Part IB Paper 8 2019 Crib

Section A The Engineer in Business

1. Answer: b. A high click-through rate indicates that the search ad is attractive, a low conversion rate indicates that the landing page is not attractive to users or not aligned with user's interests.

2. Answer: d. The second step in marketing strategic planning is market segmentation.

3. Answer: a. A brand positioning statement should include the target customers, the competitive set, the unique value proposition offered by the brand (or point of difference), and the evidence or reasons to believe. Answer (a) is included in a brand positioning statement.

4. Answer: c. A customer insight is a deep understanding about the fundamental motivations that underlie customer's behaviour. Answer (b) is a description of customer behaviour. Answers (a) and (d) are stereotypical beliefs. Answer (c) explains the motivating factor that underlies customer's behaviour, thus is a customer insight.

5. Answer: d. According to Porter's Five Forces model, answers (a) (b) (c) are all relevant factors for consideration in the industry analysis. However, (a) is a factor of threat of entry; (b) is relevant to threat of substitutes; (c) is relevant to supplier power. Answer (d) is a source of rivalry among competitors within the industry. Unused capacity and barriers to exit encourage firms to offer price cuts to attract new clients (and so increase the level of internal competition).

6. Answer: b. The VRISO Framework is a tool to recognize a strategic resource within organizations: valuable, rare, non imitable, non substitutable, and fully exploited by the organization. Thus, the right answer is 'Hard to substitute'.

7. Answer: e. A differentiation strategy differentiates a firm from its competitors by providing unique products or services that are valuable to customers. A successful differentiation strategy needs to be (i) value-enhancing (economically viable); (ii) defensible (over the long term); (iii) meaningful (perceived by buyers). Therefore the correct answer is (e).

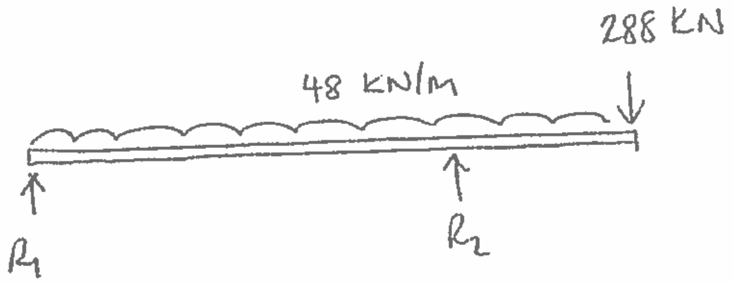
8. Answer: d. This is the case from lectures. This is predatory pricing. (a) and (c) are not anti-competitive practices per se.

9. Answer: b. This an example of manufacturer integrating backwards into upstream production.

10. Answer: c. This is a statement of the ownership theory of the firm. The other statements are incorrect. Small firms are not always at a cost disadvantage, more layers of management does not improve decision making and demand for air transport was not falling in the 1980s.

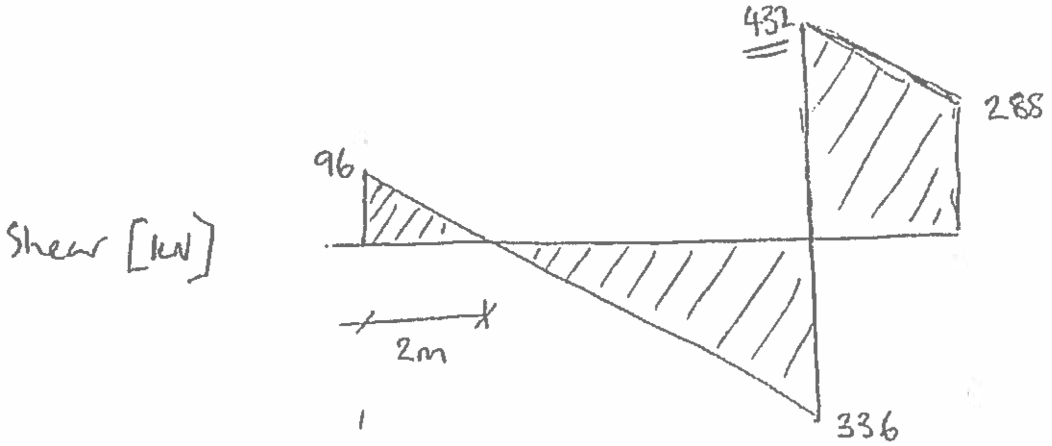
PART IB PAPER 8 SECTION B CIVIL + STRUCTURAL

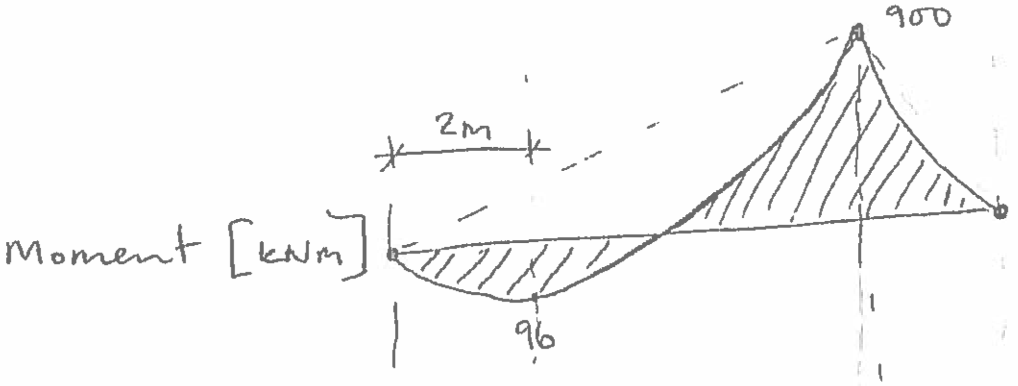
11 a]



$$R_2 = \left(48 \times 12 \times \frac{12}{2} + 288 \times 12\right)/9 = 768 \text{ KN}$$

$$R_1 = 48 \times 12 + 288 - 768 = 96 \text{ kN}$$





Max Hogging

$228 \times 3 + 48 \times 3 \times \frac{3}{2}$

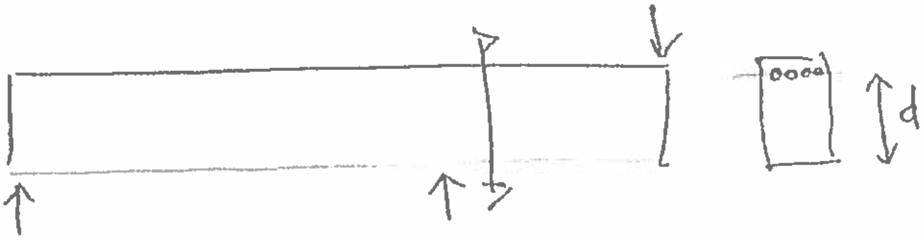
$= \underline{\underline{900 \text{ KNm}}}$

Max sagging

$96 \times 2 - 48 \times 2 \times \frac{2}{2} = \underline{96} \text{ KNm}$

b]

Maximum hogging is taken as 900 KNm at the right hand support from part a]



Section is 325 x 900 mm. Try 1% steel

$= 0.01 \times 325 \times 900 = 2925 \text{ mm}^2$

Choosing 4 x 32 mm diameter bars for $A_s = 3216 \text{ mm}^2$

Bars in one layer, check spacing:

$2 \times 40 + 4 \times 32 + 3 \times 32 = 304 < 325$ so ok for fit

Cover is 40 mm and $\phi/2 = 16$ mm

So call d = ~~[illegible]~~ 840 .

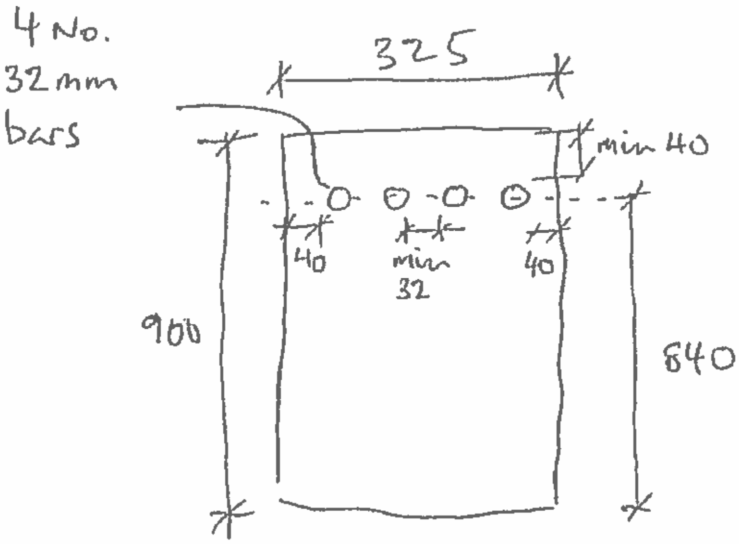
$f_{yd} = 500/1.15 = 435$ MPa

$$f_{cd} = \frac{0.85 \times 40}{1.5} = 22.67 \text{ MPa}$$

$$x = 1.67 \left(\frac{435}{22.67}\right)\left(\frac{3216}{325}\right) = 317 \text{ mm}$$

$$M = 435 \times 3216 \left(840 - \frac{317}{2}\right) = 953 \text{ KNm} \geqslant 900 \text{ KNm}$$

so ok ✓



Any reasonable and compliant arrangement is acceptable.

c]

$V = 432\,kN$ from part a]

check unreinforced:

$$k = 1 + \sqrt{200/840} = 1.49 \le 2.0$$

from part b]

$$\rho_L = 3216/(325 \times 840) = 0.012 \le 0.02$$

so

$$V_{Rd,c} = \frac{0.18}{1.5}\left(1.49\left(100 \times 0.012 \times 40\right)^{1/2}\right) 325 \times 840$$

$$= 177\,kN < 432\,kN$$

so shear reinforcement is required.

Check V_{max}:

$$V_{max} = \frac{0.4 \times 40\left(1 - (40/250)\right)}{2}\left(0.9 \times 325 \times 840\right)$$

$= 1651\,kN$ so V_{max} not exceeded

⇒ can design links

since $d = 840$ try $s = d/4 = 210$ mm

$$A_{sw} = \frac{432 \times 10^3 \times 1.15 \times 210}{435 \times 0.9 \times 840} = 317 \text{ mm}^2$$

try 12mm legs:

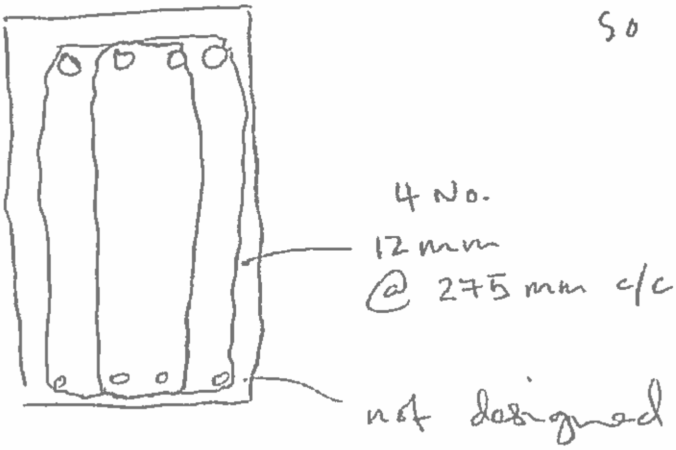
$3 \times 12\text{mm} = 3 \times 113 = 339 \text{ mm}^2$
$4 \times 12\text{mm} = 4 \times 113 = 452 \text{mm}^2$

with 4 longitudinal bars, 4 legs is tidy

so increase spacing for better use of material, say $s = 275$ mm

$$V_s = \frac{452 \times 435 \times 0.9 \times 840}{1.15 \times 275} = 470 \text{ kN} > 432 \text{ kN}$$

so ok ✓



d] Transient loads may not be present at all times. Where they provide a beneficial effect, they should ~~be~~ not be included. If the point load is reduced due to the absence of transient load, the sagging moment in the back span will increase! If this has not been designed for, the design may be unsafe.

Your reaction may depend on the design stage. If pre-construction, then the design can simply be checked and revised. If construction has begun or worse, the building is occupied then a chartered engineer in your firm must be advised immediately. If no action is taken by the firm, it may be necessary to report your concerns to the HSE ~~or~~ and/or via CROSS

12(a)

$$\gamma_d = \frac{G_s}{1+e}\gamma_w = \frac{2,76}{1.85}\times 10 = 14.9 \text{ kN/m}^3$$

$$\gamma_{sat} = \frac{G_s+e}{1+e}\gamma_w = \frac{2,76+0,85}{1.85}\times 10 = 19.5 \text{ kN/m}^3$$

[10%]

12.(b)

$$K_A = \frac{1-\sin\varphi'}{1+\sin\varphi'} = \frac{1-0,5}{1+0,5} = \frac{1}{3} = 0,\dot{3}$$

$$K_p = \frac{\cos\delta}{1-\sin\varphi'}\left[\cos\delta + \sqrt{\sin^2\varphi - \sin^2\delta}\right]e^{2\theta\,\mathrm{tg}\,\varphi'}$$

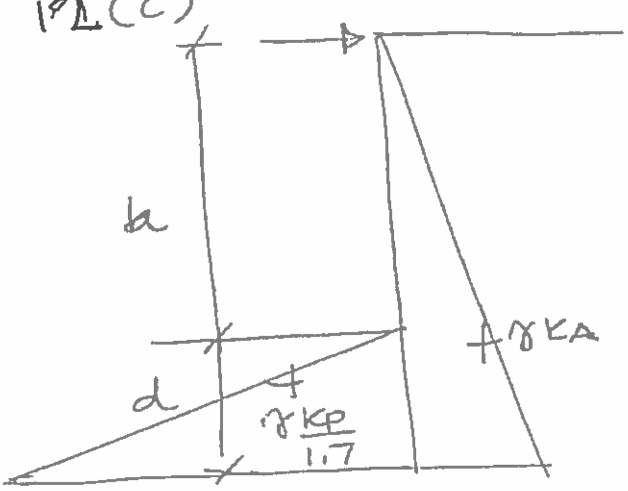
$$2\theta = \sin^{-1}\left(\frac{\sin\delta}{\sin\varphi}\right) + \delta = 20.32^\circ = 0,529$$

$$K_p = \frac{\cos 10^\circ}{0,5}\left[\cos 10^\circ + \sqrt{\sin^2 30^\circ - \sin^2 10^\circ}\right]e^{0,529\,\mathrm{tg}\,30^\circ} =$$

$$= 3.886$$

[10%]

