SECTION A

Question 1)

If an amplifier with gain A is subject to positive feedback from its output to its input via a network B, explain what conditions would result in an oscillatory output.

[4]

For oscillations to occur the loop gain should be unity and the loop phase should be 0 or 2pi, i.e. AB=1

The circuit in Fig. 1 is an oscillator based on four ideal operational amplifiers (OP1, OP2, OP3 and OP4), where a D.C. voltage $V_{ref} = 2.5$ V is provided to OP1 to provide a D.C. offset for the signal to allow for operation with a single-ended 5V power supply.

If V_X and V_Y denote the voltages at the points X and Y marked on the circuit in Fig. 1, by considering the signal passing through OP1, show that

$$V_Y = -\frac{R_2}{R_1}(V_X - V_{ref}) + V_{ref}$$

[3]

Current at inverting input is

$$\frac{V_X - V_{ref}}{R_1} = \frac{V_{ref} - V_Y}{R_2}$$

Hence

$$V_Y = -\frac{R_2}{R_1}(V_X - V_{ref}) + V_{ref}$$

As required

By considering the signal passing clockwise around the circuit from Y to X, passing through OP2, OP3 and OP4, show that

$$V_X = \left(\frac{1}{1 + j\omega RC}\right)^4 V_Y$$

[4]

Simple potential divider at the input of the OP2 gives

$$\frac{V_+}{V_Y} = \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{1}{1 + j\omega RC}$$

So output of OP2 is $V_- = V_+ = \frac{V_Y}{1+j\omega RC}$.

This voltage is then divided at the input to OP3, so that the output of OP3 is $\frac{V_Y}{(1+j\omega RC)^2}$.

Hence the input to OP4 is $\frac{V_Y}{(1+j\omega RC)^3}$. This is also the output of OP4 which is then divided by the RC circuit to give

$$V_X = \frac{V_Y}{(1 + j\omega RC)^4}$$

as required

Derive an expression for the oscillation frequency for the circuit in Fig. 1 and the required value for R_2/R_1 for stable oscillations at this frequency.

[7]

For an A.C. signal the effect of V_{ref} may be neglected since it only corresponds to a D.C. offset, therefore the loop gain is

$$AB = -\frac{R_2}{R_1(1 + j\omega RC)^4}$$

We require $|AB| = 1$ and $\angle AB = 0$

The phase shift for the loop is

$$\pi - 4\tan^{-1}(\omega RC)$$

Which should be total of zero phase

So $\tan^{-1}(\omega RC) = \pi/4$ and hence $\omega_r = 1/RC$

Therefore for a round trip unity gain gives

$$\left|\frac{1}{1 + j}\right|^4 \frac{R_2}{R_1} = 1$$

Therefore $\frac{R_2}{R_1} = 4$

Due to finite resistor values the amplifier gain is 4% higher than the ideal, calculate the corresponding percentage change in the oscillation frequency. You may assume that this small deviation has a minimal impact on the loop phase.

[7]

$$[1 + (\omega RC)^2]^2 = 4 \times 1.04$$

So

$$1 + (\omega RC)^2 = 2 \times 1.02$$

So

$$(\omega RC)^2 = 1.04$$

Therefore

$$\omega RC = 1.02$$

So the oscillation frequency will increase by 2%.

Question 2)

In the amplifier circuit shown in Fig. 2, the transistor is biased at an operating point such that $I_B = 10$ µA, $I_c = 2$ mA, V_{BE} =0.7 V, and $V_{CE} = 5$ V. At this operating point $h_{fe} = 250$, $h_{ie} = 5$ kΩ and the other transistor parameters h_{re} and h_{oe} may be neglected.

(a) Assuming the transistor is biased to maximise the output signal before clipping occurs, i.e. $V_C = V_{CC}/2$, determine appropriate values for the R_C and R_E stating any assumptions made.

