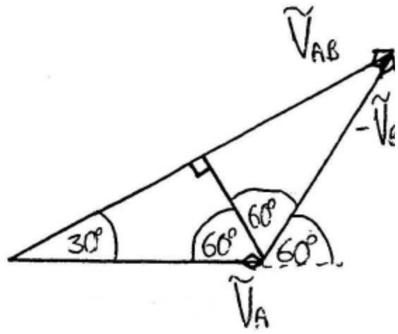
[Examiner's Note: The analysis was generally very good, but many candidates did not realise that a feedback resistor was required in parallel with the feedback capacitor to fix the d.c. gain as required.]

SECTION B

3 (a) Phase Voltage: The voltage of a line with respect to ground.
Line Voltage: The voltage difference between a pair of lines.

(b) The supply is balanced. So the phase difference between the phase voltage at two lines is $120°$.





From the cosine rule

$$|\tilde{V}_{AB}|^2 = |\tilde{V}_A|^2 + |\tilde{V}_B|^2 - 2|\tilde{V}_A||\tilde{V}_B|\cos(120°) = |\tilde{V}_A|^2(2 - 2\cos(120°)) = 3|\tilde{V}_A|^2$$

Also
$\angle\tilde{V}_{AB} = \angle\tilde{V}_A + 30°$

(c) $P = 3PV_l I_{ph}\cos(\phi)$

$I_{ph} = I_l/\sqrt{3}$, which follows from the fact that the phase currents form a balanced three- phase set, and hence a relation analogous to that in (b) holds. Therefore

$$P = 3V_l I_l/\sqrt{3}\cos(\phi) = \sqrt{3}V_l I_l\cos(\phi)$$

(d)

(i) We consider a single phase circuit where the phase voltage $V_{ph} = \frac{11}{\sqrt{3}}\ kV$ is applied to the parallel combination of impedances Z_1, Z_2 where
$Z_1 = 400 + 2\pi 50 \times 2j$ and $Z_2 = 1500/3/2 = 250\ \Omega$.

Hence

$$Z_{tot} = \frac{Z_1 Z_2}{Z_1 + Z_2} = 200 + 480j$$

and for the supply we have

$$I_{ph} = \frac{V_{ph}}{Z_{tot}} = 30 - 7.2j\ kA$$

We then have

$$P = 3Re[\tilde{V}_{ph}\tilde{I}^*_{ph}] = 3\times\frac{11}{\sqrt{3}}\times 30 = 0.571\ MW$$

$$Q = 3Im[\tilde{V}_{ph}\tilde{I}^*_{ph}] = 3\times\frac{11}{\sqrt{3}}\times 7.2 = 0.137\ MVA$$

[Examiner's Note: The most common error was simply adding two currents where one is complex.]

(ii)

$$\cos(\phi) = 0.99 \Rightarrow \tan(\phi) = 0.1425$$

$$Q_{new} = P\tan(\phi) = 0.0814$$

Let X_C be the reactance of a capacitor C to be added per phase and Q_{X_C} the magnitude of the aggregate reactive power from the capacitors.