12(c)



$\varphi = 30^\circ$

$\delta = 10^\circ$

$\gamma_d = 14.9 \text{ kN/m}^3$

$K_A = \frac{1}{3}$

$K_p = 3,886$

$$F = \frac{K_p}{K_{mob}} = 1,7$$

$$K_{mob} = \frac{3,866}{1,7} = 2,274$$

12.(c) continued...

take moments about top of he wall:

driving moment $M_D = \frac{1}{3}\gamma K_A (h+d)^3 = 1.66(6+d)^3$

restoring moment $M_R = \frac{1}{2}\gamma K_{mob} d^2 (\frac{2}{3}d + h) =$

$= 17.03\, d^2 (6 + \frac{2}{3}d)$

find d: $\Delta M = M_R - M_D = 0$ (or very slightly positive)

d (m)	M_R (kNm/m)	M_D (kNm/m)	ΔM (kNm/m)
2	499.5	849.9	−350.4
3	1226.2	1210.1	16.1
2.9	1136.2	1170.2	−34
2.97	1198.8	1198.1	0.7 ✓

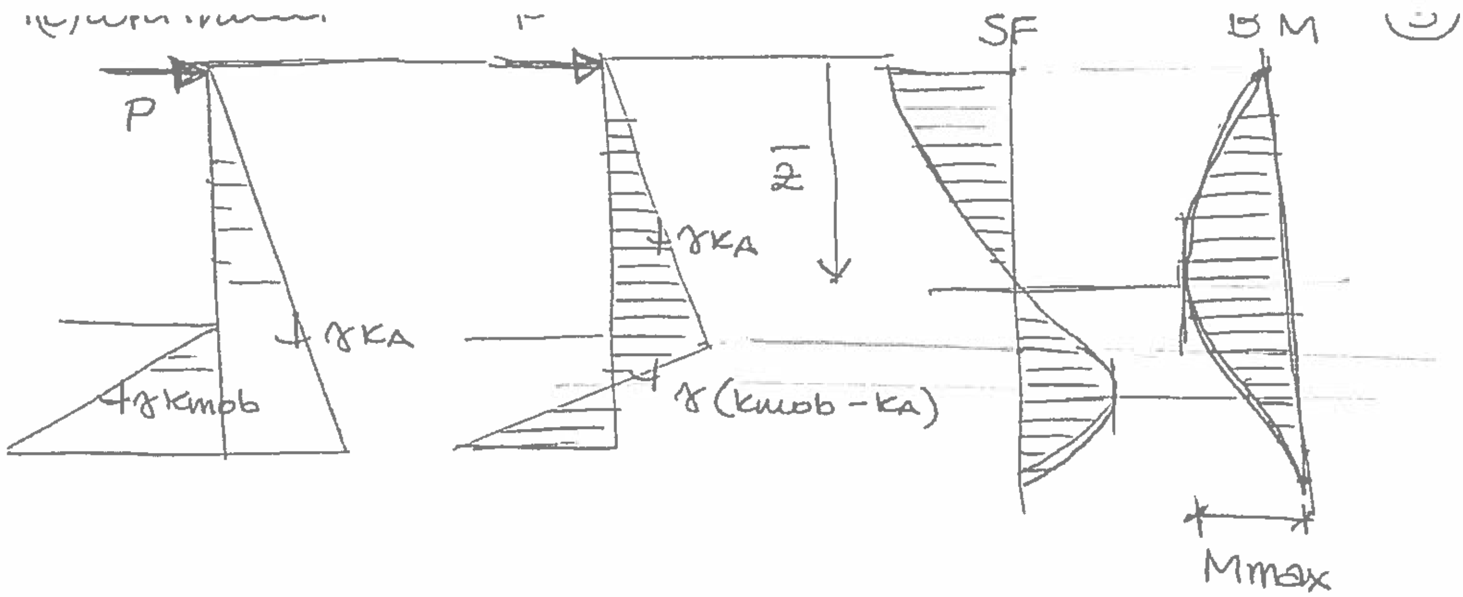
prop force from equilibrium:

$$P = \frac{1}{2}\gamma K_A (h+d)^2 - \frac{1}{2}\gamma K_{mob} d^2 =$$

$$\frac{1}{2}\gamma \left[K_A (h+d)^2 - K_{mob} d^2\right] =$$

$$\frac{14.9}{2}\left[\frac{1}{3}(6+2.97)^2 - 2.274 \cdot 2.97^2\right] =$$

$$\simeq 50.4 \text{ kN/m}$$



Bending moment is maximum where shear force is SF=0. For z<h:

$$SF(z) = P - \frac{1}{2}\gamma K_A z^2 = 50.4 - \frac{14.9}{2}\cdot\frac{1}{3}z^2$$

$$SF = 0 \qquad \bar{z} = \sqrt{\frac{50.4\times 6}{14.9}} \cong 4.5\,m$$

$$M(\bar{z}) = 50.4\times 4.5 - \frac{1}{2}\frac{\gamma K_A}{3}(4.5)^3 =$$

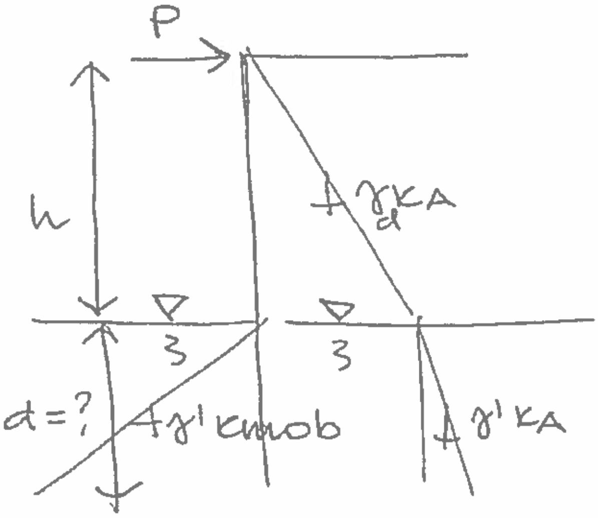
$$50.4\times 4.5 - \frac{14.9}{2}\cdot\frac{1}{9}\cdot 4.5^3 =$$

$$\cong 151 \text{ kNm/m}$$

[40%]

12 (d)

(4)



$\gamma = \gamma_d = 14.9 \text{ kN/m}^3$

$\gamma' = \gamma_{sat} - \gamma_w = 19.5 - 10 = 9.5 \text{ kN/m}^3$

$K_{mob} = \frac{3.866}{1.7} = 2.274$

moments about top of wall:

$$M_D = \frac{1}{3}\gamma_d K_A h^3 + \gamma_d K_A h d\left(h + \frac{d}{2}\right) + \frac{1}{2}\gamma' K_A d^2\left(h + \frac{2}{3}d\right) =$$

$$= 357.60 + 29.8d\left(6 + \frac{d}{2}\right) + 1.58d^2\left(6 + \frac{2}{3}d\right)$$

$$M_R = \frac{1}{2}\gamma' K_{mob} d^2\left(h + \frac{2}{3}d\right) = 10.8d^2\left(6 + \frac{2}{3}d\right)$$

d	M_R	M_D	$\Delta M = M_R - M_D$
3	1141.86	777.6	−364.26
4	1530.29	1497.60	−32.69
4.1	1585.5	1573.1	+12.4 ✓

find P from equilibrium of forces:

$$P = \frac{1}{2}\gamma_d K_A h^2 + \gamma_d K_A h d + \frac{1}{2}\gamma' K_A d^2 - \frac{1}{2}\gamma' K_{mob} d^2 =$$

$$\frac{14.9}{3\times2}\times6^2 + \frac{14.9}{3}\times6\times4.1 + \frac{9.5}{2}\times\frac{1}{3}\times4.1^2 - 9.5\times2.274\times4.1^2 =$$

$$= 56.6 \text{ kN/m}$$

1(a) continued

find $\bar{z}$ where shear force is $= 0$

$$SF(z) = 56.6 - \frac{14.9}{2} \times \frac{1}{3} \bar{z}^2$$

$$\bar{z} = \sqrt{\frac{56.6 \times 6}{14.9}} = 4.77 \text{ m}$$

$$M(\bar{z}) = M_{max} = 56.6 \times 4.77 - \frac{14.9}{2} \times \frac{1}{9} \times 4.77^3 =$$

$$\simeq 180 \text{ kNm/m}$$

13.

(a)

$$\gamma = \frac{G_s + e}{1+e}\gamma_w = \frac{2,75+1,3}{2,3} \times 10 = 17.6 \text{ kN/m}^3$$

$$w = \frac{W_w}{W_s} = \frac{\gamma_w e V_s}{G_s \gamma_w V_s} = \frac{e}{G_s} = \frac{1,3}{2,75} = 47.27\,\%$$

[5%]

(b)

$$\sigma_v = \gamma z = 17.6 \times 15 = 264 \text{ kPa}$$

$$u = \gamma_w z_w = 10 \times 14 = 140 \text{ kPa}$$

$$\sigma_v' = \sigma_v - u = 124 \text{ kPa}$$

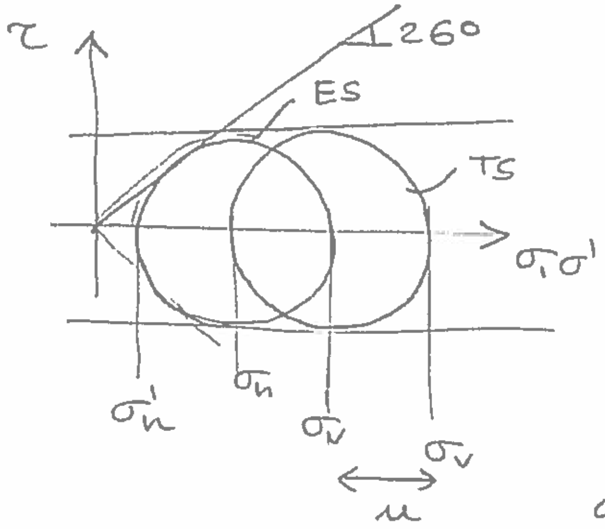
$$\sigma_h' = k_o \sigma_v' = (1 - \sin\varphi)\sigma_v' = 0,56 \times 124 = 69,6 \text{ kPa}$$

$$\sigma_h = \sigma_h' + u = 69.6 + 140 = 209.6 \text{ kPa}$$

[10%]

(c) $\sigma_v = 264$ kPa

$$\sigma_h = \sigma_v - 2s_u = 264 - 120 = 144 \text{ kPa}$$



@ failure effective stress circle is tangent to effective stress envelope

$$|\tau| = \sigma \, tg\,\varphi$$

and has same diameter as total stress circle.

hence

$$\frac{\sigma_v' - \sigma_h'}{2} = s_u \qquad \frac{\sigma_v' + \sigma_h'}{\cancel{2}} = \frac{\sigma_v' - \sigma_h'}{\cancel{2}\sin\varphi'}$$

$$\Downarrow$$

$$\sigma_v' = \sigma_h' = 2s_u \qquad \sigma_v' + \sigma_v' - 2s_u = \frac{2s_u}{\sin\varphi'}$$

$$\Downarrow$$

$$\sigma_h' = \sigma_v' - 2s_u \qquad \cancel{2}\sigma_v' = \cancel{2}s_u + \cancel{2}\frac{s_u}{\sin\varphi'}$$

(c) continued

$$\sigma'_v = 60 + \frac{60}{\sin 26^\circ} = 196.9 \text{ kPa}$$

$$u = \sigma_v - \sigma'_v = 264 - 196.9 = 67.1 \text{ kPa}$$

$$u = u_0 + \Delta u$$

$$\Delta u = u - u_0 = 67.1 - 140 = -72.9 \text{ kPa}$$

[25%]

(d) $\Delta u = 0 \quad u = u_0$

$$\sigma'_v = 124 \text{ kPa}$$

$$\sigma'_h = K_A \sigma'_v = \frac{1-\sin\varphi}{1+\sin\varphi} \cdot \sigma'_v = 0.39 \times 124 = 48.4 \text{ k}$$

$$\sigma_h = \sigma'_h + u = 48.4 + 140 = 188.4 \text{ kPa}$$

[5%]

(e)

i) The stability ratio is defined as:

$$N = \frac{\sigma_v - \sigma_T}{S_u}$$

where: σ_v is the total vertical stress at the tunnel axis ($\sigma_v = \gamma z$)

σ_T is the support pressure applied to the tunnel face

S_u is the undrained shear strength of the soil

For open face tunnelling $\sigma_T = 0$

If $N > 5$ the tunnel is unlikely to be stable and will need face support

[10%]

ii) At 20 m depth, the stability ratio would be:

$$N = \frac{17.6 \times 20}{60} = \frac{352}{60} = 5.87$$

This is quite high. If N>5 it is impossible to excavate the tunnel by open shield, because the tunnel face would not be stable. Mechanised tunnelling by closed face shields (either slurry shield or eart pressure balance shield) would be necessary to provide continuous support to the face during excavation

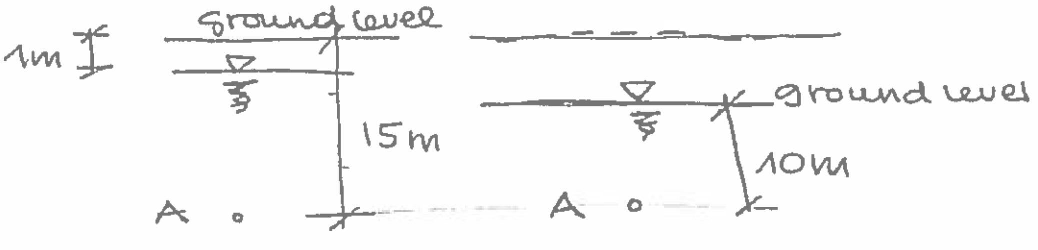
[10%]

iii) Tunnelling induced ground movements transmit to existing buildings as settlements, rotations, and distortions of their foundations. These can induce damage affecting appereance and aesthetics, serviceability or function, and, in extreme cases, stability. Masonry buildings are particularly affected by differential settlements that can cause tensile strains in the masonry, leading to cracking. Compensation grouting is a mitigation measure where horizontal steel tubes are inserted in the ground at an intermediate level between the tunnel crown and the foundations of the building, and cement grout is injected from the tubes to compensate for settlements caused by tunnelling. Instrumentation and monitoring of ground and building response are crucial to inform &

decisions about where and when to grout to prevent or limit damaging differential settlements.