[4]

Set $V_C = V_{CC}/2 = 10$ V therefore

$$R_C = \frac{10}{2 \times 10^{-3}} = 5\ \text{k}\Omega$$

If V_{CE} =5 V and $V_C = 10$ V then $V_E = 5$ V. Assume that $I_E \approx I_C$ therefore

$$R_E = \frac{5}{2 \times 10^{-3}} = 2.5\ \text{k}\Omega$$

(b) If the current flowing through R_2 is designed to be $10I_B$, determine appropriate values for R_1 and R_2.

[4]

$V_E = 5$ V therefore V_B =5.7 V. The voltage across R_2 is therefore $20 - 5.7 = 14.3\ V$.
Current through $R_2 = 10 \times I_B = 100$ µA therefore

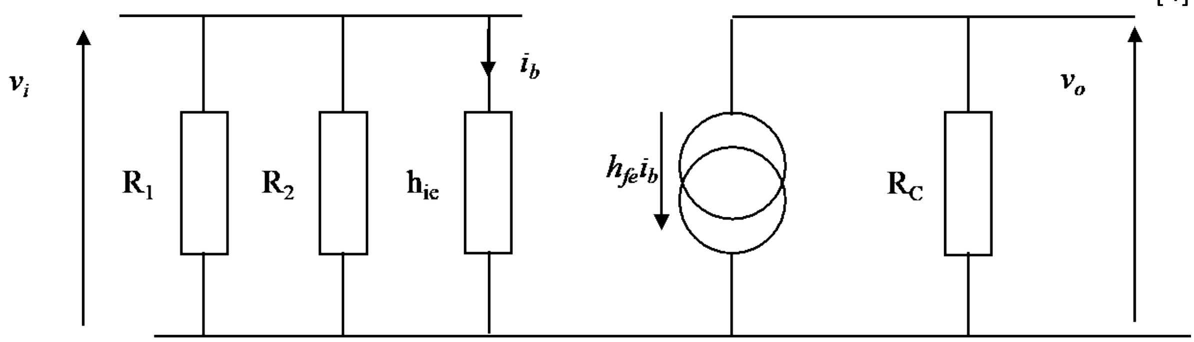
$$R_2 = \frac{14.3}{100 \times 10^{-6}} = 143\ \text{k}\Omega$$

the current flowing through R_1 will be $9 \times I_B = 90$ µA and the voltage across R_1 will be 5.7V therefore

$$R_1 = \frac{5.7}{90 \times 10^{-6}} = 63.3\ \text{k}\Omega$$

Draw a small-signal equivalent circuit for the amplifier valid for mid-band frequencies (where the reactance of the capacitors C_i, C_o and C_E may be assumed to be zero).

[4]



Using the small-signal circuit, calculate

(i) The small-signal voltage gain

(ii) The small-signal input resistance

(iii) The small-signal output resistance

[6]

i)

$$i_b = \frac{v_i}{h_{ie}}$$

$$v_o = -h_{fe} i_b R_C = -\frac{h_{fe} v_i R_C}{h_{ie}}$$

Therefore small signal gain is

$$\frac{v_o}{v_i} = -\frac{h_{fe} R_C}{h_{ie}} = \frac{-250 \times 5 \times 10^3}{5 \times 10^3} = -250$$

ii) Small signal input resistance is

$$R_1||R_2||h_{ie} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{h_{ie}}\right)^{-1} = \left(\frac{1}{63.3 \times 10^3} + \frac{1}{143 \times 10^3} + \frac{1}{5 \times 10^3}\right)^{-1} = 4.5\ \mathrm{k\Omega}$$

iii) Small-signal output resistance is

$v_i = 0$, therefore $i_b = 0$ therefore current source becomes an open circuit such that

