Q_1 a) i) $V_B = 1.7V$ $V_E = 1V$ $I_C = 2mA$ P5 2001

h_{FE} large ⇒ neglect I_B as it is ≪ I_C (or can calculate it if need be) $I_B = I_C / h_{FE}$ $= 5\mu A$

$V_E = R_4 I_C = 1V$ ⇒ $R_4 = 500\Omega$

$V_C = 1+4 = 5V$ ($V_{CC}/2$ V) $= 10 - I R_3$ ⇒ $R_3 = 2.5k\Omega$

[3]

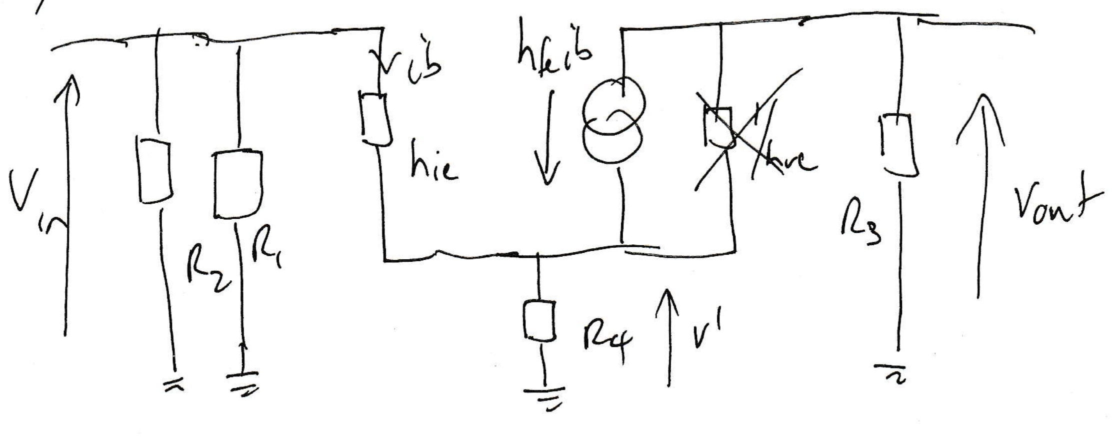
ii) $V_B = 1.7V$ neglecting I_B ⇒ $1.7V = \frac{R_2}{R_2+R_1} \times 10$

gives ratio of R_2/R_1

need R_{in} to find R_1 & R_2 (input Resistance)

[2]

b) JSM



neglect C_1 & C_2

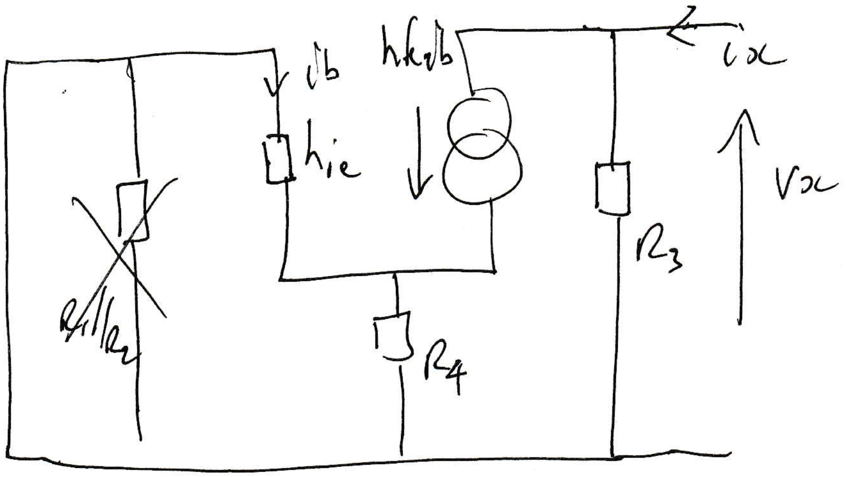
at output $0 - V_{out} = h_{fe} i_b R_3$ $V_{out} = -h_{fe} i_b R_3$

at emitter $V' = (1+h_{fe}) i_b R_4$ ⇒ $V_{in} = h_{ie} i_b + (1+h_{fe}) i_b R_4$

$$\Rightarrow \frac{V_{out}}{V_{in}} = \frac{-h_{fe} R_3}{h_{ie} + (1+h_{fe}) R_4}$$

[4]