[10%]

13 (f)



before erosion

after erosion

before erosion :

$\sigma'_v = \sigma'_{v\,max} = 124$ kPa from Q. 2(b)

after erosion

$\sigma_v = \gamma z = 17.6 \times 10 = 176$ kPa

$u = \gamma_w \times z = 10 \times 10 = 100$ kPa

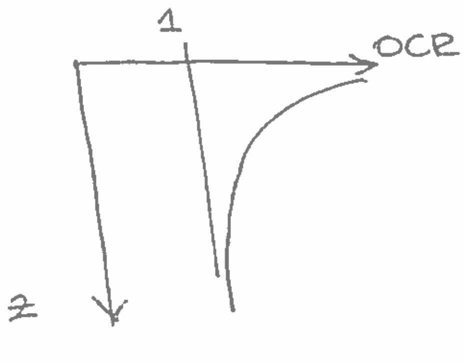
$\sigma'_v = \sigma_v - u = 76$ kPa

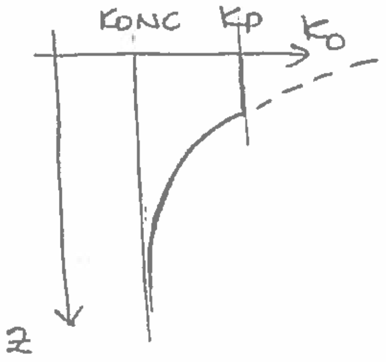
$OCR = \frac{\sigma'_{v\,max}}{\sigma'_v} = \frac{124}{76} = 1.63$

$K_{o\,oc} = K_{o\,NC} \times OCR^{0.5} = (1 - \sin\varphi') \times \sqrt{1.63} = 0.72$

$\sigma'_h = K_{o\,oc} \times \sigma'_v = 0.72 \times 76 = 54.5$ kPa

$\sigma_h = \sigma'_h + u = 54.5 + 100 = 154.5$ kPa





[25%]

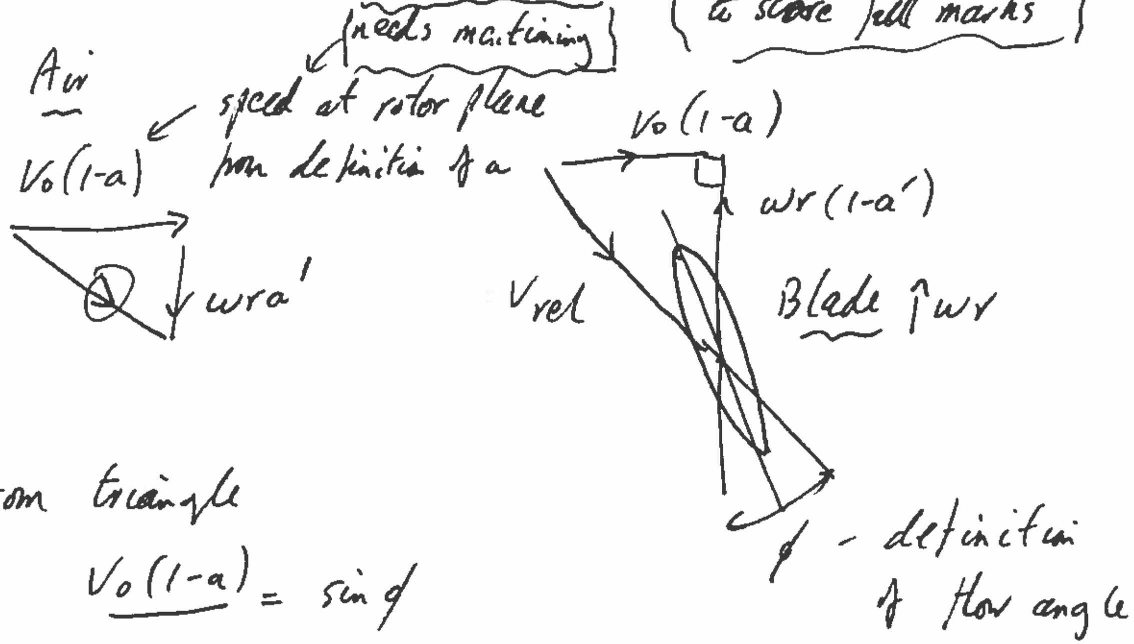
Engineering Tripos Part IB
Paper 8, Selected Topics, Section C   MECHANICAL
Crib 2018/19 (Michael Sutcliffe)

16 (a) Aerodynamics - tip speed increases towards edge so need to reduce twist to maintain optimum tip speed ratio.

Structural - forces at end of blade generate more torque but also higher bending moments causing high stress & tip deflection. Depending on material / geometry the trade-off between power, weight, cost will dictate the optimum taper.

Vibration - mass at the tip will disproportionately affect low frequency vibration. {Both elements needed to score full marks}

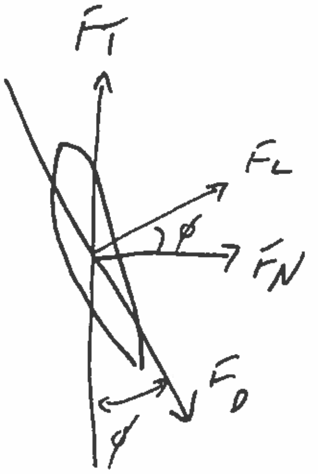
(b)



From triangle

$$\frac{V_0(1-a)}{V_{rel}} = \sin\phi$$

14 (c)(i)



$$F_N = F_L\cos\phi + F_D\sin\phi$$

$$F_T = F_L\sin\phi - F_D\cos\phi$$

But $C = \frac{F}{\underbrace{\frac{1}{2}\rho V_{rel}^2 c}} = \frac{F}{967.8\ \mathrm{kg\,m^{-3}\,m^{2}\,s^{-2}\,m}}$

This elements sometimes wrong

$$V_{rel} = \left(\frac{\sin 13.2^\circ}{10(1-0.180)}\right)^{-1} = 35.91\,\mathrm{ms^{-1}}$$

$$\Rightarrow F_N = 967.8\,(C_L\cos\phi + C_D\sin\phi) = 832\ \mathrm{Nm^{-1}}$$

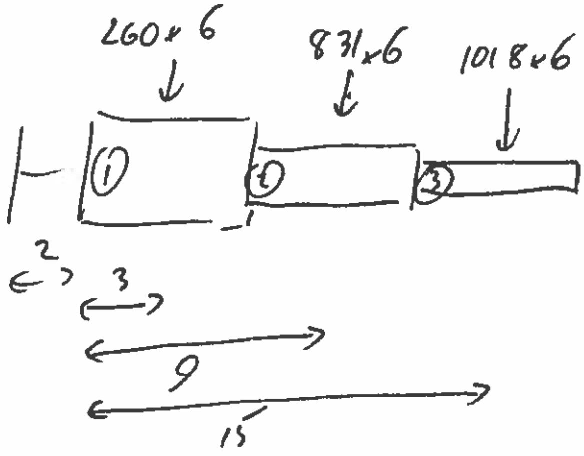
$$F_T = 967.8\,(C_L\sin\phi - C_D\cos\phi) = 188\ \mathrm{Nm^{-1}}$$ (To within rounding errors)

{Not a good exam tactic to jump to the answer without correct working}

(ii) Power generated $= \overset{\text{no. of blades}}{3}\,\omega \sum F_T\, l\, \bar{r}$

$$= 30 \times \frac{30 \times 2\pi}{60}\begin{bmatrix} 139 \times 6 \times 5 \\ +188 \times 6 \times 11 \\ +155 \times 6 \times 17 \end{bmatrix} = 305\,\mathrm{kW}$$

1c (d)



Neglecting twist – only considering flapwise bending (though candidates who included edgewise terms were credited)

$BM = \Sigma F \times r$

$I = \frac{1}{12} b \left((d+2t)^3 - d^3 \right)$ (or use approximation $\frac{1}{2} b^2 bt$)

$\sigma = \frac{My}{I}$ where $y = \frac{d}{2} + t$

Maximum BM at hub (NB not at r=0) but need to consider all three sections:

	①	②	③
BM	$(260 \times 3 + 831 \times 9 + 1018 \times 15) \times 6$ $= 141$ kNm	69.9 kNm	18.3 kNm
I	$\frac{1}{12} \cdot 0.5 (0.24^3 - 0.2^3)$ $= 2.43 \times 10^{-4} m^4$	$4.49 \times 10^{-5} m^4$	$2.82 \times 10^{-6} m^4$
y	0.12 m	0.085 m	0.04 m
σ_{max}	$141 \times 10^3 \times 0.12 / 2.43 \times 10^{-4}$ $= 69.7$ MPa	132 MPa	260 MPa

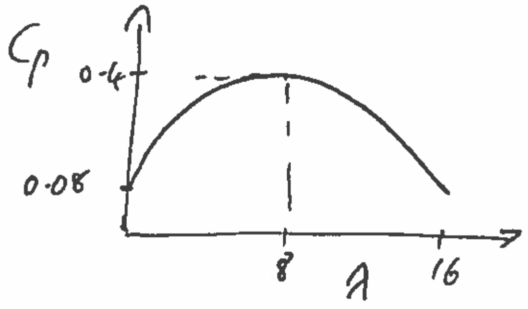
Critical stress.

Many candidates assumed max σ was at root. But question mostly done well.

15 (a) $C_p = -0.005\lambda^2 + 0.08\lambda + 0.08$

$$\frac{dC_p}{d\lambda} = -0.01\lambda + 0.08 \Rightarrow \lambda_{opt} = 8$$

$$C_{p\,max} = -0.005.8^2 + 0.08\times8 + 0.08 = 0.4$$



Since $\lambda = \frac{\omega R}{v}$, to keep $C_p = C_{p\,max}$ we require λ to be fixed at λ_{opt} $\Rightarrow \frac{\omega}{v} = $ constant $\Rightarrow \omega$ should be proportional to wind speed.

(b) (i) $P = \frac{1}{2}C_p \rho A v^3 = \frac{1}{2}.0.4.1.23.40^2.7^3 = 424$ kW

$\lambda = 8 = \omega\times\frac{40}{7} \Rightarrow \omega = 1.4$ rad/s (13.4 rpm)

$T\omega = P \Rightarrow T = 424\times10^3/1.4 = 303$ kNm

(ii) $\omega_g = \frac{2\pi\times50}{9} = 34.9$ rad/s $\Rightarrow N_g = 25$

(c) $T_g = T_c/_{25} = -12.1$ kNm $\approx 3V^2s/_{R_2'\omega_s}$

$$= 3\times\left(\frac{3300}{\sqrt{3}}\right)^2 s/_{0.5\times34.9} = s = -0.0194$$

$$\bar{\bar{I}} = \frac{V_{ph}}{\left(R_1 + \frac{R_2'}{s}\right) + j\left(X_1 + X_2'\right)} = \frac{3300/\sqrt{3}}{0.6 + \frac{0.5}{-0.0194} + j\,4.4}$$

$$= \frac{1905}{25.6\angle{+170^\circ}} = 74.4\text{ A}\angle{-170^\circ}$$

$\cos\phi = \cos(-170^\circ) = 0.985$ lagging

(a) and (b) done well, (c) less so. Common errors: use turbine torque to determine s, forgetting that s is negative

15 (d) (i) The Triple Bottom Line refers to companies, governments or other organisations assessing their operations not only in relation to the usual *financial* performance, but also in terms of their environmental performance and social/ethical performance.

The associated Capitals are:

- Natural capital (planet): atmosphere, land, fresh water, oceans, bio-sphere, material and energy resources
- Manufactured and financial capital (prosperity): built environment, industrial capacity, financial health, GDP
- Human and social capital (people): education, health, skills, knowledge, happiness

(ii) Potential positive consequences of expansion in wind power capacity [any 3 required]:

Natural capital:
- favourable energy payback
- reduce carbon emissions to the environment

Manufactured capital:
- stronger national energy industry
- new jobs, replacing declining sectors

Human and social capital:
- greater energy security
- lower dependence on imported fossil fuels
- contribution to commitments to reduce carbon emissions, supported by some of the public

Potential negative consequences of expansion in wind power capacity [any 3 required]:

Natural capital:
- large demand for some critical materials (e.g. rare earths)
- occupy large land area

Manufactured capital:
- dependence on a few powerful nations for critical materials
- new infrastructure needed to manage mixed power sources on the grid
- currently more expensive than fossil fuel

Human and social capital:
- visual and acoustic intrusion
- subsidies increase household bills
- strongly opposed by some of the public

(iii) Possible consequences to quantify, and suitable metrics:

(1) Energy payback: quantify the time for output energy to match embodied energy to build and install the wind farm

Associated data:
- bill of materials (mass used) in construction of wind turbines, including power distribution infrastructure to grid
- embodied energy (J/kg) in each material used
- manufacturing energy associated with each component
- transport energy to bring materials to site of manufacture, and turbines to site of wind farm
- energy associated with maintenance over lifetime (usually very small)
- on output side: nominal power rating, and typical capacity factor (fraction of time operating a nominal power)(2) Critical materials consumption (e.g. rare earth metals): proportion of world production of key rare earth metals associated with wind turbines, both currently and in relation to projected growth rates in use of these elements

Associated data:
- quantity of rare earth metal consumed per turbine (effectively per MW installed)
- current and future rates of installation of new turbines and their power ratings
- annual production figures for the critical materials, with breakdown by country

A significant minority of candidates were not able to define the Triple Bottom Line and associated capitals. (d) (ii) was well done but answers to (d)(iii) tended to be vague, lacking specific metrics and associated data.

16(a) (i) Different loads in different areas, shell needs to cope with bending, shear, compression. So put strength + stiffness in a mix of directions. Also lightweight, impact resistant, easy to mould.

(ii) Much depends on size with hand lay-up or RTM feasible for smaller blades but automated open mould processes are needed for larger parts. Effect of cost and production rate important.

(iii) Uni-directional material gives best strength and stiffness along spar, carrying the predominant and predictable bending loads. For large blades stiffness becomes important so CFRP, though more expensive, wins out over CFRP.

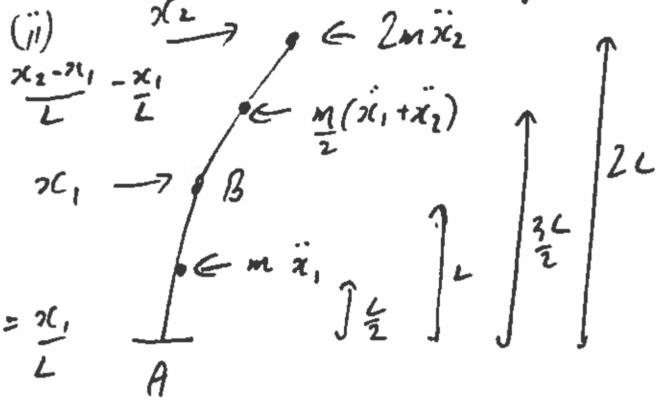
(iv) Cost always important but weight of tower is not as critical as moving blades or tower head components. Still steel has reasonable/good properties, is cheap and easy to manufacture to the required shape.