$$r_o = R_C = 5\ \mathrm{k\Omega}$$

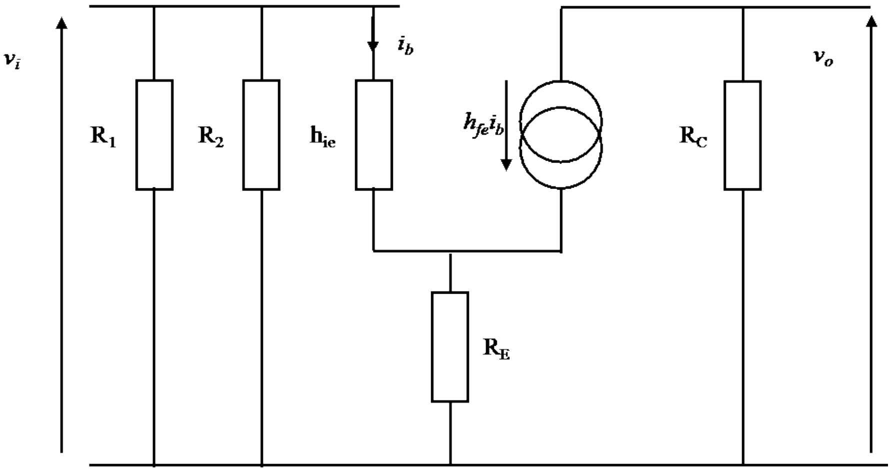
If the capacitor C_E is now omitted from the circuit such that the impedance between the emitter and the ground is R_E, draw the small-signal equivalent circuit for this modified circuit and calculate

(i) The small-signal voltage gain

(ii) The small-signal input resistance

(iii) The small-signal output resistance

[7]



(i) The small-signal voltage gain

$$v_o = -h_{fe} i_b R_C$$

$$v_i = i_b h_{ie} + (1 + h_{fe}) i_b R_E$$

$$\frac{v_o}{v_i} = -\frac{h_{fe} R_C}{h_{ie} + (1 + h_{fe}) R_E} = -\frac{250 \times 5 \times 10^3}{5 \times 10^3 + 251 \times 2.5 \times 10^3} = -1.97$$

ii) The small-signal input resistance

Input current $i_i = \frac{v_i}{R_1} + \frac{v_i}{R_2} + i_b$

$$v_i = h_{ie} i_b + (1 + h_{fe}) i_b R_E$$

Therefore

$$i_i = \frac{v_i}{R_1} + \frac{v_i}{R_2} + \frac{v_i}{h_{ie} + (1 + h_{fe})R_E}$$

Therefore input impedance

$$r_i = \frac{v_i}{i_i} = \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{h_{ie} + (1 + h_{fe})R_E}\right)^{-1}$$

$$= \left(\frac{1}{63.3 \times 10^3} + \frac{1}{143 \times 10^3} + \frac{1}{5 \times 10^3 + 251 \times 2.5 \times 10^3}\right)^{-1}$$

$$= 41 \text{ k}\Omega$$

iii) The small-signal output resistance $v_i = i_b h_{ie} + (1 + h_{fe}) i_b R_E$

Set $v_i = 0$ therefore $i_b = 0$, hence current source becomes open circuit as before and thus

$$r_o = R_C = 5 \text{ k}\Omega$$

As before

Question 3)

(a) (i)

Start with Load D:

R and X parallel, so simply

P = 3 V^2/R = 3 x $(415 \text{ V})^2/12\ \Omega$ = 43 kW

Q = 3 V^2/X = 3 x $(415 \text{ V})^2/8$ V/A = 65 kVar

S = sqrt($P^2 + Q^2$) = 78 kVA

Continue with Load Y:

R and X in series and different in each arm!

Calculate current of the first two arms

$I_{Y1,2}$ = 240 V/sqrt($(20\ \Omega)^2$ + $(10 \text{ V/A})^2$) = 10.7 A

Calculate current of the third arm

I_{Y3} = 240 V/sqrt($(10\ \Omega)^2$ + $(5 \text{ V/A})^2$) = 21.5 A

Power is now the sum (with the respective resistance):

P = 2 x ($I_{Y1,2}^2$ 20 Ω) + 1 x (I_{Y3}^2 10 Ω) = 4608 W + 4608 W = 9.2 kW

Reactive power would be

Q = -2 x ($I_{Y1,2}^2$ 10 V/A) - 1 x (I_{Y3}^2 5 V/A) = 2304 Var + 2304 Var = 4.6 kVar

S = 10.3 kVA

(ii)

Calculate the needed compensation in star configuration.