$$Q_{X_C} = Q - Q_{new} = 0.0556$$

$$Q_{X_C} = 3V_{ph}^2/X_C$$

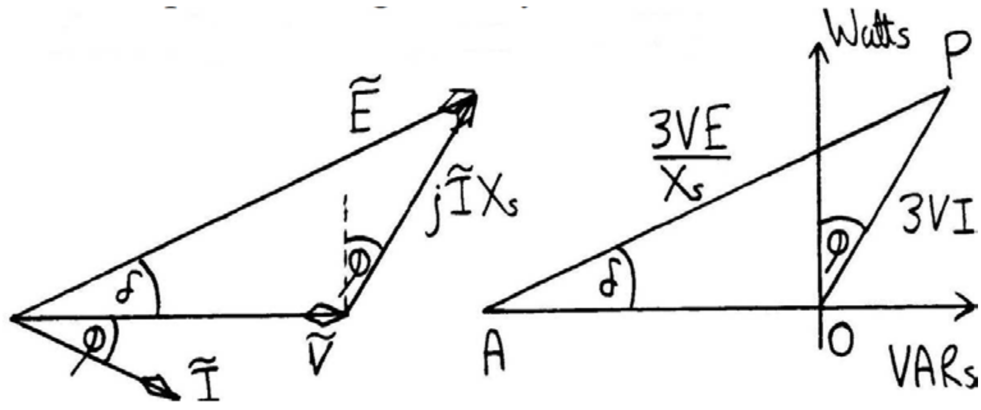
So $X_C = 2175$. Noting that

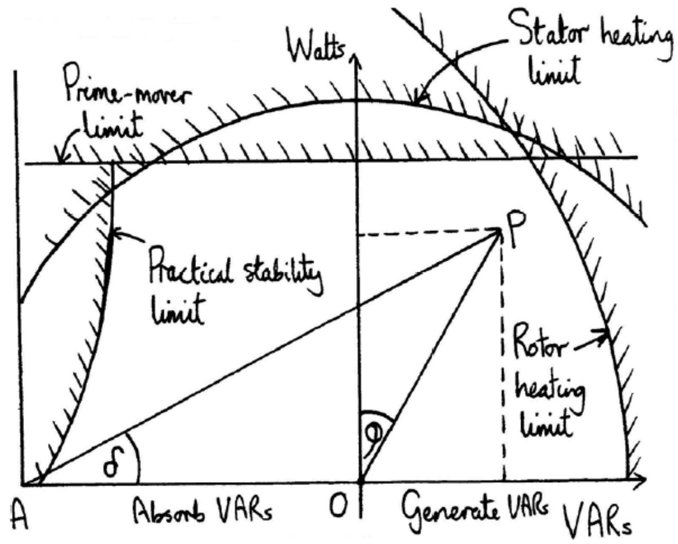
$$X_C = \frac{1}{2\pi f C}$$

we get

$C = \frac{1}{2\pi f X_C} = 1.46\ \mu F$ (per phase)

4 (a) The operating chart of a generator is a diagram that shows the generator operating limitations. This is obtained by scaling the generator phasor diagram by $3V/X_s$





Real power limited by prime mover (horizontal line at P_{max}).

$S = 3VI$ (length OP) limited by the rated VA of the generator – stator heating limit.

Length AP limited by maximum excitation E_{max} – rotor heating limit.

(b) (i) $P = 3V_{ph}I_{ph}\cos(\phi)$, so

$$I_{ph} = \frac{P}{3V_{ph}\cos(\phi)} = \frac{200\ M}{3\times\frac{11\ k}{\sqrt{3}}\times 0.75} = 14\ kA$$

$$E = \sqrt{\left(I_{ph}X_s cos\phi\right)^2 + \left(V_{ph} + I_{ph}X_s sin\phi\right)^2} = 9.656\ kV$$

$$\sin(\delta) = \frac{I_{ph}X_s cos\phi}{E} = 0.3261$$

So $\delta = 19.0°$.

(ii)

$$I_{ph} = \frac{P}{3V_{ph}} = 10.5\ kA$$

$$E = \sqrt{V_{ph}^2 + \left(I_{ph}X_s\right)^2} = 7.09\ kV$$

(iii) E_{max} – rotor heating limit

$S = 3VI$ – stator heating limit

5 (a) The per-unit system is a representation where physical quantities are expressed in dimensionless form (p.u.) by normalizing them with corresponding base values. Its significance is that it simplifies the analysis and avoids changes in voltages due to transformers.

(b) Faults modelled as a connection of all phases to ground at the point where the fault occurs. Properties required from protection system:
- Discrimination: Only isolates the faulty part of the system.
- Stability: Breaker stays closed when the fault is outside the protected zone.
- Sensitivity: Ability to set the current level at which the beaker opens.