Mostly good comments, but marks dropped by omitting key elements

16 (b) (i) Tower head mass associated with blades, gearbox, nacelle can be important compared with tower mass and significantly affects the vibration response.

(ii)

$\phi_B = \frac{x_2 - x_1}{L} - \frac{x_1}{L}$



$\phi_A = \frac{x_1}{L}$

Some candidates used Lagrangian mechanics but tended to get the wrong answer. Inclusion of rotational inertia terms not expected. Many good answers - method generally good.

M_B on top section ↺ $\frac{K}{L}(x_2 - 2x_1) + \frac{L}{2}m\left(\frac{\ddot{x}_2+\ddot{x}_1}{2}\right) + 2mL\ddot{x}_2 = 0$

M_A ↺ $\frac{2kx_1}{L} + \frac{mL}{4}\ddot{x}_1 + m\frac{3L}{2}\left(\frac{\ddot{x}_2+\ddot{x}_1}{2}\right) + 2m\,2L\,\ddot{x}_2 = 0$

$$\frac{k}{L^2}\begin{pmatrix}-2 & 1\\ 2 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\end{pmatrix} + \frac{m}{4}\begin{pmatrix}1 & 9\\ 4 & 19\end{pmatrix}\begin{pmatrix}\ddot{x}_1\\ \ddot{x}_2\end{pmatrix} = 0$$

One candidate remarked, correctly, that it is easier in Matlab

$[M]^{-1}[K][x] = \omega^2[x]$ $m^{-1}K = \frac{\cancel{4m}k}{mL^2}\cdot\frac{4}{17}\begin{pmatrix}19 & -9\\ -4 & 1\end{pmatrix}\begin{pmatrix}2 & 1\\ 2 & 0\end{pmatrix} = \frac{k}{mL^2}\begin{pmatrix}13.177 & -4.471\\ 2.353 & 0.941\end{pmatrix}$

$$\begin{vmatrix}13.177-\lambda & -4.471\\ 2.353 & 0.941-\lambda\end{vmatrix} = 0 \Rightarrow \lambda^2 - 14.118\lambda + 1.879 = 0$$

$$\lambda = 7.059 \pm 6.925\ \frac{k}{mL^2}$$

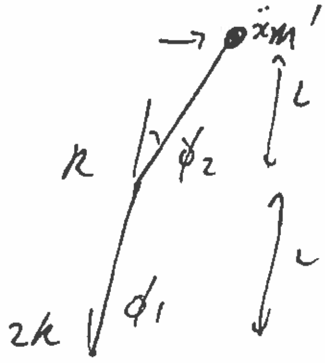
Lowest natural freq: $\omega^2 = 0.134\frac{k}{mL^2}$

$$\omega = \frac{0.366}{L}\sqrt{\frac{k}{m}}$$

$$f = \frac{\omega}{2\pi} = \frac{0.058}{L}\sqrt{\frac{k}{m}}$$

16 (b)(i)

Or <u>Estimate</u>



from notes

$\ddot{x}m' = (2m + 0{\cdot}26 \times 2m)\ddot{x}$

All mess at top
0·26 × 2m is effective
mess of tower.

BACK OF ENVECOPE SOLUTION

see below

$$\phi_1 = \frac{2Lm'\ddot{x}}{2k}$$

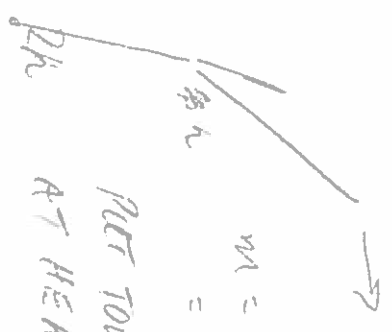
$$\phi_2 = \frac{m'L\ddot{x}}{k}$$

$$x = 2L\phi_1 + L\phi_2$$

$$= \frac{3m'L^2\ddot{x}}{k}$$

$$\omega = \sqrt{\frac{k}{m'}}\frac{1}{L\sqrt{3}} = \frac{1}{L}\sqrt{\frac{k}{m}}\frac{1}{\sqrt{3\times2{\cdot}48}} = \frac{0{\cdot}367}{L}\sqrt{\frac{k}{m}}$$

BACK OF ENVECOPE SOLN.



FROM NOTES

$m = 2m + 2m \times 0{\cdot}26$

$= 2{\cdot}48m'$

PUT TOWER MASS AT HEAD

$$\phi_1 = \frac{2m\ddot{x}L}{2k}$$

$$\phi_2 = m L\ddot{x}/k$$

$$x = 2L\phi_1 + L\phi_2$$

$$= \frac{2mL^2\ddot{x}}{k} + \frac{mL^2}{2k}\ddot{x}$$

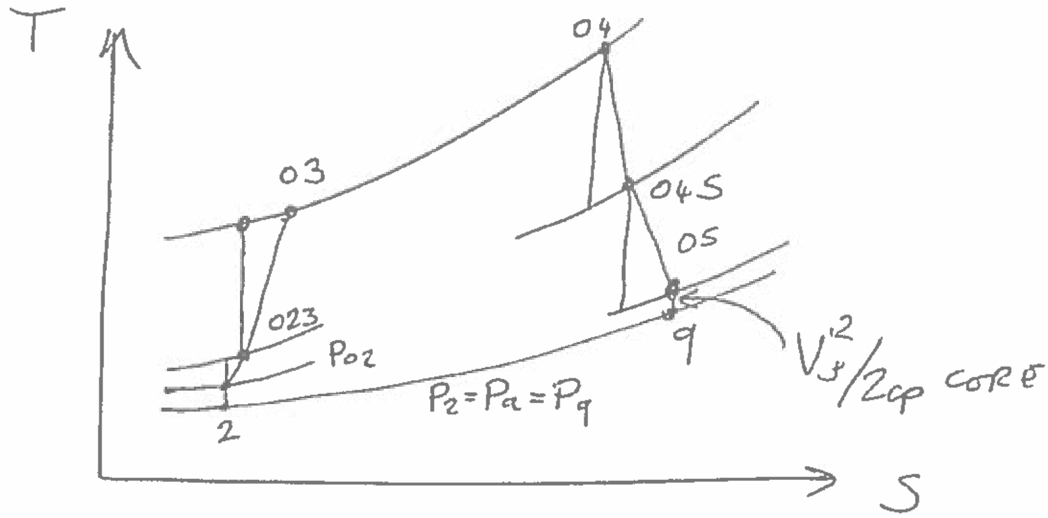
$$\omega = \sqrt{\frac{k}{m}}\frac{1}{L}\sqrt{\frac{1}{3}} = \sqrt{\frac{k}{m}}\frac{1}{L}\sqrt{\frac{1}{2{\cdot}48\times3}}$$

$$= 0{\cdot}367\frac{1}{L}\sqrt{\frac{k}{m}}$$

SECTION D ROB MILLER AERO THERMAL

(17)

(a)



IMPROVING EFFICIENCY IMPROVES/INCREASES THE SPECIFIC WORK OUTPUT AVAILABLE FROM LP TURBINE $(T_{045} - T_{05})$ I.E. POWER TO FAN RISES. THE JET VELOCITY IS FIXED SO THE FAN TEMPERATURE RATIO AND PRESSURE RATIO ARE FIXED. HENCE ONLY WAY OF RAISING FAN POWER IS TO RAISE FAN MASS FLOW. THIS MEANS FAN BYPASS RATIO AND FAN DIAMETER WILL RISE.

THE SFC OF THE ENGINE WILL IMPROVE (DROP). THIS IS CAUSED BY THE CORE THERMAL EFFICIENCY RISING. PROPULSIVE EFFICIENCY CONSTANT DUE T FIXED V_j.

THE NET THRUST WILL RISE BECAUSE $\dot{m}$ FAN IS RISING.

(b)

$$R_{ISC} = 0.9 = \frac{\left(\frac{P_{03}}{P_{023}}\right)^{\frac{\gamma-1}{\gamma}} - 1}{\frac{T_{03}}{T_{023}} - 1} \Rightarrow$$

$$\frac{T_{03}}{T_{02}} = \frac{(25)^{\frac{0.4}{1.4}}}{0.9} + 1 = \underline{\underline{2.68}}$$

$$T_{03} = \underline{\underline{7761k}}$$

ⓑ CONT

⑰

$$\psi = \frac{\Delta h_o}{U^2} \qquad U^2 = \frac{\Delta h_o}{\psi} = \frac{C_p(T_{03}-T_{023})}{N_{STAGE} \times \psi}$$

$$= \frac{1005 \times (776.1-290)}{8 \times 0.4}$$

$$= 1.5267 \times 10^5$$

$$U = 390.7\,ms^{-1}$$

$$r\Omega = U = 390.7$$

$$\Omega = \frac{U}{r} = \frac{390.7}{0.3} = 1.3023 \times 10^3$$

$$N = \frac{\Omega \times 60}{2\pi} =$$

$$\underline{\underline{12{,}440\,RPM}}$$

ⓒ
$$\frac{\dot{m}\sqrt{C_p T_{023}}}{A\,P_{023}} = \frac{\gamma}{\sqrt{\gamma-1}} M \left(1+\frac{\gamma-1}{2}M^2\right)^{-\left(\frac{\gamma+1}{2(\gamma-1)}\right)}$$

$$M = 0.7$$

$$\frac{\dot{m}\sqrt{C_p T_{023}}}{A\,P_{023}} = \underline{\underline{1.1705}}$$

$$A = \frac{40 \times \sqrt{1005 \times 290}}{1.1705 \times 70 \times 10^3} = 0.2636$$

$$A = \pi\left((r_m+\Delta R)^2 - (r_m-\Delta R)^2\right)$$

$$A = \pi\, 4 r_m \Delta R$$

70mm ↙

ATV $2\Delta R$
$$\Delta R = \frac{0.2636}{\pi \times 4 \times 0.2} = \underline{0.07}$$

SPAN = 0.14

⑰ⓓ $C_p(T_{03} - T_{023}) = C_p(T_{04} - T_{045})$

COMP HP TURB

$T_{045} = T_{04} - (T_{03} - T_{023})$

$= 1900 - (776.1 - 290)$

$= \underline{\underline{1414k}}$

$U = 390.7\ ms^{-1}$ FROM PART ⓑ

$\psi = \frac{C_p\ (1900-1414)}{390^2 \times N_{STAGE}}$

$N_{STAGE} = \underline{\underline{1.6 \text{ STAGES}}}$

THE HP TURBINE IS COOLED BECAUSE OF HIGH TET, THIS MEANS THAT LOADING MAY BE LIMITED BECAUSE COOLING FLOWS LIMIT BOUNDARY LAYER DEVELOPMENT

<u>BEST TO GO FOR 2 STAGES</u>

⑱ ⓐ HEIGHT 10,000 m

$$\frac{P}{P_{SL}} = 0.2615 \qquad \frac{T}{T_{SL}} = 0.7748$$

$$P_{SL} = 1.01325\,\text{bar} \qquad T_{SL} = 288.15\,K$$

$$\underline{\underline{P = 0.265\,\text{bar}}} \qquad \underline{\underline{T = 223.26\,K}}$$

$$P_0 = P\left(\frac{T_0}{T}\right)^{\gamma/\gamma-1} \qquad T_0 = T\left(1+\frac{\gamma-1}{2}M^2\right)$$

$$= 223.26\left(1+\frac{0.4}{2}0.8^2\right)$$

$$= \underline{\underline{251.8\,K}}$$

$$P_0 = 0.265\left(\frac{251.8}{223.3}\right)^{\gamma/\gamma-1}$$

$$P_0 = \underline{\underline{0.4\,\text{bar}}}$$

TEMP THE FLOW IS ASSUMED TO BE BROUGHT TO REST ADIABATICALLY WITH ZERO WORK AND AT CONSTANT ALTITUDE.

PRESS ALL OF THE ABOVE BUT AN EXTRA CONDITION OF REVERSIBILITY IS REQUIRED.

$$V = \cancel{a}\,M\sqrt{\gamma R T_a} = 0.8\times\sqrt{1.4\times223.26\times287}$$

$$= \underline{\underline{239.6\,ms^{-1}}}$$

ⓑ

$$F_G = \dot{m}V_j = 600\times380 = 228\,kN$$

$$F_N = \dot{m}(V_j - V) = 600\times(380-239.6)$$

$$= \underline{\underline{84.2\,kN}}$$

⑱ ⓒ APPLYING CONSERVATION OF MOMENTUM BETWEEN THE PROPELLING NOZZLE AND FAR DOWNSTREAM

$$\dot{m}_a (V_j - V_{PN}) = A_N (P_{PN} - P_A)$$

HENCE $\underline{\dot{m}_a V_{PN} + A_N P_{PN}} =$

$$\dot{m}_a V_j + A_N P_a = \underline{\underline{F_G + P_a A_N}}$$

$F_G + P_a A_N$ THEREFORE ONLY DEPENDS ON CONDITIONS INSIDE THE ENGINE $\underline{\underline{m_A\ V_{PN}\ P_{PN}}}$ PROVIDED THE PROPULSION NOZZLE IS CHOKED. THE CHOKED CONDITION STOPS THE ENGINE BEING INFLUENCE BY THE DOWNSTREAM CONDITIONS

$$\tilde{\dot{m}} = \frac{600 \times \sqrt{1005 \times 251.8}}{0.4 \times 10^5 \times 3} = \underline{\underline{2.52}}$$

$$\tilde{F} = \frac{228\overset{\times 10^3}{4} + \overset{0.265 \times 10^5}{\cancel{0.640}} \times 3}{0.4 \times 10^5 \times 3} = \qquad 2.5625$$

(18) (d)

$$F_N = F_G - \dot{m}V$$

$$\frac{F_N}{P_a A_N} = \frac{F_G}{P_a A_N} - \frac{\dot{m}V}{P_a A_N}$$

$$= \frac{\tilde{F} P_0 A_N - P_a A_N}{P_a A_N} - \frac{\tilde{\dot{m}} P_0 A_N}{\sqrt{C_p T_0}\, P_a A_N} V$$

$$= \hat{F}\frac{P_0}{P_a} - 1 - \tilde{\dot{m}}\frac{P_0}{P_a}\frac{1}{\sqrt{C_p T_0}} V$$