For compensation, the three arms would need to become equal. Increasing with a parallel load is not possible, so can only bring the arms with larger resistance and reactance down:

Add 20 Ω in series with 10 V/A to the first two arms in star configuration, open circuit to the third.

(iii)

Simply three times 10 Ω active and 5 V/A reactive in series.

(b)(i)

Line current of Load D
$I_{d,act}$ = 415 V/12 Ω = 34.6 A
$I_{d,react}$ = 415 V/8 V/A = 51.9 A
Power (which would also lead to line current of Load R):
$P_r = 3 \times I_{Y3}^2$ 10 Ω = 13.8 kVA
$Q_r = 3 \times I_{Y3}^2$ 5 V/A = 6.9 kVar
=> $I_{r,act} = P_r/(3 \times 240$ V) = 19.2 A
=> $I_{r,react} = Q_r/(3 \times 240$ V) = 9.6 A
(check total current magnitude: 21.5 A as in (a))

Full line current through power:
P_{total} = 43 kW + P_r = 56.8 kW
Q_{total} = 65 kVar – Q_r = 58 kVar
=> $I_{total,act} = P_{total}/(3 \times 240$ V) = 78.9 A
=> $I_{total,react} = Q_{total}/(3 \times 240$ V) = 80.5 A
I_{total} = 112.7 A

(ii)

tan ϕ = 80.5 A/ 78.9 A = 1.02 => ϕ = 45.58°
cos ϕ = 0.7

(c)(i)

Unity power factor needs -58 kVar = 3 V^2/X_c with three capacitors
=> X_C = -3 (240 V)²/58000 Var = -3.0 V/A = -1/(jωC)
=> C = 1.06 mF

(ii)

cos ϕ = 0.95 => tan ϕ = 0.329
=> Q_{target} = P tan ϕ = 56.8 kW x 0.329 = 18.7 kVar
=> Q_{comp} = 58 kVar – 18.7 kVar = 39 kVar
=> X_C = -3 (240 V)²/39 kVar = -4.4 V/A
=> C = 0.72 mF

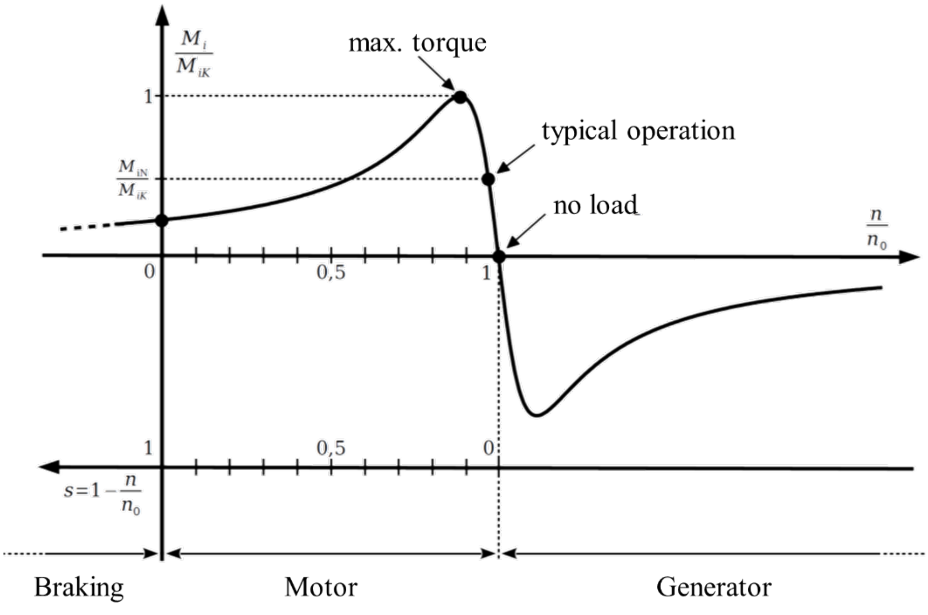
(d)