c) to measure R_o, short input & apply v_x, i_x at o/p

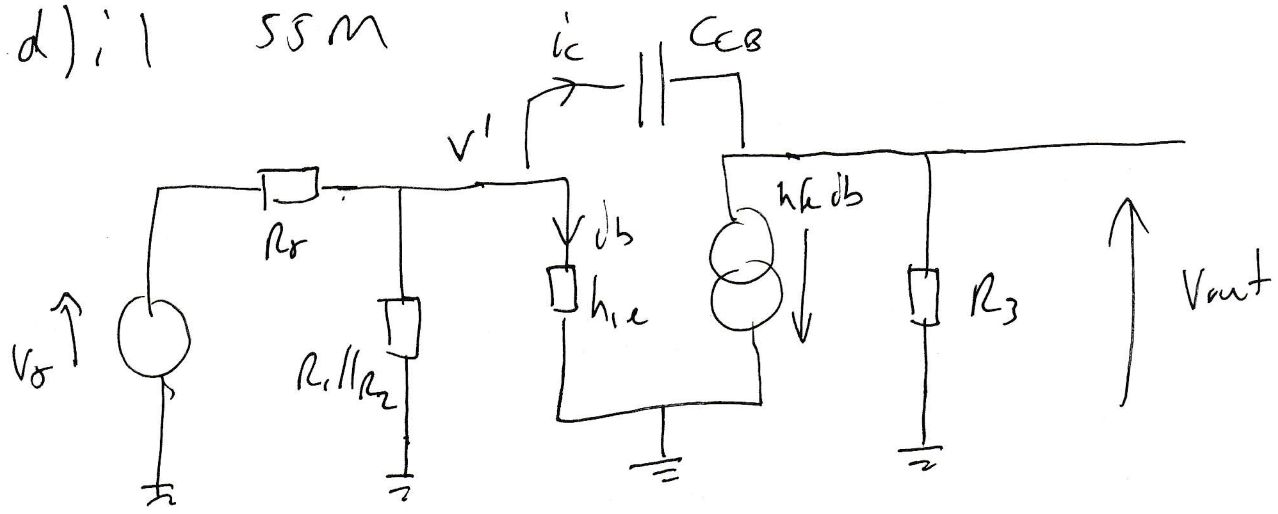


Cannot assume $i_b = 0$

$$R_o = \frac{v_x}{i_x}$$
$(\approx R_3)$

Cannot solve this circuit as the current source has no parallel impedance. Have to include $1/h_{oe}$ in order to solve for R_o. [4]

d) i) SSM



[3]

ii) assume $i_c \approx i_b \Rightarrow$ neglect i_c at output node.

$$\Rightarrow V_{out} = -h_{fe} i_b R_3$$

assume R_{in} large $\Rightarrow R_1 // R_2 \gg h_{ie}$ so can neglect current through R_1 & R_2.

$v' = h_{ie} i_b \qquad i_b = -v_{out}/h_{fe} R_3$

at v' node

$$\frac{V_\sigma - h_{ie} i_b}{R_\sigma} = i_b + \frac{(h_{ie} i_b - V_{out})}{1/j\omega C_{CB}}$$

$$\Rightarrow V_\sigma = i_b\left[h_{ie} + R_\sigma + j\omega C_{CB} h_{ie} R_\sigma\right] - j\omega C_{CB} R_\sigma V_{out}$$

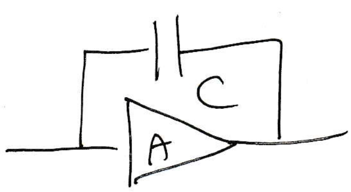
$$= -V_{out}\left[\frac{1}{h_{fe} R_3}\left(h_{ie} + R_\sigma + j\omega C_{CB} h_{ie} R_\sigma\right) + j\omega C_{CB} R_\sigma\right]$$

$$\frac{V_{out}}{V_\sigma} = \frac{-h_{fe} R_3}{h_{ie} + R_\sigma + j\omega C_{CB} h_{ie} R_\sigma + j\omega C R_\sigma h_{fe} R_3}$$

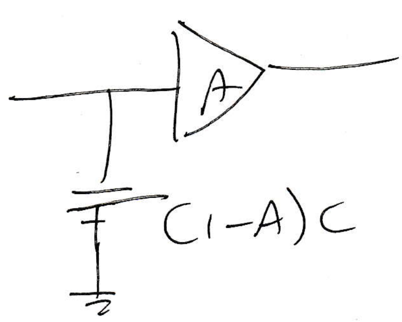
$$= \frac{-h_{fe} R_3}{h_{ie} + R_\sigma + j\omega C_{CB} R_\sigma (h_{ie} + h_{fe} R_3)} \qquad [7]$$

note if not neglecting current through R_1 & R_2 denominator will include the extra term $h_{ie} R_\sigma / R_1 \| R_2$

iii) Use the miller effect

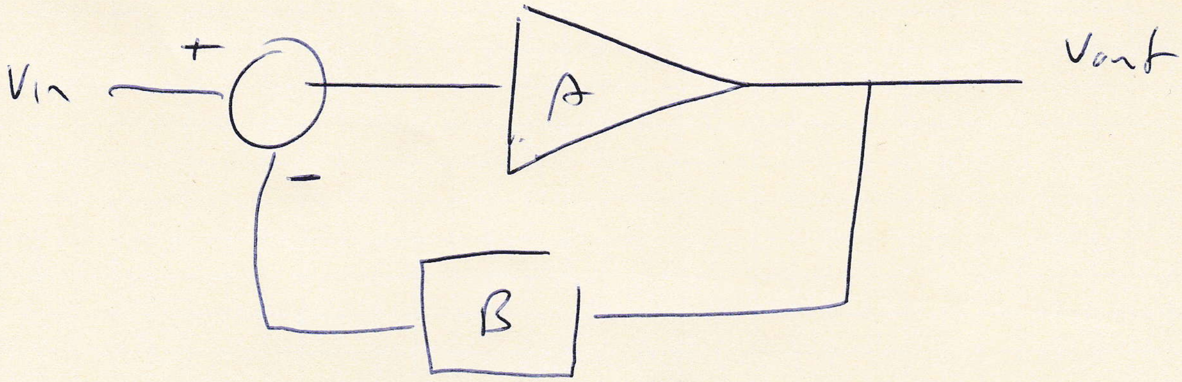


$\equiv$



[2]