$V = M\sqrt{\gamma R T_a}$

$$= \left(\hat{F}\frac{P_0}{P_a} - 1\right) - \hat{\dot{m}}\frac{P_0}{P_a}\sqrt{\frac{R\gamma}{C_p}}\sqrt{\frac{T_a}{T_0}}\, M$$

$$\frac{F_N}{P_a A_N} = \left(\hat{F}\frac{P_0}{P_a} - 1\right) - \dot{m}\frac{P_0}{P_a}\sqrt{\frac{T_a}{T_0}}\sqrt{\gamma - 1}\, M$$

$R = C_p - C_v$

$$\frac{R}{C_p} = \frac{C_p - C_v}{C_p} = 1 - \frac{1}{\gamma}$$

(e) 16000m

$$\frac{T}{T_{SL}} = 0.7519 \qquad \frac{P}{P_{SL}} = 0.1398$$

$$T_{New} = 216.6K \qquad P_{New} = 0.1417 bar$$

$$T_{0New} = \underline{\underline{244.3K}} \qquad P_{0New} = \underline{\underline{0.216 bar}}$$

$$\dot{m}_{New} = \frac{\tilde{\dot{m}} P_{0New} A_N}{\sqrt{C_p T_{0New}}} = \frac{2.52 \times 0.216 \times 10^5 \times 3}{\sqrt{1005 \times 244.3}}$$

$$= \underline{\underline{329.6 kg s^{-1}}}$$

⑱ ⓔ cont

$$\frac{F_N}{P_a A_N} = \left(2.5625 \times \frac{0.216}{0.1417} - 1\right)$$

$$- 0.8 \times 2.52 \times \frac{0.216}{0.1417}\sqrt{\frac{216.6}{2443}} \times \sqrt{0.4}$$

$$= 1.0761$$

$$F_N = 0.1417 \times 10^5 \times 3 \times 1.0761$$

$$= \underline{\underline{45.7\ kN}}$$

⑲ ⓐ THERE IS AN OPTIMUM C_L FOR MINIMISING FUEL BURN AT CRUISE.

$$C_L = 0.5$$

REARRANGING GIVES

$$V_{CRUISE} = \sqrt{\frac{W_{CRUISE}}{0.5\,\rho_a C_L}}$$

CRUISING AT HIGH ALTITUDE LOWERS THE DENSITY. THIS IS REQUIRED IF THE CRUISE MACH NUMBER IS TO BE OPTIMISED AT $M=0.85$.

THROUGH THE FLIGHT WEIGHT DROPS AS FUEL IS BURNT. THIS MEANS FOR C_L TO BE HELD CONSTANT ρ_a MUST DROP. ~~THIS~~ MEANS RAISING ALTITUDE.

ⓑ ~~$F' = \frac{W_f}{SW_p} = \frac{1}{S}\left(1+\frac{W_e}{W_p}\right)\left(e^{\left(\frac{Sg\,SFC}{V\,L/D}\right)}-1\right)$~~

$$S = \frac{V\,L/D}{g\,SFC}\ln\left(\frac{W_{START}}{W_{END}}\right) = \frac{V\,L/D}{g\,SFC}\ln\left(\frac{W_e+W_p+W_f}{W_e+W_p}\right)$$

$$\text{So}\quad \frac{W_e+W_p+W_f}{W_e+W_p} = e^{\left(\frac{Sg\,SFC}{V\,L/D}\right)}$$

$$1+\frac{W_f}{W_e+W_p} = e^{\left(\frac{Sg\,SFC}{V\,L/D}\right)}$$

$$W_f = (W_p+W_e)\left(e^{\left(\frac{Sg\,SFC}{V\,L/D}\right)}-1\right)$$

$$F' = \frac{W_f}{SW_p} = \frac{1}{S}\left(1+\frac{W_e}{W_p}\right)\left(e^{\left(\frac{Sg\,SFC}{V\,L/D}\right)}-1\right)$$

19) (c) MAKE ENGINE + AIRFRAME LIGHTER (IE A380 COMPOSITES)
AVOID HALF EMPTY PLANES. MINIMISE SFC
THROUGH HIGH THERMAL + PROPULSIVE EFFICIENCY.
MAXIMISE $V\frac{L}{D}$.

(d) $$F' = \frac{W_F}{S W_P} = \frac{1}{S}\left(1 + \frac{W_E}{W_P}\right)\left(e^{\left(\frac{Sg\,SFC}{VL/D}\right)} - 1\right)$$

$$S = \frac{V\,L/D}{g\,SFC}\ln\left(\frac{W_{START}}{W_{END}}\right)$$

$$V = M\sqrt{\gamma R T} = 0.85\sqrt{1.4\times287\times223}$$
$$= 254.4\ \mathrm{ms^{-1}}$$

$$S = \frac{254.4\times 20}{9.81\times1.7\times10^{-5}}\ln\left(\frac{400+70+250}{400+70}\right)$$

$$S = \text{~~[illegible]~~}\ 13{,}013\ \mathrm{km}$$

$$F' = \frac{1}{1.3\times10^{7}}\left(1+\frac{400}{70}\right)\left(e^{\left(\frac{1.3\times10^{7}\times9.81\times1.7\times10^{-5}}{254.4\times20}\right)} - 1\right)$$

= ~~6.10×10~~^{9} ~~[illegible]~~ 2.74 $\times10^{-8}$ kg/kgm

= ~~[illegible]~~ 0.274 $\times10^{-3}$ kg/kgkm

$M_{CO_2} = 8\times44$ $\qquad M_{FUEL} = 8\times12 + 18$
$= 114$

$$\frac{\dot{M}_{CO_2}}{S N_{PASS}} = \left(\frac{M_{FUEL}}{S M_{PASS}}\right)\left(\frac{M_{CO_2}}{M_{FUEL}}\right)\left(\frac{M_{PASS}}{N_{PASS}}\right)$$

NB $\frac{W_{FUEL}}{W_{PASS}} = \frac{M_{FUEL}}{M_{PASS}}$

⑲ ⓓ CONT

$$\frac{\dot{M}_{CO_2}}{S\,N_{PASS}} = \left(\overset{0.274}{\cancel{0.364}} \times 10^{-3}\right) \times \left(\frac{352}{114}\right) \times 100$$

$$= 0.\overset{085}{\cancel{\text{[illegible]}}}\ \text{kg / PASSENGER km}$$

$$\underline{\underline{= 85\cancel{\text{[illegible]}}\ \text{g}\ CO_2\ \text{PER PASSANGER km}}}$$

SIMILAR TO A CAR WITH TWO PASSANGERS

BUSES AND TRAINS ARE MUCH LOWER.

SECTION E : ELECTRICAL

Solutions to Paper 8 – 2019.

⑳ (a)(i) They are related by the De-Broglie's equation

$\lambda = h/p$, h is the Planck's constant $\simeq 6.62 \times 10^{-34}$ Js.

(a)(ii) The resolution scales as $\simeq$ the wavelength λ.

$$\lambda = h/p$$

$$E = p^2/2m \Rightarrow p = \sqrt{2mE}.$$

$$\therefore \lambda \simeq h/\sqrt{2mE} \simeq \text{resolution} \simeq \underline{6.139\,\text{pm}}$$

However due to relativistic corrections.

resolution

$$\simeq \frac{h}{\sqrt{2mE}} \cdot \frac{1}{\sqrt{1+\frac{E}{2mc^2}}}.$$

~ nm.

(b)(i) Region I : $\dfrac{-\hbar^2}{2m}\dfrac{\partial^2\psi}{\partial x^2} = E\psi.$

Region II : $\dfrac{-\hbar^2}{2m}\dfrac{\partial^2\psi}{\partial x^2} + \left[\dfrac{u_1-u_2}{L_1-L_2}x + \dfrac{u_2L_1-V_1L_2}{L_1-L_2}\right]\psi = E\psi.$

Region III : $\dfrac{-\hbar^2}{2m}\dfrac{\partial^2\psi}{\partial x^2} + u_2\psi = E\psi.$

(ii) No we cannot ~~hay~~ have $\psi(x) = Ae^{\beta x}$.

Any one / both answers are ok.

- If $\beta > 0$, $|\psi(x)| \to \infty$ as $x \to \infty$ in Region III.. This violates the normalizability of the wavefunction.
- Since $E > u_2$, β is complex

(c.) (i) Quantum mechanics allows the particle to exist in a barrier whose potential energy is greater than the total energy of the particle. The particle can therefor pass through such a barrier and exist on either sides of it. This is called tunneling.

(ii) There are several applications

The student can suggest any one answer.

(i) Scanning tunneling microscopes :- Where the dependence of the tunneling probability (and hence the current) on the width of the barrier [e.g. $\propto e^{-kx}$ for rectangular barriers] is used to profile a conductive sample

(ii) S/D contacts in MOSFETs (or any Schottky junction) are heavily doped to reduce the barrier width and allow tunneling to in principle build Ohmic contacts.

(iii) Flash memories use tunneling to trap charge in a burried semiconductor. The charge cannot escape at equilibrium conditions and offer non volatile memories.

iv) Displays..

etc

(iii) $\psi(x) = A x (L-x)$

Since $\int_0^L |\psi|^2 dx = 1$, $\int_0^L A^2 x^2 (L-x)^2 . dx = 1$

$$\Rightarrow A^2 L^2 \frac{x^3}{3} + \frac{A^2 x^5}{5} - \frac{2A^2 L x^4}{4} \Big|_0^L = 1$$

$$\Rightarrow A^2 \left[\frac{L^5}{3} + \frac{L^5}{5} - \frac{L^5}{2}\right] = 1 \Rightarrow A^2 \left[\frac{L^5}{30}\right] = 1 \Rightarrow A = \frac{\sqrt{30}}{L^{5/2}}.$$

② (A) i) If $N_A = 1e15$

$N_A >> n_i \Rightarrow p \simeq N_A = 1e15/cc$

$n \simeq \frac{n_i^2}{p} = 1e5/cc.$

If $N_A = 1e10$, we need to calculate using the exact equation for charge neutrality since $N_A \simeq n_i$

$n + N_A = p \Rightarrow n + N_A = \frac{n_i^2}{n} \Rightarrow n^2 + N_A n - n_i^2 = 0$

$$n = \frac{-N_A \pm \sqrt{N_A^2 + 4n_i^2}}{2} \Rightarrow n = \frac{-1e10 + \sqrt{5}\,e10}{2.} \simeq 0.6e10/cc$$

$p = n_i^2/n \simeq 1.6\ e10/cc$ [or equivalently $p \simeq N_A + n \simeq 1.6\ e10/cc$].

(ii) Time constant = RC

~~Conductivity~~ Resistivity $= \frac{1}{q n \mu_n + q \mu_p p}$, since $\left.\begin{matrix} n \simeq 1e15/cc \\ p \simeq 1e5/cc \end{matrix}\right\} n >> p.$

$\therefore$ Resistivity $= \frac{1}{q n \mu_n}$

$$\mu_n = \frac{q\tau_{col}}{m_e} = \frac{1.6e{-19} \times 1e{-13}}{1.6e{-31}} \simeq 0.1\ m^2/Vs = 1000\ cm^2/Vs.$$

$$\text{Resistivity} \simeq \frac{1}{(1.6e{-19})(1000)(1e15)} \simeq 6.25\ \Omega cm. = \rho$$

$$R = \rho L/A = \frac{\rho \times 5e{-4}}{5e{-4} \times 1e{-4}} \simeq 62.5\ k\Omega., \quad C = 100e{-12}\ F$$

$\Rightarrow \tau = RC \simeq 6.25\ \mu s.$

(b.) Poisson's Equation:

$$\frac{d\mathcal{E}}{dx} = \frac{\rho}{\epsilon_s} \Rightarrow \frac{d\mathcal{E}}{dx} = \frac{-qN_0 e^{-x/\lambda}}{\epsilon_s}$$

$$\mathcal{E} = \frac{qN_0\lambda}{\epsilon_s} e^{-x/\lambda} + \text{const}$$

$$\text{at } x=d, \quad \mathcal{E}=0 \Rightarrow \text{const} = -\frac{qN_0\lambda}{\epsilon_s} e^{-d/\lambda}.$$

$$\therefore \mathcal{E}(x) = \frac{qN_0\lambda}{\epsilon_s}\left[e^{-x/\lambda} - e^{-d/\lambda}\right]$$

$$\therefore \mathcal{E}(d/2) = \frac{qN_0\lambda}{\epsilon_s}\left[e^{-d/2\lambda} - e^{-d/\lambda}\right]$$

$$\frac{d\varphi}{dx} = -\mathcal{E}(x) = \frac{qN_0\lambda}{\epsilon_s}\left[e^{-d/\lambda} - e^{-x/\lambda}\right]$$

$$\varphi = \frac{qN_0\lambda}{\epsilon_s}\left[e^{-d/\lambda}x + \lambda e^{-x/\lambda}\right] + \text{const}$$

$$\therefore \varphi(x=d/2) - \varphi(x=0) = \frac{qN_0\lambda}{\epsilon_s}\left[e^{-d/\lambda}\frac{d}{2} + \lambda e^{-d/2\lambda} - \lambda\right].$$

(c.) (i) Constant voltage scaling:-

$t_{ox} \to t_o/k$, $L \to L/k$, $W \to W/k$

$\Rightarrow$ $C_{ox} = \epsilon_{ox}/t_{ox} \to C_{ox}k$, $(V_{GS}, V_{DS}) \to (V_{GS}, V_{DS})$

$I \simeq \mu C_{ox}\frac{W}{L} V_{GS}V_{DS} \longrightarrow Ik$

Power $= I \cdot V \cdot \to$ Power $\cdot k$.

$\therefore$ Power increases by a factor k.
(consumption)

(ii) Constant field scaling:

$t_{ox} \to t_{ox}/k$, $L \to L/k$, $W \to W/k$.

$\Rightarrow C_{ox} = \epsilon_{ox}/t_{ox} \to C_{ox}k$, $(V_{GS}, V_{DS}) \to \left(\frac{V_{GS}}{k}, \frac{V_{DS}}{k}\right)$.

$I \sim \mu C_{ox} W/L \; V_{GS} V_{DS} \to I/k$

Power $= I \cdot V =$ Power$/k^2$ $\Rightarrow$ Power reduces by a factor of k^2.