(c) (i) Choose base values for VA and voltages

$VA_b = 200\ MVA$

$V_b = 11\ kV, 132\ kV, 33\ kV$

Perform changes of base for the reactances of the transformers:

11/132 *kV* transformer $X_{pu(200)} = X_{pu(100)} \times \frac{200}{100} = 0.2\ pu$

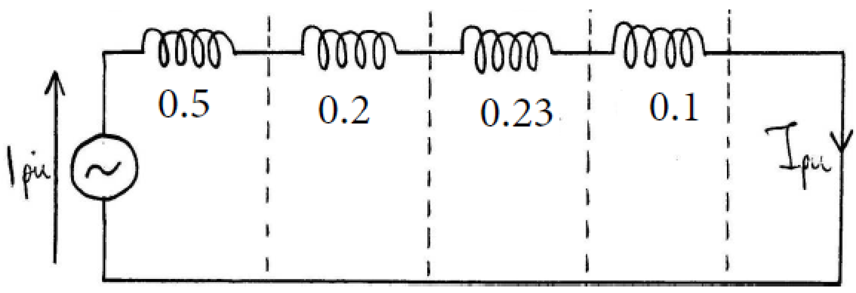
132/33 *kV* transformer $X_{pu(200)} = X_{pu(100)} \times \frac{200}{100} = 0.1\ pu$

Find the feeder impedance in pu

$$Z_b = \frac{V_b^2}{VA_b} = \frac{(132\ k)^2}{200\ M} = 87.12\ \Omega$$

$$Z_{pu} = \frac{Z}{Z_b} = \frac{20}{87.12} = 0.2296$$

The single phase line diagram in pu is therefore



Total reactance $X_{F(pu)} = 0.5 + 0.2 + 0.23 + 0.1 = 1.03\ pu$

$$I_{pu} = \frac{V_{pu}}{X_{F(pu)}} = \frac{1}{1.03} = 0.9709\ pu$$

At 11 kV bus

$$I_b = \frac{VA_b}{\sqrt{3}V_b} = \frac{200M}{\sqrt{3}\times 11k} = 10.5\ kA$$

$$I_F = I_{pu}\times I_b = 0.9709\times 10.5 = 10.2\ kA$$

(ii) Add reactance $X_{pu} = X_F = 1.03\ pu$

$$Z_b = \frac{V_b^2}{VA_b} = \frac{(33\ k)^2}{200\ M} = 5.445\ \Omega$$

$$X = X_{pu}\times Z_b = 1.03\times 5.445 = 5.608\ \Omega\ (per\ phase)$$

SECTION C

6 (a) The Poynting Vector is defined as $\mathbf{N} = \mathbf{E} \times \mathbf{H}$. Its direction is the direction of propagation of an electromagnetic wave, and its magnitude is the instantaneous power per unit area in the electromagnetic wave. [4]

(b) Figure 5 shows a waveguide.

(i) Ampère's Circuital Law is

$$\oint_L \mathbf{H.dl} = I_T$$

where I_T is the total current enclosed by the loop. Hence

$$\underline{I = H_y w}$$

Also from

$$V = -\int E dx$$

we have that

$$\underline{V = -E_x d}$$ [4]

(ii) The average power per unit area in an electromagnetic wave is

$$\left|\overline{\mathbf{N}}\right| = \frac{\left|\mathbf{E} \times \mathbf{H}\right|}{2}$$

As **E** and **H** are perpendicular, this simplifies using the answers to part (i) to

$$\left|\overline{\mathbf{N}}\right| = \frac{\left|E_x\right|\left|H_y\right|}{2} = \frac{VI}{2wd}$$

Therefore, the power being transmitted along the line is

$$\text{Power} = \left|\overline{N}\right| \times \text{Area} = \frac{VI}{2}$$

As V and I are related to their rms values by $V_{rms} = V/\sqrt{2}$ and $I_{rms} = I/\sqrt{2}$, then substitution into the above shows that the power is just $V_{rms}I_{rms}$. [4]