2 a) Negative feedback



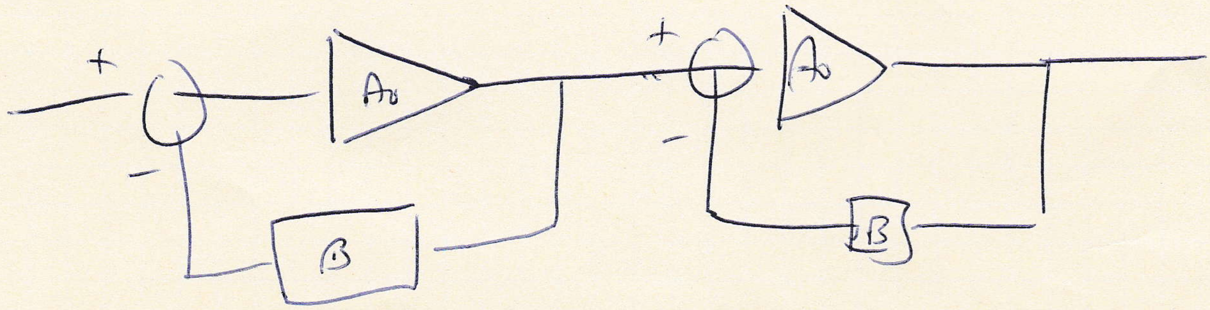
$$V_{out} = A V_{in} - AB V_{out}$$

$$V_{out}(1 + AB) = A V_{in}$$

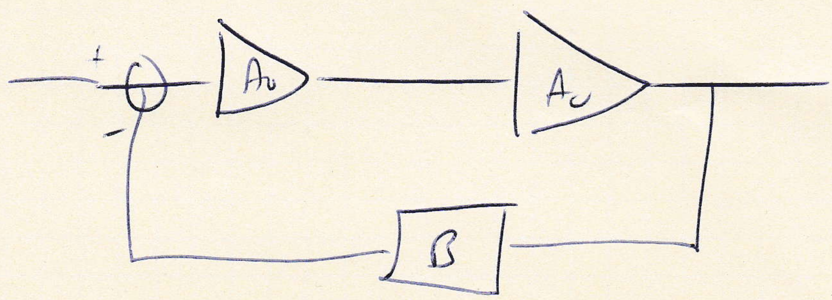
$$\frac{V_{out}}{V_{in}} = \frac{A}{1 + AB}$$ [4]

b) 2 amplifiers ~~ta~~ can be either

(1)



(2) or



scenario (1) Total gain is $\left(\frac{A_0}{1+A_0B}\right)^2$

variation for stage 1 = $\frac{\Delta A}{1+A_0B}$

but this is multiplied by stage 2 $\frac{\Delta A}{1+A_0B} \times \frac{A_0}{1+A_0B}$

for scenario (2) total gain is

$$G = \frac{A_0^2}{1+BA_0^2}$$

variation of (2) is now reduced by a factor of $(1+A_0^2B)$ which is better than scenario (1)

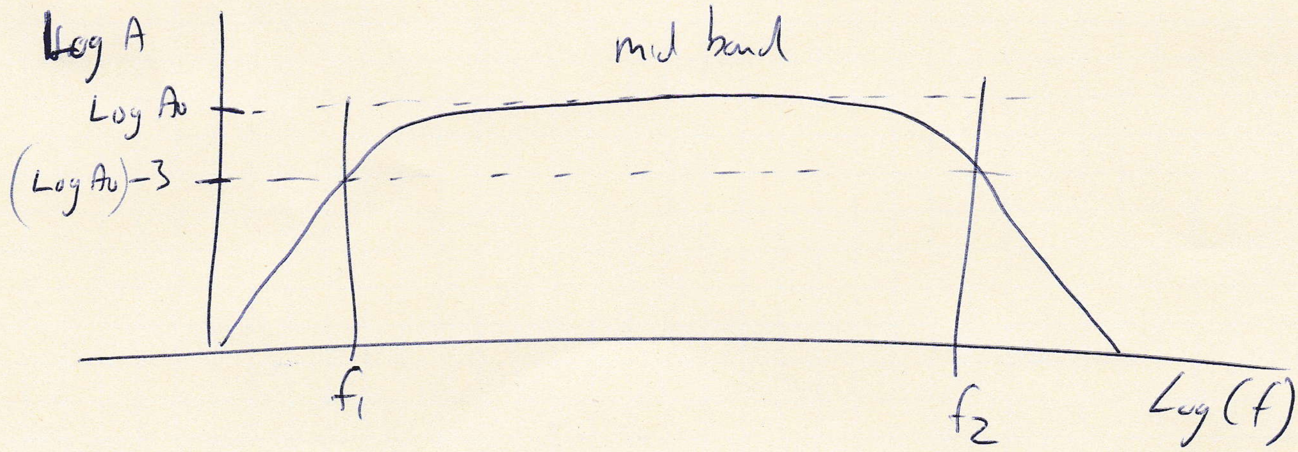
Gain Variation $\delta G/G$

$$\frac{dG}{dA} = \frac{2A(1+A^2B) - 2AB(A^2)}{(1+A^2B)^2} = \frac{2A}{(1+A^2B)^2}$$

$$\frac{\delta G}{G} = \frac{\delta A \, 2\cancel{A}}{(1+A^2B)^{\cancel{2}}} \times \frac{\cancel{(1+A^2B)}}{A^{\cancel{2}}} = \frac{2\,\delta A}{A(1+A^2B)}$$

[6]

c) Bode plot



Assumption is that $f_2 \gg f_1$ so that there is no overlap of the two roll-offs.

Low pass cutoff freq f_2

$$\Rightarrow \quad A(f) = \frac{1}{1 + jf/f_2}$$

High pass $\quad A(f) = \dfrac{1}{1 + \dfrac{f_1}{jf}}$

Total gain expression will be

$$A(f) = \frac{A_0}{\left(1 + \frac{jf}{f_2}\right)\left(1 + \frac{f_1}{jf}\right)} \qquad [5]$$

d) i) $f_1 = 0$ Hz

$$A(f) = \frac{A_0}{1 + jf/f_2}$$

$$B(f) = R /\!/ C = \frac{R \cdot \frac{1}{j\omega C}}{R + 1/j\omega C} = \frac{R}{1 + j\omega CR}$$

negative feed back $\Rightarrow$ Gain $= \dfrac{A(f)}{1 + B(f)A(f)}$

$$= \frac{A_0 / 1 + jf/f_2}{1 + B(f) \, A_0/(1 + jf/f_2)}$$

$$= \frac{A_0}{(1 + jf/f_0) + A_0 B(f)}$$

$$= \frac{A_0}{(1 + jf/f_0) + \frac{A_0 R}{1 + j\omega CR}}$$

$$= \frac{A_0(1 + j\omega CR)}{A_0 R + (1 + jf/f_2)(1 + 2\pi f CR)}$$

[6]