(22.) (a) (i) Sputtering and Evaporation are both physical vapor deposition techniques

However :

In Sputtering :- Atoms from the sample are knocked out by ion bombardment that cause collision cascades. The knocked out atoms are there deposited on the surface in a relatively directionless manner, and without the need for heating the sample. Works as long as the energy exchange > binding energy.

e-beam evaporation: A directed e-beam heats the sample and the sample vaporizes. This allows for a dedicated, directed deposition on the substrate.
Only sample heated and not substrate (unlike thermal evaporation).

For step coverage - sputtering → ~~non~~ directionless.

(ii) Many methods → generally liftoff or etching
Etching method would go like this (Better to get finer features)

Piranha clean of glass ⟶ chome deposition (ebeam or sputtering) ⟶ +ve photo resist spin coat. (here if needed HMDS can be put on cr + Bake before spinning photoresist.)

↓

Bake. (soft 110°, 1min) ⟵ Expose to UV. ⟵ developer. ⟵ Hard Bake say 110°, 3min

if under developed (Hard Bake → developer)

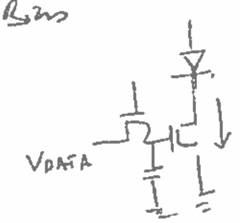
↓

Now Etch the unwated chrome ⟶ PR removal with acetone/IPA.

22) (b) (i) ITO → conductive transparent films
ctd
useful in optoelectronic devices using visible wavelength.
transparent due to wide Bandgap.

(ii) High k ⇒ $C_{ox}\uparrow$ ⇒ Better field effect. ⇒ Better g_m.
If $C_{ox}\uparrow$ by reducing t_{ox}, the Electric field can get high causing breakdown or leakage current.
∴ We keep $t_{ox}\uparrow$ and increase $k\uparrow$.

(iii) OLED driving TFT is in saturation Bias



$$\Rightarrow \quad I = \frac{\mu C_{ox}}{2} \frac{W}{L} (V_{gs} - V_T)^2$$

V_{gs} = VDATA = 78V for full brightness.
I = 40 µA for full Brightness
μ = 20 cm^2/Vs , V_T = 2V
C_{ox} = 20 e-9 F/cm^2

$$\therefore \frac{W}{L} = \frac{40e-6 \times 2}{20 \times 20e-9 \times 25} = 8.$$

(c) (i) Inductive reading :⇒ Change in B cause current
∴ transition from ↑ to ↓ or ↓ to ↑
defines current direction.
these transitions are sensed and recorded.
↑ ↑ or ↓ ↓ ⇒ previous state maintained

Magnetoresistance ⇒ Resistance change based on direction of B field.
∴ unlike the transition at edges the B field ↑ or ↓ is sensed in each cell
More sought after approach as we trend to solid state drives.

(ii) $2d\sin\theta = n\lambda$ ⇒ d = 5e 1.5e-10/(2*sin(10deg))
=4.3 A

Paper 8 – Section F crib

Q. 23

(a)(i) Differentiation amplifies high f noise, ∴ low-pass filter with gaussian.

$$\frac{dI}{dx} \xrightarrow{FT} j\omega I(j\omega) = sI(x)$$

(ii)

$$S(x,y) = \sum_{-n}^{n} \sum_{-n}^{n} g_\sigma(u)\, g_\sigma(v)\, I(x-u, y-v)$$

$N = 2n+1$ filter samples of $g_\sigma(x)$

(iii) $S(x,y)$

$$S(x+\Delta x, y) \simeq S(x,y) + \Delta x \frac{\partial S}{\partial x}(x,y) + \frac{\Delta x^2}{2} \frac{\partial^2 S}{\partial x^2}$$

$$S(x-\Delta x, y) \simeq S(x,y) - \Delta x \frac{\partial S}{\partial x} \ldots$$

$$\therefore \quad \frac{\partial S}{\partial x} \simeq \frac{S(x+\Delta x, y) - S(x-\Delta x, y)}{2\Delta x} \quad \therefore \quad \boxed{\begin{array}{|c|c|c|} \frac{1}{2} & 0 & -\frac{1}{2} \end{array}}$$

$$\frac{\partial S}{\partial y} \simeq \frac{S(x, y+\Delta y) - S(x, y-\Delta y)}{2\Delta y} \quad : \quad \boxed{\begin{array}{c} \frac{1}{2} \\ \hline 0 \\ \hline -\frac{1}{2} \end{array}}$$

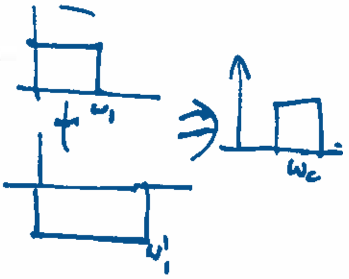
(b)

(i) $\nabla^2 S(x,y) \simeq S(x,y,k\sigma) - S(x,y,\sigma)$

$k \simeq 1.3 - 1.6$

where $S(x,y) = G_\sigma(x,y) * I(x,y)$

Difference of low-pass filter is a band-pass filter.



(ii)

Scale-space is sampled discretely (log)

$$S(x,y,\sigma_i) = G_{\sigma_i}(x,y) * I(x,y)$$

- Sample $\sigma_i = 2^{\frac{i}{s}} \sigma_0$, $\sigma_{i+1} = \sigma_i 2^{\frac{1}{s}}$

log-scale
geometric progression

- s images per octave

- Sub-sample once σ_i doubles $\sigma_i = 2^{\frac{i}{s}} \sigma_0$ $i = s, 2s \ldots$
  $\frac{1}{4}$ size Image

- Incremental blur $\sigma_k = \sigma_i \sqrt{2^{\frac{2}{s}} - 1}$. in each octave (same size)

(iii) Look for max/min of $\nabla^2 S \simeq S(k\sigma) - S(\sigma)$

$$\simeq S(\sigma_{i+1}) - S(\sigma_i)$$

Consider 26 nearest neighbours for max/min to localize $\underline{\underline{(x,y) \text{ and } \sigma_i}}$

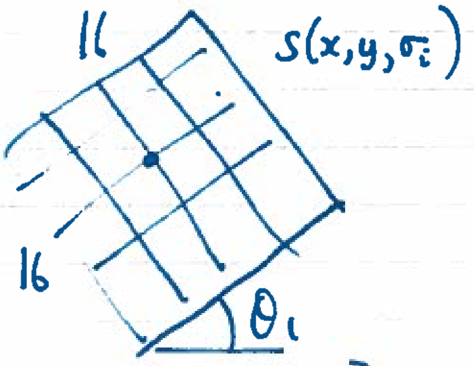
(c) (i) Scale of keypt in image — look at $S(x, y, \sigma_i)$

σ_i is scale

Orientation - Look x 16x16 pixels around keypoint

- compute HOG, 10° bins, smooth
- look for max., θ_i - ref. orientation

<u>16x16 sampled at (x, y) in $S(x, y, \sigma_i)$ at θ_i</u>



[Geometric invariance $\sigma_i + \theta_i$]

SIFT = 128D vector used for matching (iii) by N.N.

(ii) - 4x4 grids of 4x4 pixels ie 16x16 pixels

- compute ∇S
- weight by g_σ
- produce 4x4 cells (16 cells)
- each cell HOG at 45° (8 dirs)
  add mag $|\nabla S|$ to bin

- concatenate to give $8 \times 16 = 128$ D
- normalize to unit length
- remove effect of outliers if value > 0.2 truncate to 0.2 + renormalize

invariance to photometric effects + exact alignment

a) **Q24**

i)

$$\log p(\{y_n\}_{n=1}^{N} \mid \{\underline{z}_n\}_{n=1}^{N}, \underline{w}, b) = \sum_n \log p(y_n \mid \underline{z}_n, \underline{w}, b)$$

$$= \sum_n \left[y_n \log p(y_n = 1 \mid \underline{z}_n, \underline{w}, b) + (1-y_n) \log p(y_n = 0 \mid \underline{z}_n, \underline{w}, b) \right]$$

$$= \sum_n \left[y_n \log \underbrace{\sigma(\underline{z}_n, \underline{w}, b)}_{\triangleq \sigma_n} + (1-y_n) \log (1 - \sigma(\underline{z}_n, \underline{w}, b)) \right]$$

ii)

$$\frac{d}{d\underline{w}} \log p(\{y_n\}_{n=1}^{N} \mid \{\underline{z}_n\}_{n=1}^{N}, \underline{w}, b) = \frac{d}{d\underline{w}} \mathcal{L}(\underline{w}) = \sum_n \frac{d}{d\sigma_n} \mathcal{L}(\underline{w}) \frac{d\sigma_n}{d\underline{w}}$$

$$(\mathrm{I}) \quad \frac{d\mathcal{L}(\underline{w})}{d\sigma_n} = \frac{y_n}{\sigma_n} - \frac{(1-y_n)}{1-\sigma_n} = \frac{y_n(1-\sigma_n) - (1-y_n)\sigma_n}{\sigma_n(1-\sigma_n)} = \frac{y_n - \sigma_n}{\sigma_n(1-\sigma_n)}$$

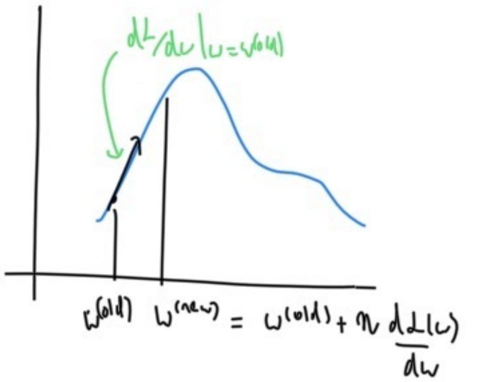
$$(\mathrm{II}) \quad \frac{d\sigma_n}{d\underline{w}} = \frac{d}{d\underline{w}} \frac{1}{1+e^{-\underline{w}^T \underline{z}_n - b}} = \frac{e^{-\underline{w}^T \underline{z}_n + b}}{(1+e^{-\underline{w}^T \underline{z}_n - b})^2} \underline{z}_n = \sigma_n(1-\sigma_n)\underline{z}_n$$

$$\therefore \quad \frac{d}{d\underline{w}} \mathcal{L}(\underline{w}) = -(\sigma_n - y_n)\underline{z}_n$$

iii)

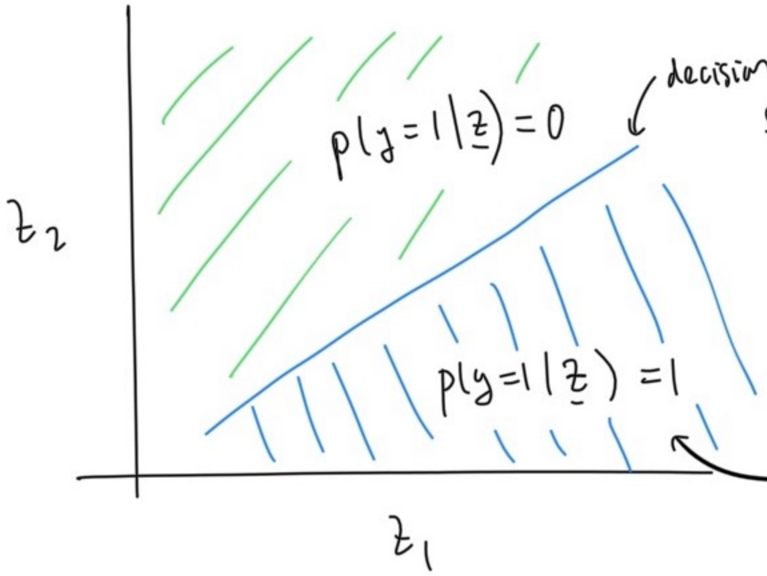
$$\underset{\substack{\uparrow \\ \text{new setting} \\ \text{of weight}}}{\underline{w}} \leftarrow \overset{\text{old setting of weight}}{\underline{w}} - \underset{\substack{\uparrow \\ \text{learning rate}}}{\eta} \frac{d}{d\underline{w}} \underset{\substack{\uparrow \\ \text{gradient pointing in the direction of} \\ \text{higher likelihood}}}{\mathcal{L}(\underline{w})}$$

gradient ascent



b) i)

data set 1

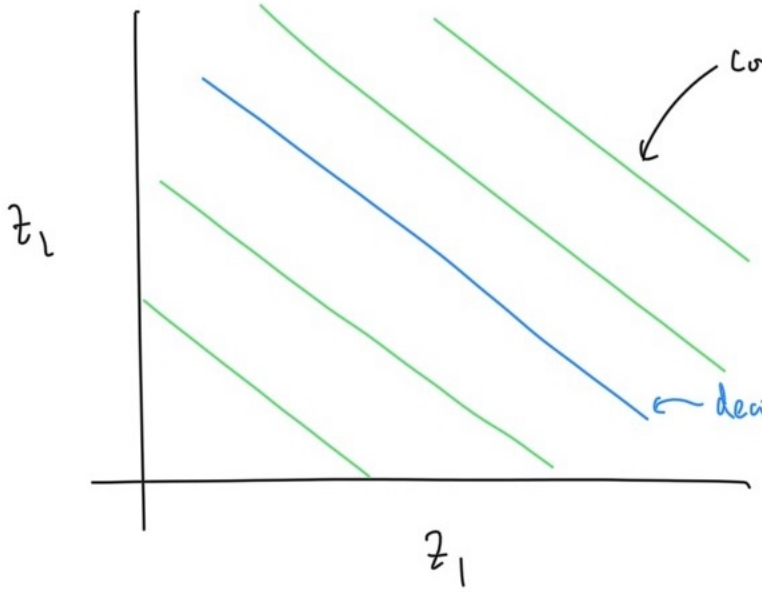


decision boundary always straight line for this model

as classes are linearly separable the model will be completely confident either side of boundary
=) regularisation needed

=) overfitting

dataset 2



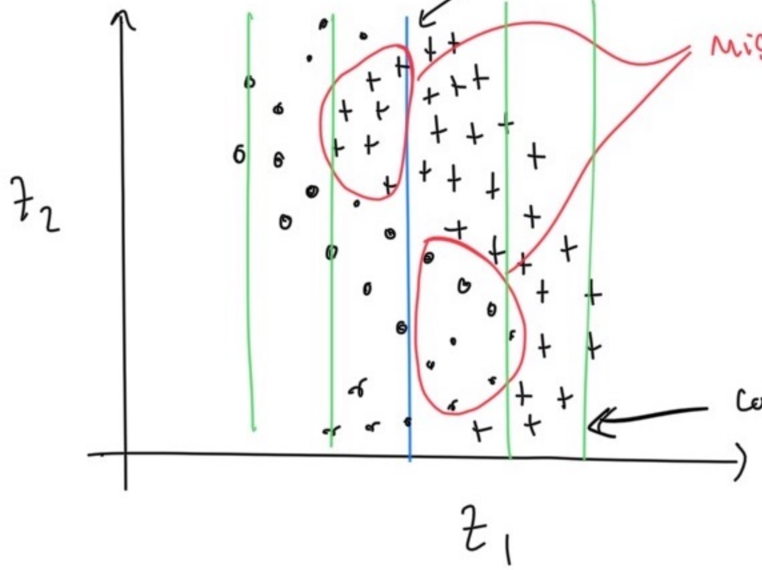
confidence slowly grows away from decision boundary as classes overlap

← decision boundary

well fit model

dataset 3

decision boundary (non-linear model needed!)



misclassified

confidence will grow but only slowly either side of boundary due to misclassified points

under fit model

ii) Two innovations

(I) add regularisation to log-likelihood cost (prior over weights)

$$\underset{\theta}{\arg\max}\left[\log p(\{y_n\}_{n=1}^{N} \mid \{\underline{z}_n\}_{n=1}^{N}, \underline{w}, b) + \underset{\substack{\uparrow \\ -\frac{1}{2}\beta\,\underline{w}^T\underline{w}}}{\log p(\underline{w})}\right]$$

This will help with the overfitting to dataset $\underline{1}$

(II) use a nonlinear model like a single hidden layer neural net

This will help w/ dataset 3 but regularisation will also be needed to prevent overfitting as the classes are non-linearly separable.