Unity case: S = P => I_L = 56.8 kW/(3 x 240 V) = 78.9 A from previously 112.7 A
=> 70% of previous current or -30%
cos ϕ = 0.95:
$I_{L,react} = Q_{target}/(3 \times 240$ V) = 26 A
=> I_L = sqrt((26 A)² + (78.9 A)²) = 83.1 A
=> 74% of previous current or -26%

SECTION B

Question 4)

(a)



Slip in braking mode <0

At synchronous speed (no load, no friction) s = 0

At speed 0, s = 1

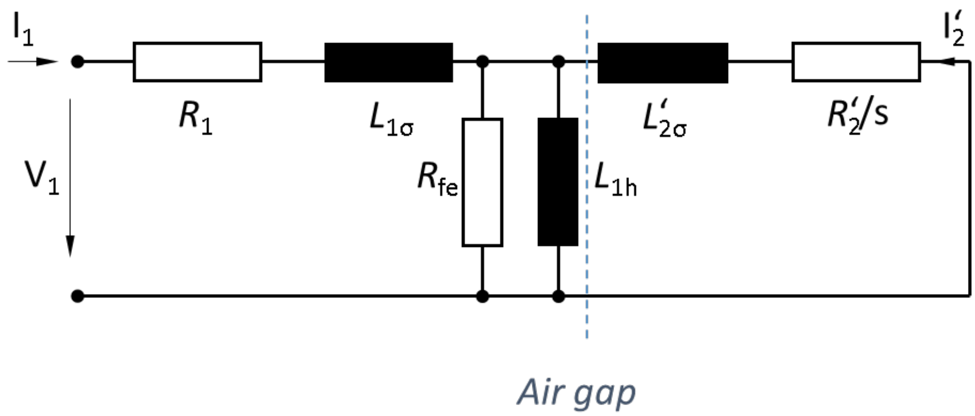
When braking, torque points in opposite direction compared to speed => neg. mechanical power

(b)(i)

Blocked rotor: s = 1

Other point must be synchronous at exactly 3000 rpm => s = 0

(ii)



R_1: copper resistance in stator

L_1: stator stray inductance/reactance

R_{fe} or R_i: equivalent iron-loss resistance

L_{1m}: magn. reactance

L'_2: rotor stray inductance/reactance (referred to stator)
R': rotor resistance (can be split into loss and the equivalent part converted into mechanical power, both referred to the stator)

(iii)

No-load test at 3000 rpm, R_2'/s infinitely large
R_i and X_1 large compared to X_1 and R_1 => entire voltage drop on them
=> $3\ V^2/R_i = 1200\ W$ => $R_i = 430.6\ \Omega$
=> $I_{Ri} = 1/\sqrt{3}\ 415\ V/R_i = 0.556\ A$; $I_{Xh} = \text{sqrt}(\ (8\ A)^2 - I_{Ri}^2) = 7.98\ A = 1/\sqrt{3}\ 415\ V/X_h$
=> $X_h = 30\ V/A$
Locked rotor with s = 1, neglect R_i and X_i
$X_1 = X_2'$
$3\ (I/\sqrt{3})^2\ (R_1 + R_2') = 667\ W$ => $R_1 + R'_2 = 2.61\ \Omega$
$(16\ A/\sqrt{3})^2\ (4\ X_1^2 + (R_1 + R_2')^2) = (30\ V)^2$
=> $X_1^2 = \frac{1}{4}(\ (30\ V)^2/(16\ A/\sqrt{3})^2 - (R_1 + R_2')^2) = 0.934\ V^2/A^2$
=> $X_1 = X_2' = 0.966\ V/A$

From sole stator measurement
$(R_1 16\ A/\sqrt{3})^2 + (X_1 16\ A/\sqrt{3})^2 = (22.6\ V)^2$
=> $R_1^2 = (22.6\ V)^2 - (X_1 16\ A/\sqrt{3})^2\ 3/(16\ A)^2 = 3/(16\ A)^2\ (22.6\ V)^2 - (X_1)^2 = 4.99\ \Omega^2$
=> $R_1 = 2.2\ \Omega$
=> $R_2' = 2.61\ \Omega - 2.2\ \Omega = 0.41\ \Omega$