(c) (i) We can start by using the Ampère Law to calculate the magnetic field

$$\oint_L \mathbf{H.dl} = I_T$$

$$H.2\pi R = NKt$$

$$H = \frac{NKt}{2\pi R}$$

We can then calculate the energy stored to be

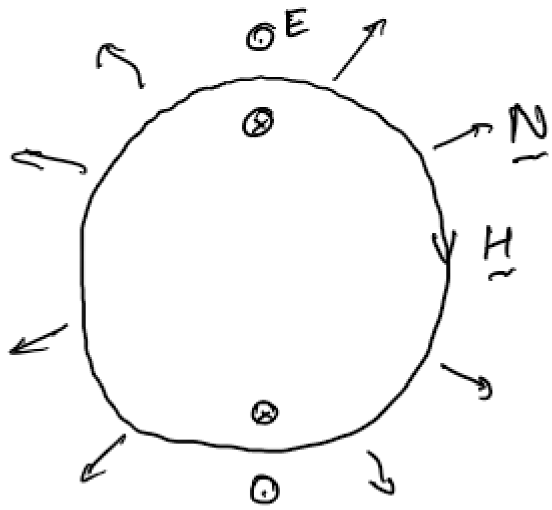
$$\text{Energy} = \frac{1}{2}\int \mathbf{B.H}dv$$

where the integral is over the volume of the toroid. The toroid if filled with air, so $\mathbf{B}=\mu_0\mathbf{H}$, and hence

$$\text{Energy} = \frac{1}{2}\mu_0 H^2 \times \text{volume} = \frac{\mu_0 N^2 K^2 r^2 t^2}{8\pi^2 R^2} \times \pi r^2 \times 2\pi R$$

$$\text{Energy} = \frac{\mu_0 N^2 K^2 r^2 t^2}{4R}$$ [6]

(ii) From the Maxwell equations, $\nabla \times \mathbf{H} = \mathbf{J} + \dot{\mathbf{D}}$,but inside the toroid $\mathbf{J}$ = 0, so $\nabla \times \mathbf{H} = \dot{\mathbf{D}}$. The electric field will point locally perpendicular to the plane of the toroid. Also, $|\mathbf{E}| = \eta|\mathbf{H}|$. Hence, the Poynting vector will point radially outwards, as shown in the following sketch looking from above the toroid



The magnitude of the Poynting vector will then be approximately

$$|N| = \eta |H|^2 = \frac{\eta N^2 K^2 r^2 t^2}{4\pi^2 R^2} \qquad [7]$$

7 (a) We are given that the expression for the radial electric field **E** in the coaxial cable is

$$\mathbf{E} = \frac{E_{0\rho}}{\rho}\mathbf{e}_{\rho}\exp(j[\omega t - \beta z])\exp(-\alpha z)$$

Using the Maxwell equations and the vector calculus expression of curl in cylindrical polar coordinates on page 14 of the Mathematics Data Book (2008 edition), we have that

$$-\dot{\mathbf{B}} = \nabla \times \mathbf{E} = \frac{1}{\rho}\begin{vmatrix} \mathbf{e}_{\rho} & \rho\mathbf{e}_{\theta} & \mathbf{e}_{z} \\ \partial/\partial\rho & \partial/\partial\theta & \partial/\partial z \\ \frac{E_{0\rho}}{\rho}\exp(j[\omega t - \beta z])\exp(-\alpha z) & 0 & 0 \end{vmatrix}$$

$$-\dot{\mathbf{B}} = \frac{1}{\rho}\frac{\partial}{\partial z}\left\{\frac{E_{0\rho}}{\rho}\exp(j[\omega t - \beta z])\exp(-\alpha z)\right\}\rho\mathbf{e}_{\theta}$$

$$\dot{\mathbf{B}} = \frac{\mathbf{e}_{\theta}E_{0\rho}}{\rho}[(\alpha + j\beta)\exp(j[\omega t - \beta z])\exp(-\alpha z)]$$