Q25 Consider the following two problems (both of which are to be solved offline). For each problem choose a suitable algorithm from the areas of Reinforcement Learning and Optimal Control (a different algorithm for each problem). In each case describe the algorithm and its application to the problem in detail, explaining why it is appropriate to the problem. Included in your answers should be definitions of the terms *value function*, *policy iteration*, *value iteration* and the action-value function, Q. You may make reasonable assumptions.

25 (a) Finding the optimal trajectory for reversing a car into a parking space while minimising the distance travelled (see Fig. 1). An accurate model of the kinematics of the car is available. [10]

The state-space is of low dimension, x, y and θ, and the problem is deterministic, so it is feasible to solve this directly by dynamic programming. The value function $V(s)$ might be the minimum distance along the optimal path of from state s. Choose a suitable time step, let $V(s_{parked}) = 0$ and work backwards. A complete answer would include a description of both value and policy iteration and an argument about which is to be preferred here (which would depend on assumptions that the student makes)

25 (b) Determining the ability of a car to avoid obstacles when driven at speed on wet surfaces. A highly detailed simulation environment is available, which incorporates important nonlinear and stochastic characteristics from a low level model of the tyre/road surface interaction. [15]

The problem has a high state dimension and is stochastic, so some form of RL would be appropriate here, with Q-learning being the expected answer. The action-value function should be defined. The reward should be a combination of a reward for avoiding the obstacle plus a reward for maintaining control of the vehicle. The state-space is of too high a dimension for a tabular approach, so some way of approximating Q is required, eg a deep neural net.

SECTION G: ***Bioengineering*** CRIB

26 (a) Both ultrasound (US) and optical coherence tomography (OCT) make use of *pulse-echo* signals to image the fundus. Compare and contrast US and OCT in the following areas:

(i) lateral focusing of the signal (i.e. across the direction of travel); [3]

Answer: For OCT, light is focused laterally using convex lenses, which have a different refractive index from the surrounding medium, or gradient-index lenses with varying refractive indices. The achievable focal point width is about 10 microns, depending on the power of the lens and the wavelength of the light.

For US, the sound is focused in one lateral direction (across the probe face) using a similar lens with a varying sound speed, and in the other direction (along the probe face) using electronic focusing, by sending out the sound pulses from multiple elements at slightly different times. The focal width is nearer to 1 mm, mostly due to the much larger wavelength of sound.

(ii) attenuation of the signal and maximum imaging depth; [3]

Answer: For OCT, light is very strongly scattered, and this is the source of nearly all the attenuation of the signal. Strong scattering means that it is quite likely that there will be multiple scattering events, which are a problem for pulse-echo systems which presume only a single scatter. Attenuation is such that the imaging depth is only about 1 mm, and is higher for higher frequencies, but lower for blood/water than for tissue.

For US, sound is only weakly scattered, so most of the attenuation is by absorption. This means the echo is much weaker, but less likely to be scattered more than once. Attenuation with depth is much less than with light, and hence the imaging depth is a few cm. However, like light, sound is also attenuated more at higher frequencies, and is less attenuated by blood/water than tissue.

(iii) detection of the echo and subsequent determination of the signal depth. [4]

Answer: Light is extremely fast (2×10^8 ms^{-1}) and hence the entire signal (most likely comprising several echos) is only registered as a single intensity at a photo-diode, though this can be split across frequencies using spectral OCT. Depth resolution is only possible by using an interferometer and changing the reference path length whilst sending multiple pulses, in which case the signal depth is a function of the reference path length. Alternatively, in spectral OCT the signal depth can be deduced from the frequency spectrum of the intensity.

Sound is much slower (1.5×10^3 ms^{-1}) and it is possible to record the echo signal as it arrives, using a piezoelectric crystal. In this case the depth is calculated from the time at which the echo was recorded, compared to the original pulse, and presuming a constant speed of sound in the medium.

This presumption of a constant speed of sound is only accurate to about 5%.

(b) For an ideal OCT system, with no dispersion, the intensity I at the output of the interferometer, as a function of frequency ω, is given by:

$$I(\omega) = S(\omega) + S(\omega)\left|\int_{-\infty}^{\infty} r_s(l_s)\, e^{j\frac{\omega}{c}l_s}\, \mathrm{d}l_s\right|^2 + 2\Re\left\{S(\omega)\int_{-\infty}^{\infty} r_s(l_s)\, e^{j\frac{\omega}{c}(l_r-l_s)}\, \mathrm{d}l_s\right\}$$

(i) Define the terms S, r_s, l_r, l_s and c in this equation. [3]

Answer: S is the power spectrum of the laser pulse which is sent into the eye, usually a fairly broad bandwidth low-coherence pulse.

r_s is the tissue reflectivity density function, which represents the percentage of light scattered back for a unit depth of the tissue being imaged. This is the parameter we are trying to image with OCT.

l_r is the round trip distance from the laser pulse source, to the reference mirror, and back to the photo-detector.

l_s is the round trip distance from the laser pulse source, to a scatterer at a particular depth, and back to the photo-detector.

c is the average speed of light in the medium being imaged and the medium in the interferometer.

(ii) Explain how r_s can be recovered from $I(\omega)$ using Spectral OCT. Include in your answer the mathematical derivation of the equation used to calculate r_s, and the experimental measurements required in order to use this equation. [8]

Answer: The spectra of the first term in the equation is measured by blocking the object arm (i.e. the path to the eye), which returns a measurement of just the laser power spectrum. This can be deducted from the actual measurement.

The spectra of the second term can similarly be measured by blocking the reference arm, and later deducted from the actual measurement.

This leaves us with only the third term:

$$I_{\mathrm{int}}(\omega) = 2\Re\left\{S(\omega)\int_{-\infty}^{\infty} r_s(l_s)\, e^{j\frac{\omega}{c}(l_r-l_s)}\, \mathrm{d}l_s\right\} \tag{1}$$

Substituting $t = \frac{l_s-l_r}{c}$, which means that $\mathrm{d}l_s = c\mathrm{d}t$ and $l_s = ct + l_r$:

$$I_{\mathrm{int}}(\omega) = 2\Re\left\{S(\omega)c\int_{-\infty}^{\infty} r_s(ct+l_r)\, e^{-j\omega t}\, \mathrm{d}t\right\} \tag{2}$$

This integral is just a Fourier Transform, so we can express this as:

$$I_{\mathrm{int}}(\omega) = 2\Re\left\{S(\omega)c\,\mathcal{F}\left\{r_s(ct+l_r)\right\}(\omega)\right\} \tag{3}$$

Re-arranging leads to:

$$r_s(ct+l_r) = \frac{1}{c}\mathcal{F}^{-1}\left\{\frac{I_{\mathrm{int}}(\omega)}{S(\omega)}\right\} \tag{4}$$

where the 2 disappears because we only have the real part of $I_{\mathrm{int}}(\omega)$, measured by an array of photodiodes. The imaginary part is presumed zero, and the complex coefficient is twice the actual measurement.

Note that zero time in this inverse transform corresponds to $ct + l_r = 0$, hence $t = -\frac{l_r}{c}$, so the measurement is from the reference mirror path length onwards.

(iii) What are the consequences of the $S(\omega)$ term in your answer to (ii) for the design of a spectral OCT system? [4]

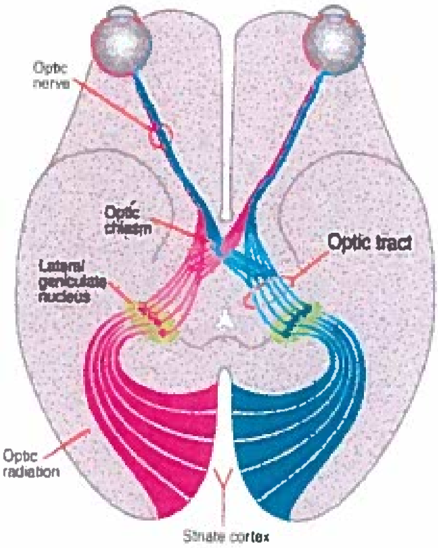
Answer: The division by $S(\omega)$ implies that we can only measure the intensity for frequencies actually contained in the laser pulse, so the diffraction grating and photodiode array need to be positioned so this is the case. This also implies that we want as broad-band a pulse as possible (larger range of frequencies) because this will increase the time resolution (increase the sampling rate) after the inverse transform. Increased time resolution then equates to better depth resolution of the r_s measurement.

27 (a) A patient no longer sees anything in their left visual field when closing the right eye, and similarly in their right visual field when closing the left eye. Comment on the plausibility of each of the following potential causes:

- A stroke damaged the left part of the visual cortex
- The right optic nerve was damaged before the optic chiasm
- An injury caused damage to the optic chiasm

[3]

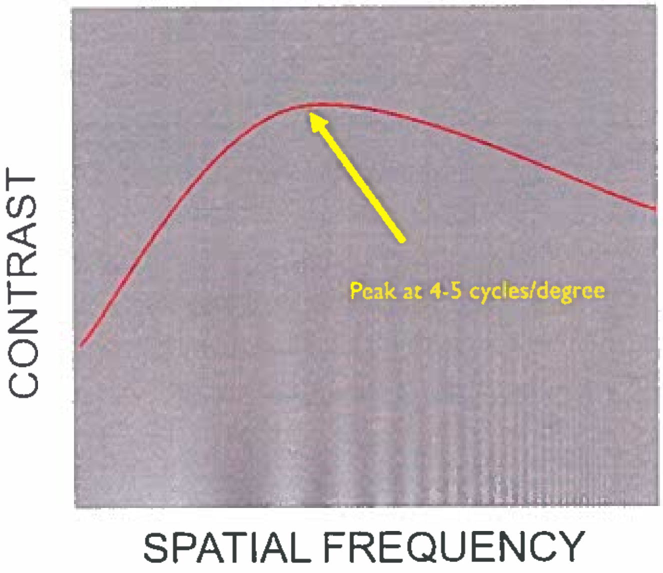
Answer: A stroke in the left visual cortex is unlikely, as it would affect the right visual field for both eyes. Damage to the right optic nerve before the optic chiasm is unlikely too, as it would not explain impaired vision when closing the right eye. An injury to the optic chiasm is very likely: this is where the parts of the left and right optic nerves carrying information about the corresponding left and right outer visual fields cross; damage to the chiasm therefore predicts exactly this type of vision loss.



(b) This question is about the contrast sensitivity function in vision, and hybrid images.

(i) In a few sentences, describe the so-called "contrast sensitivity function" in the context of sinusoidal gratings, including a brief account of how contrast thresholds can be measured experimentally. [4]

Answer: The contrast sensitivity function quantifies how sensitive we are to various spatial frequencies of visual stimuli. For sinusoidal grating stimuli, it is defined as the minimum contrast (or "contrast threshold") at which the grating becomes perceivable, e.g. yields at least 75% of correct answers in a 2-alternative-forced-choice detection task. In humans, it is U-shaped (or inverted-U-shaped as in the graph below, depending on the choice of y-axis) reflecting the fact that intermediate spatial frequencies at around 4-5 cycles/degrees can be perceived at lower contrast than lower or higher frequencies.



(ii) Look at the large and small versions of the same hybrid image below in Figure 1. In the large version, you see one British prime minister. In the small version, you see another one. Explain how this image might have been constructed, and why it is perceived differently when viewed from up-close and from far away. [5]

Fig. 1

Answer: Hybrid images are constructed by blending a low-pass filtered version of one image (here, David Cameron) and a high-pass filtered version of another one (here, Theresa May). Viewing the image from up-close (or viewing a large version at a fixed distance) effectively shifts the whole frequency spectrum towards lower frequencies (e.g. the same cycle of a grating will be contained within a larger visual angle). This is such that the high-frequency content of the hybrid image (May) will enter the perceivable range of frequencies at that contrast (cf. the U shape of the contrast sensitivity function), while the low-frequency content (Cameron) will be too low to be perceived. Vice-versa, looking at the image from afar pushes May into high-frequencies that are less easily perceived, and Cameron into intermediate frequencies that are best perceived.

(c) For each of the following terms, name one species or group of animals whose eyes are characterised by that term: negative lens, apposition eye, lens capable of protruding through the iris, multiple lenses, adjustable cornea curvature, inverse eye, flat cornea, reflective mirror, pin hole camera, scanning telescope. [5]

Answer:

- negative lens: eagle
- apposition eye: fly
- lens capable of protruding through the iris: merganser
- multiple lenses: Pontella
- adjustable cornea curvature: porpoise
- inverse eye: human
- flat cornea: seal or penguin
- reflective mirror: scallop
- pin hole camera: Nautilus
- scanning telescope: Copilia

(d) The following questions are about the information theoretic analysis of two neurons' responses about two stimuli. Table 1 below shows the (joint) probabilities with which these neurons give each of three possible responses for the two stimuli. Please provide your answers using bits as the unit of information theoretic quantities.

stimulus 1		neuron 1		
		1	2	3
neuron 2	1	0	0.05	0.2
	2	0.2	0.2	0.1
	3	0.1	0.05	0.1

stimulus 2		neuron 1		
		1	2	3
neuron 2	1	0.25	0.25	0
	2	0.25	0.25	0
	3	0	0	0

Table 1

(i) How much surprise do we get when the response of neuron 1 to stimulus 1 is 3? [1]

Answer: $-\log_2(0.2+0.1+0.1) = -\log_2 0.4 = 1.32$ bits

(ii) How much surprise do we get when the response of neuron 2 to stimulus 1 is 2? [1]

Answer: $-\log_2(0.2+0.2+0.1) = -\log_2 0.5 = 1$ bits

(iii) How much surprise do we get when, for stimulus 1, the response of neuron 1 is 3 and the response of neuron 2 is 2? [2]

Answer: $-\log_2 0.1 = 3.32$ bits

(iv) Are the responses of the two neurons to stimulus 1 independent? How is this reflected in your answers to the previous questions? [3]

Answer: They are not independent because their joint response probabilities do not factorise (they are not equal to the product of the corresponding marginal response probabilities of each neuron). This can also be seen in the fact that the surprise given by their joint response (3.32 bits) does not equal the sum of the surprises given by their individual responses (1.32 + 1 = 2.32 bits).