ii) if feedback is not dominated by the network B(f), then 3dB point will be set by f_2

ie when $2\pi f RC$ ~~...~~ $<< 1$

then $B(f) = R$

$$\Rightarrow \text{gain} = \frac{A_0}{1 + j\,f/f_2 + A_0 R}$$

which has a 3dB point ~~at~~ when

$$\frac{f}{f_2} = 1 + A_0 R \qquad [2]$$

$$f = f_2 (1 + A_0 R) \qquad (\text{ie } 1 + AB)$$

hence the frequency set by RC must be higher than $f_2(1 + A_0 B)$

iii) The feedback network must not have ~~a~~ phase of 180° when combined with the phase characteristic of the amplifier other wise this will lead to positive feedback & unstable oscillations. [2]

Q3 Consider a balanced star-connected three phase voltage supply with voltage V_{phase} at each phase and a balanced star-connected load with impedance $\bar{Z}_{phase}$ at each phase. The star point of the voltage supply and the star point of the load are connected with a conductor with impedance $\bar{Z}_c$. The power supply operates at 50 Hz with $V_{phase} = 240\ V$ and the load impedance is $\bar{Z}_{phase} = 100 + j20\Omega$.

(a) Derive the current flow through the conductor. [3]

Solution:

The balanced star connection currents form a balanced three phase set is $I_A + I_B + I_C = 0$. From Kirchhoff's laws we have $I_A + I_B + I_C + I_N = 0$. Thus, $I_N = 0$ is the current flow through the conductor.

(b) Calculate the power factor of the system and the line current. [5]

Solution:

The power factor of the system is

$\cos\phi = \cos\left(\tan^{-1}\left(\frac{20}{100}\right)\right) = 0.98$.

The phase and line current of this circuit are equal and can be calculated as follows.

$\bar{I}_{phase} = \frac{\bar{V}_{phase}}{\bar{Z}_{phase}} = \frac{240}{100+j20} = 2.31 - j0.46\ A$.

(c) Calculate the input power and VARs. [5]

Solution:

$P = 3V_{phase}I_{phase}\cos\phi = 3 \times 240 \times 2.31 = 1663.2\ W$

$Q = 3I^2_{phase}X = 3 \times 2.36^2 \times 20 = 334.17\ VAR$

(d) Derive the required delta-connected inductors/capacitors to alter the power factor to 0.99 lagging. [8]

Solution:

As the power factor will be improved from 0.98 to 0.99 lagging, capacitors are required.

Note that capacitors do not affect P. Thus, the required Q to reach power factor (PF) equal to 0.98 lagging can be calculated as follows.

$Q_{new} = \frac{P}{PF}\sin(\cos^{-1}(PF)) = \frac{1663.2}{0.99}\sin(\cos^{-1}(0.99)) = 236.99\ VAR$

For star-connected capacitors we have $X_s = \frac{V_{phase}^2}{Q_s} = \frac{240^2}{236.99-334.17} = -592.71\Omega$.

For delta-connected capacitors we need $X_d = 3X_s = -1778\Omega = \frac{1}{2\pi 50 C_d}$. Thus, the delta-connected capacitors must be $C_d = 1.79\ \mu F$.

(e) Consider that we connect the three phase outputs of the voltage supply in series and achieve the single phase voltage $V_1 - V_2 + V_3$. Calculate the amplitude of the single phase voltage. [4]

Solution:

As we have a balanced power supply, $|V_1| = |V_2| = |V_3| = V_{phase} = 240\ V$. Moreover, V_2 and V_3 can be written as $V_2 = V_1\angle 120°$ and $V_3 = V_1\angle 240°$. Thus, the components of V_2 and V_3 in the direction of V_1 can be written as

V_2 component = $V_{phase}\cos(120) = -0.5V_{phase}$

V_3 component = $V_{phase}\cos(240) = -0.5V_{phase}$

Thus, the component of $V\_1 - V\_2 + V\_3$ in the direction of V_1 is

$V_1 - V_2 + V_3$ component = $V_{phase} + 0.5V_{phase} - 0.5V_{phase} = V_{phase}$.

The components of V_2 and V_3 perpendicular to V_1 can be written as

V_2 component = $V_{phase}\sin(120) = 0.866V_{phase}$

V_3 component = $V_{phase}\sin(240) = -0.866V_{phase}$

and the component of $V_1 - V_2 + V_3$ perpendicular to V_1 is

$V_1 - V_2 + V_3$ component= $0 - 0.866V_{phase} - 0.866V_{phase} = -1.732V_{phase}$.

Thus, the amplitude of the single phase voltage is achieved as follows.

$|V_1 - V_2 + V_3| = \sqrt{\left(V_{phase}^2 + (1.732V_{phase})^2\right)} = 2V_{phase} = 480\ V$

Q4 (a) 3 phase power generation is adopted throughout most of the world.

(i) Explain why 3 phase power generation is superior to other number of phases. [4]

(ii) What is the difference between 3 phase 3 wire distribution and 3 phase 4 wire distribution? [2]

Solution:

(i) With single phase, even though less wire is used, efficiency is very low. Two phase requires the same number of wires as three phase due to three phase not needing a neutral wire, but it has lower efficiency. Higher number of phases increase efficiency only marginally but extra wires are required. Hence three-phase is adopted throughout the world.

(ii) For unbalanced loads, an extra neutral wire is included in the 3 phase 4 wire distribution systems.