(c)(i)

Rotor resistance $R_2'/s = R_2'\ (1 + 1/s - 1)$
loss resistance: R_2'
component modelling the power transfer to the mechanical domain: $R_2'\ (1/s - 1)$

(ii)

$s = 1 - 2700\ \text{rpm}/3000\ \text{rpm} = 0.1$

(iii)

Total current path: R_1 -> X_1 -> X_2' -> R_2' -> $R_2'\ (1/s - 1)$
Current: $I = \sqrt{3}\ 415\ V/\text{sqrt}(\ (R_1 + R_2'/s)^2 + 4\ X_1^2) = \sqrt{3}\ 415\ V/\text{sqrt}(\ (2.2\ \Omega + 0.41\ \Omega/0.1)^2 + 4\text{x}\ 0.934\ V^2/A^2) = \sqrt{3}\ 415\ V/6.59\ V/A = 109\ A$
$P_r = 3\text{x}\ R_2'\ (I/\sqrt{3})^2 = 4878\ W$
$P_s = 3\text{x}\ R_1\ (I/\sqrt{3})^2 = 26175\ W$

Question 5)

a)

$V=1/\sqrt{LC}$= 1/(400×10^{-9}×100×10^{-12})$^{0.5}$=1.58×10^{8} m/s

Z=(L/C)$^{0.5}$=63.25Ω

Assuming μ_r=1

$$V=\frac{1}{\sqrt{\varepsilon_0\varepsilon_r\mu_0\mu_r}}=\frac{c}{\sqrt{\varepsilon_r}}$$

With c=3×10^{8} m/s

Therefore

$\varepsilon_r = c^2/v^2$ =3.61

b)

wavelength λ = V/f = 1.58 × 10^{8}/ 250 ×10^{6}= 0.632 m

for open circuit, length= λ/4=0.158 m

for short circuit, length= λ/4=0.316 m

c)

$$\rho_L = \frac{Z_L-Z_0}{Z_L+Z_0} = \frac{150-63.25}{150+63.25} = 0.41$$

$$VSWR=\frac{1+|\rho_L|}{1-|\rho_L|} = 2.39$$

d)

V_{GEN}=150 V rms

$V_L=V_{GEN}(1+\rho_L)$=211.5V

Power=V_L^2/R=298.2W

Question 6)

a)

Peak power intensity $= \frac{Gain \cdot Power}{4\pi r^2} = \frac{2500 \cdot 30}{4\pi(3 \cdot 10^7)^2} = 6.63$ pW m^{-2}

b)

i) Power received= $A_{eff} \times$ peak power intensity= 19.9 pW

ii) Perfect matching: all power received goes to the power electronics (neglecting re-emission).

Therefore 19.9pW=0.5 VI= 0.5 I^2Z_0

Then I=$\sqrt[2]{\frac{19.9 \cdot 10^{-12}}{0.5 \cdot 50}} = 892.2$nA and I_{rms}=630.9nA

c)

The signal is broadcast isotropically. Hence the power per unit area, P, at a distance r from the source is:

$P=10000/(4\pi r^2)$ W/m^2

The receiver area is 0.03m^2, hence the power received at the antenna, P_{rec}, is:

$P_{rec}=P\times 0.03W=300/(4\pi r^2)$ W

At the maximum distance, r_{max}, $P_{rec}=10^{-9}$W

Hence $r_{max}=[300/(4\pi\times 10^{-9})]^{0.5}=154.51$ km

d)

If the receiver is placed at 85% r_{max}, then the power received is $(1/0.85)^2$ times the minimum required level

If the receiver is inclined at an angle θ to the optimum angle for picking up the signal, then the effective area is reduced by a factor $\cos(\theta)$

Hence, the antenna can be misoriented by and still pick up the minimum required signal, with

$\mathrm{Cos}(\theta_{max})=(0.85)^2$

Hence $\theta_{max}=43.74^0$