Therefore, integrating with respect to time gives

$$\mathbf{B} = \frac{\mathbf{e}_{\theta}E_{0\rho}}{j\omega\rho}[(\alpha + j\beta)\exp(j[\omega t - \beta z])\exp(-\alpha z)]$$

Therefore, as $\mathbf{B} = \mu_0\mathbf{H}$, we have

$$\mathbf{H} = \frac{\mathbf{e}_{\theta}E_{0\rho}}{j\mu_0\omega\rho}[(\alpha + j\beta)\exp(j[\omega t - \beta z])\exp(-\alpha z)]$$

This has the form

$$\mathbf{H} = \frac{H_{0\theta}}{\rho}\mathbf{e}_{\theta}\exp(j[\omega t - \beta z])\exp(-\alpha z)$$

where

$$H_{0\theta} = \frac{(\alpha + j\beta)E_{0\rho}}{j\mu_0\omega}$$ [8]

(b) To determine an expression for $E_{0\rho}$, we first need the exelctric field as a function of radius inside the coaxial cable, for which we use the Gauss Law of Electrostatics,

$$\oint_S \mathbf{E.dS} = \frac{q}{\varepsilon_0 \varepsilon_r}$$

and evaluate over a short length of the cable *dz* to give

$$E.2\pi\rho dz = \frac{\sigma dz}{\varepsilon_0 \varepsilon_r}$$

$$E = \frac{\sigma}{2\pi\varepsilon_0 \varepsilon_r \rho}$$

We can then use this to find the total potential

$$V_0 = -\int E dx = -\int_b^a \frac{\sigma}{2\pi\varepsilon_0 \varepsilon_r \rho} d\rho = \frac{\sigma}{2\pi\varepsilon_0 \varepsilon_r} \ln\left(\frac{b}{a}\right)$$

Therefore

$$E = \frac{V_0}{\rho \ln(b/a)}$$

and hence

$$\underline{E_{0\rho} = \frac{V_0}{\ln(b/a)}}$$ [6]

(c) (i) The maximum electric field is at $\rho = a$, so

$$V_{0\max} = E_{\max} a \ln(b/a)$$

Therefore

$$P_{t\max} = \pi \sqrt{\frac{\varepsilon_0 \varepsilon_r}{\mu_0}} E_{\max}^2 a^2 \ln\left(\frac{b}{a}\right)$$

For a fixed *b*, we need to calculate the optimum *a*, so

$$\frac{\partial P}{\partial a} = \frac{a}{b} \times -b + 2a \ln\left(\frac{b}{a}\right) = 0$$

$$\ln\left(\frac{b}{a}\right) = 0.5$$

$$x = \frac{b}{a} = 1.65$$ [8]

(ii) $$Z = \frac{0.5}{2\pi}\sqrt{\frac{\mu_0}{\varepsilon_0 \times 2.25}} = 20\ \Omega$$ [3]

[Examiner's Note: The majority of the students failed to set up the equation for the power correctly. Common mistakes were not spotting that Z varied with the value of x or assuming that maximum power occurred at maximum E. A surprisingly large number of students having failed to get the correct value of x then, needing a value for part (ii), chose to put it equal to 1 which would mean that the inner and outer conductors had the same radii!]

EGT1
ENGINEERING TRIPOS PART IB

Wednesday 7 June 2017 2 to 4

Paper 5

ELECTRICAL ENGINEERING – NUMERICAL ANSWERS

3	(d)	(i)	$30 - 7.2j$ kA; 0.571 MW; 0.137 MVA
		(ii)	1.46 µF (per phase)
4	(b)	(i)	19.0°; 9.656 kV
		(ii)	7.09 kV
5	(c)	(i)	10.2 kA
		(ii)	5.608 Ω (per phase)
7	(c)	(ii)	20 Ω