(b) (i) What is an infinite bus? Give an example where infinite bus assumption may not hold. [3]

(ii) A national grid system is supplied by 122 generators for a total of 56 GW. Suddenly, one of the generators operating at 7.4 GW fails. Does the infinite bus assumption still hold? Why? [2]

(iii) What could be done to restore the grid such that infinite bus assumption still holds? [2]

Solution:

(i) Infinite bus means fixed frequency and fix voltage. The bus is so strong that no single generator can change neither its frequency nor its voltage. For systems not connected to national grid, infinite bus assumption may not hold, i.e., submarine or a farm with its own generators.

Remark: Infinite bus assumption is unrelated to the number of generators. A single nuclear reactor running a small town would still be estimated as an infinite bus.

(ii) The supposed failure is big enough to introduce a gap between supply and demand. Therefore, voltage and frequency fluctuations are expected. As a result, the infinite bus assumption may not hold.

(iii) The demand should be reduced to supply level. In order to do so, either demand similar in magnitude to the failed generator should be disconnected from the grid or reserve capacity to replace the failed generator should be employed.

(c) A synchronous star-connected generator with 4 poles and a synchronous reactance of $X_s = 0.2\Omega$, is connected to a 50 Hz infinite bus with line voltage 33 kV.

(i) If the prime mover is set to 600 MW and the power factor at the terminals of the machine is 0.95 lagging, find the excitation voltage. [6]

(ii) If the power factor changes to 0.7 with line voltage and the excitation voltage that you obtained in (i) remaining unchanged, calculate the new prime mover power. [6]

Solution:

(i)

$$P = 3I_{ph}V_{ph}\cos\theta \Longrightarrow I_{ph} = \frac{P}{3\frac{V_l}{\sqrt{3}}\cos\theta} = 11.0 \text{ kA}$$

$$E = \sqrt{(V_{ph} + I_{ph}X_s\sin\theta)^2 + (I_{ph}X_s\cos\theta)^2} = 19.9 \text{ kV} = 34.4 \text{ kV}_{\text{line}}$$

(ii)

$$E = \sqrt{(V_{ph} + I_{ph}X_s\sin\theta)^2 + (I_{ph}X_s\cos\theta)^2} \Longrightarrow$$

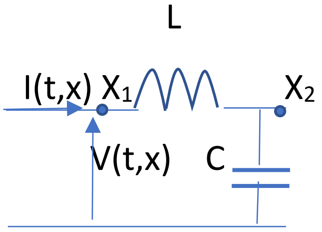
$$I_{ph}^2X_s^2 + 2I_{ph}V_{ph}X_s\sin\theta + V_{ph}^2 - E^2 = 0$$

Solving the quadratic equation and using the positive root we find $I_{ph} = 5.82$ kA. The new prime mover power then becomes:

$$P = 3I_{ph}V_{ph}\cos\theta = 233 \text{ MW}$$

END OF PAPER

Answer Q6



(a) $\delta V = -L\,(X_2 - X_1)\frac{\partial I}{\partial t} = -L\delta x\frac{\partial I(x,t)}{\partial t}$, $\quad {dV}/{dx} = -L\,{\partial I}/{\partial t} \quad$ (1)

$\delta I = -C\,(X_2 - X_1)\frac{\partial V(x,t)}{\partial t} = -C\delta x\frac{\partial V(x,t)}{\partial t}$, $\quad {dI}/{dx} = -C\,{\partial V}/{\partial t} \quad$ (2)

These are the Telegraphers equations.

(b) From (1) and (2)in (a)

$$\frac{\partial V}{\partial I} = \frac{\partial V}{\partial x}\cdot\frac{\partial x}{\partial I} = -L\frac{\partial I}{\partial t}\cdot -\frac{1}{c}\cdot\frac{\partial t}{\partial V} = \frac{L}{C}\cdot {1}\Big/{\frac{\partial V}{\partial I}}$$

$$\therefore \frac{\partial V}{\partial I} = Z_0 = \sqrt{\frac{L}{C}}$$

(c) Taking the 2nd differential of (1) in X and Substituting for *dI/dx*

$$\frac{\partial^2 V}{\partial x^2} = LC\frac{\partial^2 V}{\partial t^2}$$

Similarly takingthe 2nd differential of (2) in X and substituting for ${dV}/{dx}$

$$\frac{\partial^2 I}{\partial x^2} = LC\frac{\partial^2 I}{\partial t^2}$$

Both V and I are functions of distance and time i.e V= V(*x,t*) and I= I(*x,t*). As the equations take the form of the 2nd differential space being proportional to 2nd differential in time, they describe wave equations. Hence V and I behave like waves over the two parallel wires. In the general wave equation (data book) $\frac{\partial^2 f(x,t)}{\partial x^2} = \frac{1}{C^2}\frac{\partial^2 f(x.t)}{\partial t^2}$ where C is the wave velocity. Hence for the V and I waves

C= ${1}/{\sqrt{(LC)}}$ The voltage pulse will also travel at wave velocity in the twisted wire transmission line.

(d) (i) With values of L = 1.0 x 10^{-6} H m^{-1} and C = 2.5 x 10^{-11} F m^{-1}

C = ${1}/{\sqrt{2.5x10^{-17}}} = 2.0\ x\ 10^8\ ms^{-1}$ $\therefore$ the delay is time taken for the pulse to travel 2m.

Delay τ = 10 ns

(ii) General form of the solution to the wave equation is:

$$V(x,t) = V_1 e^{j(\omega t - \beta x)} + V_2 e^{j(\omega t + \beta x)}$$

The pulse can be considered to be decomposed in Fourier terms to the sum of many frequency components. Each of the components will travel with the same velocity as we assume the values of L and C are frequency independent. V_2 is the backward travelling wave due to reflection. In this case of a pulse one needs to consider reflection at the load and the subsequent reflection of V_2 at the source.

-

The reflection co-efficient at the load is:

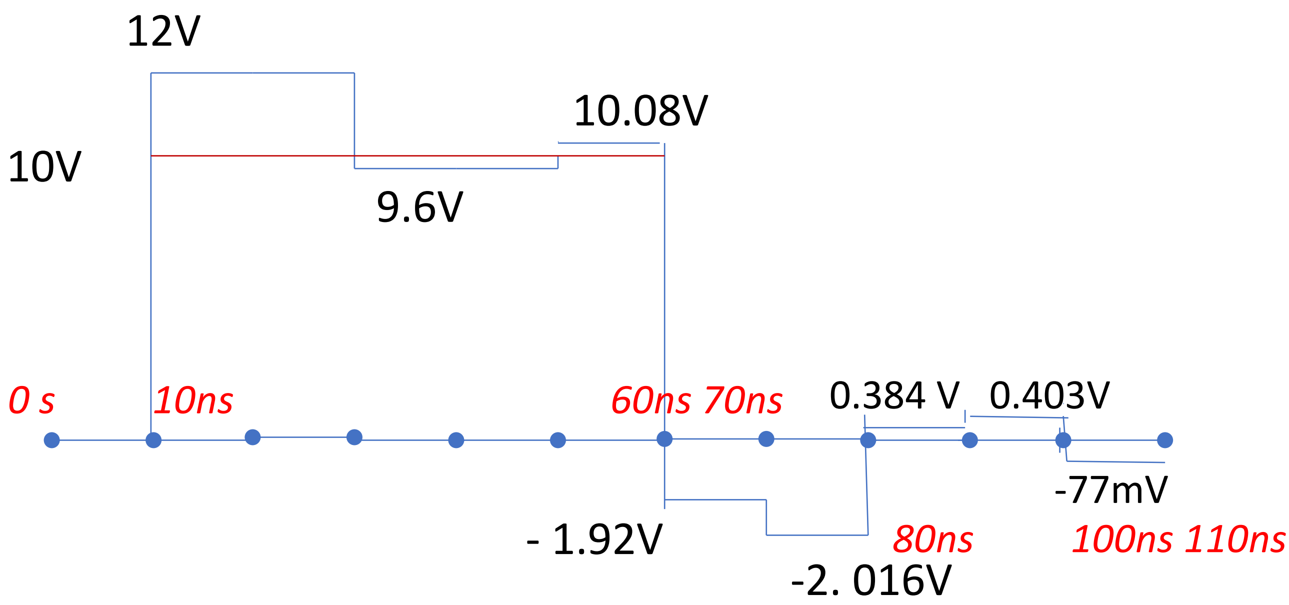
$$\bar{\rho} = \frac{\bar{Z}_L - Z_0}{\bar{Z}_L + Z_0}, \; Z_0 = \sqrt{\frac{1.0x10^{-6}}{2.5x10^{-11}}} = 200\Omega \;\; \bar{z}_L = 300\Omega$$

$$\bar{\rho} = \frac{100}{500} = 0.2{<}0 \text{ deg}$$

The source voltage is ideal , i.e Z_S=0, hence the reflection co-efficient at the source is:

$$\rho_s = \frac{\bar{Z}_s - Z_0}{\bar{Z}_s + Z_0} = -1 = 1 < 180\ deg$$

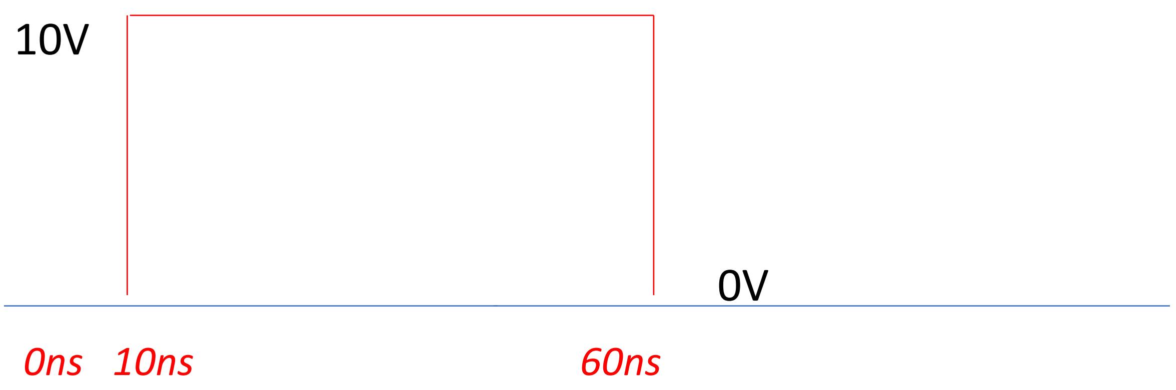
The time variation of the voltage signal at the load is given below. Time 0 is taken to be the origination of the pulse at the voltage source.



The pulse arrives at the load 10ns after it is transmitted form the source. On arrival there is reflection coefficient of 0.2V which gives and added value of 2V to the 10 V pulse. The 0.2V reflected component is transmitted to the source. At the source it is reflected by a phase angle change of 180 degrees i.e it returns to the load as -0.2 20ns later (10ns to get to the source and 10ns back). This is now added to the 12V signal, but one needs to also consider the reflected vale of the -2V signal, this is an additional -0.4V, so the total to be added is -2.4V. This process continues. The end of the transmitted pulse is taken as a second signal of -10Vwhich arrives 50ns later. After this the residual reflected signal form the 10V signal and the new reflected components form the -10V signal need to be considered. This now results in signal variation every 10ns converging towards 0V.

(iii) In order to have to replicate the transmitted pulse exactly at the load (with only a time delay τ) the initial reflection at the load must be zero. i.e ρ = 0

This condition is met if $Z_L = Z_0$ so if the load is chosen to be 200Ω then there is no reflection at the load. The new pulse waveform is then:



This shows the advantage of 'matching' the load to the transmission line impedance.

(iv) To transmit at 100mbs a bit would be equivalent to a pulse of 5ns with 50% mark-space ratio i.e a period of 10ns. The delay over 2m in the line is 10ns and for network cable the data would have to reach many loads of varying distances. But if the loads at the network load are all matched to the characteristic impedance of the transmission line as in d (iii) above, the data can be transmitted in principle at any rate. However, there will be varying delays in the data reaching different nodes on the network.

So YES it is in principle possible to use the twisted pair transmission line for 100mbs data communication in a network, PROVIDED all the loads at the nodes on the network are matched to the transmission line.

Q7

$\bar{\nabla} \times \bar{E} = -\partial/\partial t\,(\bar{B})$ and $\bar{\nabla} \times \bar{H} = \bar{J} + \epsilon_o\ \partial/\partial t\,(\bar{D})$

Also $\bar{B} = \mu_o \bar{H}$

$\bar{\nabla} \times (\bar{\nabla} \times \bar{E}) = -\partial/\partial t(\bar{\nabla} \times \bar{B})$

Using the vector identity

$\bar{\nabla} \times (\bar{\nabla} \times \bar{E}) = \bar{\nabla}\bar{\nabla}.\bar{E} - \bar{\nabla}^2 \bar{E}$

And noting that $\bar{\nabla}.\bar{E} = \epsilon_o\,(\bar{\nabla}.\bar{D}) = \rho$ and that $\rho = 0$ and $J = 0$ in free space

$\bar{\nabla}^2 \bar{E} = \mu_o\ \partial/\partial t(\bar{\nabla} \times \bar{H})$

Substituting the 2nd Maxwell equation

$\bar{\nabla}^2 \bar{E} = \epsilon_o\ \mu_o\ \partial^2/\partial t^2\ (\bar{E})$

Similarly noting that $\bar{\nabla}.\bar{B} = \mu_o\,(\bar{\nabla}.\bar{H}) = 0$

$\bar{\nabla}^2 \bar{H} = \epsilon_o\ \mu_o\ \partial^2/\partial t^2\ (\bar{H})$

These are both wave equations. Hence the electric field and magnetic field propagate through free space as waves when there is a time varying electric or magnetic field (one induces the other according to Maxwell's equations) in free space.

Taking the general case of $\bar{E} = E_x\,\bar{a}_x + E_y\,\bar{a}_y + E_z\,\bar{a}_z$ as all components have to conform to the wave equations, a general solution will have the form:

$E_x\,\bar{a}_x = E_{xo}\,e^{(\pm j(\omega t - \beta z))}\,\bar{a}_x + E_{xo}\,e^{(\pm j(\omega t - \beta y))}\,\bar{a}_x + E_{xo}\,e^{(\pm j(\omega t - \beta x))}\,\bar{a}_x$

(as we are considering propagation in free space we do not need to consider any waves travelling in the -*x* direction as there are no boundaries to reflect from).

Therefore substituting any given component into the wave equation one gets

$\beta^2 = \epsilon_o\ \mu_o\,\omega^2$ and $\omega/\beta = 1/\sqrt{(\epsilon_o\ \mu_o)}$. The velocity of any wave component is obtained by considering an observer travelling at the same speed as the wave. In which case the value of the electric (magnetic) field will be constant. Hence $(\omega t - \beta_x x) = constant$

$\partial/\partial t\,(\omega t - \beta_x x) = 0;\ \partial x/\partial t = \omega/\beta_x$ is the velocity of the electromagnetic wave in the x direction. But form the general solution we see that this is a constant, which is the same in any direction.

OR

One could note that the general solution to the wave equation is of the form

$$\frac{\partial^2 f(r,t)}{\partial r^2} = \frac{1}{c^2}\frac{\partial^2 f(r,t)}{\partial t^2}$$

With c = velocity of the wave.

Hence for the $\bar{E}$ and $\bar{H}$ waves c = $1/(\sqrt{\varepsilon_0\,\mu_0})$

Using the data book values for ϵ_o and μ_o,
$1/\sqrt(\epsilon\_o\ \mu\_o) = 1/\sqrt(8.854x10^{\wedge}(-12).4\pi x10^{\wedge}(-7)) = 2.998\ x\ 10^{\wedge}8\ ms^{\wedge}(-1)$. This is the same as the measured velocity of visible light (see data book for velocity of light). Hence a general radio wave which are coupled waves of E^{-} and H^{-} must also be a form of light.
(i) Considering the Maxwell equation $(\nabla)^{-}XE^{-} = -\mu\_o\ \partial/\partial t(H)^{-}$

$$((\partial E\_z)/\partial y - (\partial E\_y)/\partial z)a^{-}\_x + \left((\partial E\_x)/\partial z - \frac{\partial E_z}{\partial x}\right)a^{-}\_y + \left(\frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y}\right)\bar{a}_z$$
$$= -\mu\_o\ \partial/\partial t(H\_x\ (a\_x)^{-} + H_y\overline{a_y} + H_z\overline{a_z})$$

As $\bar{E} = E_x\overline{a_x}$ as it is confined in the x-y plane and defined only in one direction, and it does not vary in the x-y plane $\frac{\partial E_x}{\partial y} = \frac{\partial E_x}{\partial x} = 0$,

$$\frac{\partial E_x}{\partial z}\bar{a}_y = -\mu_o\frac{\partial}{\partial t}(\overline{H_y a_y})$$

And

$$-\frac{\partial H_y}{\partial z}\bar{a}_x = \epsilon_o\frac{\partial}{\partial t}(\overline{E_x a_x})$$

From the general solution above with only one component of $\bar{E}$ and $\bar{H}$

For a plane wave one gets

$$\frac{E_x}{H_y} = \frac{\omega\mu_o}{\beta}$$

From the solution to b) we know $\frac{\omega}{\beta} = 1/\sqrt{\epsilon_o\,\mu_o}$

$$\frac{E_x}{H_y} = \sqrt{\frac{\mu_o}{\epsilon_o}} = 377\ \Omega$$

Which has units of ohms (Vm^{-1}/ Am^{-1}) and is defined as the characteristic or intrinsic impedance of free space. It allows one to express the magnitude of magnetic field as a constant factor of the orthogonal electric field magnitude and vice versa.

ii)

Restating Maxwell's equations:

$\bar{\nabla} X\bar{E} = -\frac{\partial}{\partial t}\bar{B}$ and $\bar{\nabla} X\bar{H} = \bar{J} + \epsilon_o\frac{\partial}{\partial t}\bar{D}$

Also $\bar{B} = \mu_o\bar{H}$. In a conducting medium J≠0 and $\bar{J} = \sigma\bar{E}$. Note however in a conductor there is no net electrostatic charge. Though there is charge movement electrical neutrality is maintained e.g. moving electrons in conduction band always balanced by nuclear charge locally. Hence the ρ = 0 condition is maintained.

∴

$$\overline{\nabla^2}\bar{E} = \mu\frac{\partial}{\partial t}(\bar{\nabla}X\bar{H}) = \mu\sigma\frac{\partial\bar{E}}{\partial t} + \mu\varepsilon\frac{\partial^2\bar{E}}{\partial t^2}$$

By substituting for a plane wave where E is only defined in the x-direction

$$E_x = E_{x0}e^{j(\omega t - \gamma_c z)} \ then \quad -\gamma_c^2 E_x = (j\omega\sigma\mu - \mu\varepsilon\omega^2)E_x$$

we use a general propagation constant γ_c *for the conductor as opposed to that in free space* β

$$\therefore j\gamma_c = \sqrt{(j\omega\mu(\sigma + j\omega\varepsilon)} = \alpha + j\beta$$

$$Using \ \frac{\partial E_x}{\partial z} = -\mu\frac{\partial H_y}{\partial t}$$

$$\frac{E_x}{H_y} = \frac{j\omega\mu}{j\gamma_c} = \frac{j\omega\mu}{\sqrt{(j\omega\mu(\sigma + j\omega\varepsilon)}} = \sqrt{\frac{j\omega\mu}{(\sigma + j\omega\varepsilon)}} = \bar{\eta}$$

η is the intrinsic impedance, which is complex, for a conducting medium where σ is the conductivity.

Note that when $\sigma \gg \omega\varepsilon \ \ \eta \cong \sqrt{\frac{j\omega\mu}{\sigma}} = \sqrt{\frac{\omega\mu}{\sigma}} \ \angle 45 \ deg$

d) (i)

Intrinsic impedance of sea water. From

$\sqrt{\frac{j\omega\mu}{(\sigma+j\omega\varepsilon)}} = \eta$ and noting that σ = 4 Sm^{-1} which is ≫ than $2\pi 10^6$ x 81X $8.85X10^{-12}$= 4.5 x 10^{-3}

Hence η = $\sqrt{\frac{2\pi 10^6 x 4\pi 10^{-7}}{4}} . \frac{1}{\sqrt{2}}(1 + j) \ \ \lceil\eta\rceil = \ 1.4\Omega$

From c) (ii) above

$J\gamma_c = \alpha + j\beta$ and $E_x = E_{x0}e^{j(\omega t - \gamma_c z)}$

Hence $E_x = E_{x0}e^{-\alpha z + j(\omega t - \beta z)} \ and \ \alpha + j\beta = \sqrt{(j\omega\mu(\sigma + j\omega\varepsilon)}$

$$since \ \sigma \gg \omega\varepsilon \ \ \alpha = \beta = \sqrt{\frac{\omega\mu\sigma}{2}} = \ 3.97$$

Electric field intensity in the sea water is given by the fraction transmitted into the sea at the sea air interface. The transmission co-efficient is given as:

$$\widehat{\mathrm{T}} = \frac{2\hat{\eta}}{\hat{\eta} + \widehat{\eta_0}} = \frac{2.8\angle 45}{2.8\angle 45 + 377} = 7.43X10^{-3}\angle 44.8$$

The electric field in sea water is therefore

$$E_x = \left|\hat{T}\right| E_{x0} e^{-\alpha z + j(\omega t - \beta z)}$$

There is an exponential decay term with -αz. The maximum when z=0. Therefore the maximum of the sinusoidally time varying electric field at z=0 is;

$E_{xmax-sea} = 7.43X10^{-3} . 4000 = 29.72\, Vm^{-1}$

(iii)

The average power density for a plane wave resulting from Poynting's theorem in free space is

$\bar{S} = \bar{E}X\bar{H} = \frac{1}{2} . \frac{E_{x0}^2}{\eta}$

In sea water η is complex .i.e there is a phase shift between $\bar{E}$ *and* $\bar{H}$.

Also E_{xo} *varies as* $\left|\hat{T}\right| E_{xo} e^{-\alpha z}$

Therefore the average power density in the sea water at any location z normal to the surface is

$\bar{S} = \bar{E}X\bar{H} = \mathrm{Re}(\frac{1}{2} . |\widehat{T}|^2 \frac{E_{x0}^2}{\bar{\bar{\eta}}} e^{-2\alpha z}) = \frac{1}{2} . 29.72^2 . \frac{\cos 45}{1.4} . e^{-2(3.97)z} = 223.1 e^{-7.94z}$

The receiving antenna on the submarine has an area of 1m^2 and has to collect 12pW

$\therefore ln\left[\frac{12\ X\ 10^{-12}}{223.1}\right] = -7.94z \quad -30.55 = -7.94z \quad and\ \boldsymbol{z = 3.9\ m}$