(v) What is the entropy of the joint responses of these two neurons for stimulus 1? [1]

Answer: $-(2\cdot 0.05\,\log_2 0.05 + 3\cdot 0.1\,\log_2 0.1 + 3\cdot 0.2\,\log_2 0.2) = 2.82$ bits

(vi) When each stimulus is used with an equal probability, what is the conditional entropy of the joint responses of these two neurons? [1]

Answer: $0.5\cdot 2.82 - 0.5\cdot(4\cdot 0.25\,\log_2 0.25) = 2.41$ bits

(vii) When each stimulus is used with an equal probability, what is the mutual information between the stimuli and the responses of these two neurons? [1]

Answer:

$-(2\cdot 0.025\,\log_2 0.025 + 3\cdot 0.05\,\log_2 0.05 + 3\cdot 0.1\,\log_2 0.1 + 4\cdot 0.125\,\log_2 0.125) - 2.41 = 1$ bit

28 (a) Discuss why the mechanical properties of biological materials, such as cornea of the eye, are difficult to measure compared to conventional engineering materials. [3]

Answer: A variety of reasons (answer out of six points)

- small features/inhomogeneity
- anisotropy
- inconvenient shape
- ethical/practical procurement problems
- complex loading
- in vitro behaviour not representative of in vivo
- time-dependence
- specimen/person variability
- species variability

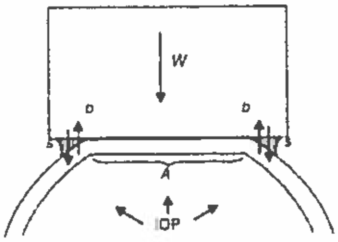
(b) State the role played by intra-ocular pressure (IOP) in the eye. How is IOP in the eye maintained? [3]

Answer: IOP has the role to maintain the shape and hence the optical function of the eye. Excessive pressure may lead to large stresses and in particular damage leading to glaucoma. IOP is mainly determined by the coupling of the production of aqueous humor and the drainage of aqueous humor mainly through the trabecular meshwork located in the anterior chamber angle.

(c) Goldmann tonometry is a common testing method to measure IOP. Assisted by a sketch of the testing geometry of Goldman tonometry, describe how the value of IOP is determined and how this can be used to detect a potential disease state of the eye. [5]

Answer: Goldmann tonometry is considered to be the gold standard IOP test and is the most widely accepted method. A special disinfected prism is mounted on the tonometer head and then placed against the cornea. The examiner then uses a cobalt blue filter to view two green semi circles. The force (W in the diagram) applied to the tonometer head is then adjusted using a dial connected to a variable tension spring until the inner edges of the green semicircles in the viewfinder meet. When an area (A in the diagram) of diameter of 3.06 mm has been flattened, the opposing forces of corneal rigidity and the tear film are roughly approximate and cancel each other out allowing the pressure in the eye to be determined from the force applied. Like all non-invasive methods, it is inherently imprecise and may need to be adjusted.

High IOP can lead to disease such as Glaucoma, which is an eye condition where the optic nerve is damaged by the pressure of the fluid inside the eye. High IOP is often not noticeable in patients, thus tonometry can provide an early detection method for glaucoma.



(d) To evaluate whether potential damage on the optic nerves can occur in the eye, two methods are used to model the deformation of lamina cribrosa: one being the plane membrane model, and the other being the elastic plate model. (For the following questions, you can take 1 mmHg = 133 Pa, and Poisson's ratio = 0.5.)

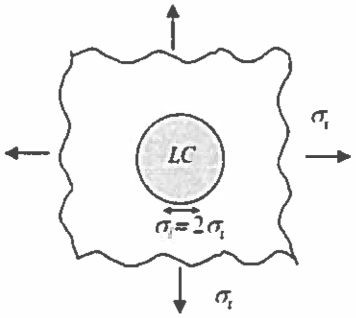
(i) Describe how the plane membrane model can be used to estimate the exerted strain on lamina cribrosa, given that the radial strain ϵ_r is 0.5% for IOP = 40mmHg. State your assumptions. [5]

Answer: Assume that modulus of LC is much less than that of the sclera (c.f. foams, relative density $\rho^* / \rho_s = 0.1 \rightarrow E^* / E_s = 10^{-3}$).

- Biaxial remote stresses. No radial stress at LC interface.
- Assume that the in-plane stretch of LC is determined by stretch of surrounding sclera

$$\epsilon_l = \frac{\sigma_l}{E} = \frac{2\,\sigma_r}{E} = \frac{2\,\epsilon_r}{1-\nu} \approx 4\,\epsilon_r \tag{5}$$

Hence $\epsilon_l \sim 2\%$.



(ii) The elastic plate model is indicated in Figure 2, where the central deflection d, is given by $\frac{d}{h} + 1.85\left(\frac{d}{h}\right)^3 = 0.7\,\frac{p}{E}\left(\frac{R}{h}\right)^4$; and central membrane stress (σ_m) and bending stress (σ_b) given by $\sigma_m = 0.91\,E\left(\frac{d}{R}\right)^2$, $\sigma_b = 1.78\,E\,\frac{d\,h}{R^2}$. Taking $p = 40$ mmHg, $R = 600\ \mu$m, $h = 110\ \mu$m, $d = 128\ \mu$m, calculate the strain. [6]

Answer: Inserting numbers, yields $E = 809$ kPa, $\sigma_m = 33.5$ kPa, $\sigma_b = 56.4$ kPa, $\sigma = \sigma_m + \sigma_b \sim$

90 kPa, strain is $\epsilon = \sigma\ (1 - \nu)/E \sim \sigma/2E \sim 5.5\%$

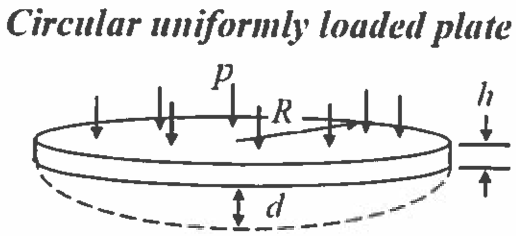


Fig. 2

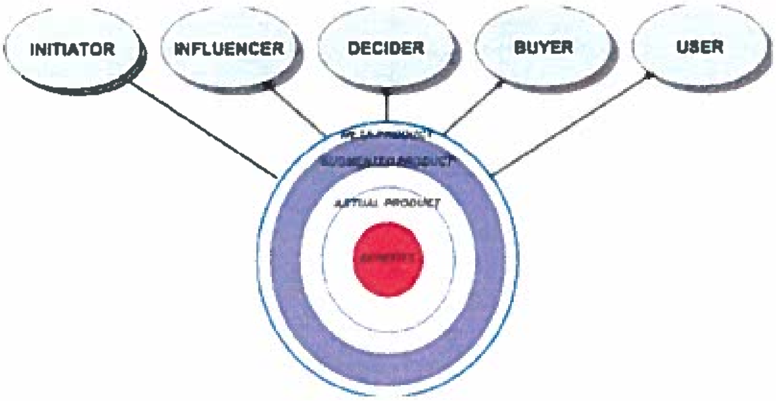
(iii) It is assumed that a strain of 5-8% in the lamina cribrosa can induce potential damages in the optic nerve. Briefly discuss the applicability of the above two models for modelling optical nerve damage. [3]

Answer: It seems that the membrane model underestimates the exerted strain. Out-of-plane effect may be a critical element. Thus the plate model may be more appropriate.

2P8- 2019 Section H   Manufacturing and Management CRIB

29a -

The key stakeholders are represented by the generic fig below



- Initiator(s) - Begins the buying process & gathers information
- Influencer(s) - Persuades or guides, has some role in influencing the decision, but is not the primary decision maker
- Decider(s) - Holds the power / purse strings. May not however be the ultimate user.
- Buyer - Conducts the transaction
- User - The actual end user, who interacts with the product

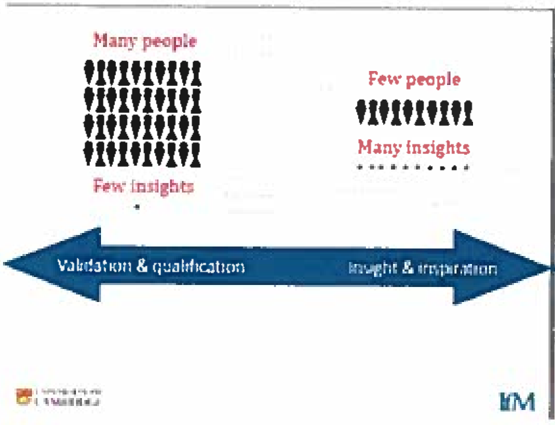
For the particular application of the technology the team has in mind, the interpretation of the stakeholder could vary. For example, for a low grade medical device which can be used to analyse gut microbiota imbalance, the users could be, for instance the pharmacist or the patients. Pharmacists could be also the buyers of the technology (e.g. of the equipment to be used in the pharmacy to deliver the innovative services). Influencers could be both medical practitioners and nutritional experts. The deciders/buyers could be the pharmacy brand chains (e.g. Boots, Lloyds …)

Any of the versions is acceptable. More than one version makes an improved to excellent answer. **(up to 5 points)**

29 b – The approaches and the choice of the techniques to understand the customer depend on whether the researchers would like to validate and qualify their ideas or would need more open-ended insight and inspiration. For the first circumstance, the researchers need to reach out to many people , but using techniques that are focused on few specific insights (e.g. asking very specific and targeted questions in surveys or interviews). In the second circumstance, the researchers need to adopt techniques which can help them understand many aspects of the users' perception of the innovation and then need to use techniques such as observation.

Other criteria include the cost (and hence the resources and time available for deploying the techniques).

The question refers to these slides:



Many people

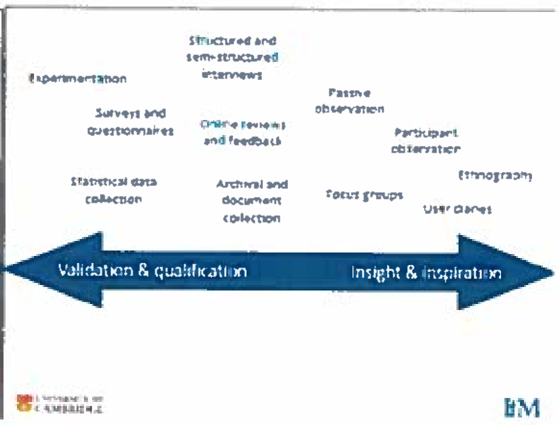
Few insights

Few people

Many insights

Validation & qualification

Insight & inspiration



Structured and semi-structured interviews

Experimentation

Passive observation

Surveys and questionnaires

Online reviews and feedback

Participant observation

Ethnography

Statistical data collection

Archival and document collection

Focus groups

User diaries

Validation & qualification

Insight & inspiration

The student might mention several techniques and then describe **2 in detail** highlighting **the advantages. For example:**

Interviews: these are useful for capturing basic information on behaviours (frequency, where, when, how much questions) and attitudes (but need to be very careful with wording of questions). Interviews could be used with the pharmacists, pharmacy patients, (and maybe GPs and specialists as influencers) to review potential needs and concerns. Advantages of interviews are that they are relatively quick and cheap.

Observation: these are useful for uncovering issues and needs that consumers might not be able to identify if just asked a question. As an equivalent technology does not exist, the challenge would be to identify how people behave in pharmacies or in other similar contexts. Once this is identified, the Researcher team could be observing how people install and manage the dispensation of tests or personalised services and how different type of people (patients) behave around testing and technologies. The downside of observation is that it can be time consuming and costly to analyse.

Creation of archetypes or Personas: Developing a character, with a life story (biography) and life goals helps the designers highlight the requirements and needs of potential customers whilst diminishing the risk of personal interpretation.

29 c

The alternative business models available range from

1) Focus on the development of a physical product – i.e. the personal gut microbiota device (which could be sold directly to the pharmacy to use on their clients or indirectly as a personal care appliance to the user via the pharmacy or other stores)
2) Focus on the product sale but with the idea to link it to the supply of a complete nutritional supplements and disposable accessories range (e.g. monetising on the consumables rather than the appliance).
3) Focus on the development of a service enabled by this product (e.g. the appliance is available to the stores as the opportunity to test the clients for microbiota imbalance)

Also available, are the business models which consider ways to progress the maturity of the early stage technology (which imply that the choice of business model will be influenced by the availability of resources, and this, in turn, is influenced by the funding available).

1) Do everything yourself and try to raise money to complete the next
2) Selling the invention to a large company (such as medical personal devices such as Philips or a nutritional company)
3) Partner with one of these large organisations to commercialise the innovation in their sector.
4) Licensing the invention (maybe trying to rethink the range of potential applications and expand the possible segments the invention can be used for)

Excellent answers, will be complete and with examples (see brackets above).

Better answers would also attempt a review of the relative merits of each of these business models in the context of this technology and speculate on the availability of funding (e.g. this is a good option if the team has obtained funding for manufacturing), discussing different funding options for each of the preferred business models. giving a very rough indication of which would need the most money, what the money would be spent on, and where it might be sourced. There may need to be iterations between the design of the preferred business model, the resources required to implement that business model, and the availability of funding to access the resources

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30 a

The choice of axes provides the ability to segment the market into groups that would be real and substantial.

Potential axes:
- family –business vs short range –long range
- hybrid-electric vs short range – long range
- young single vs family – hybrid-electric

By benefits delivered

By user / consumer

By product attributes

By usage

The choice of axes in the first map allows grouping of potential customers based upon characteristics of product use, and product attributes. The second map compares characteristics of two types of product attributes. Etc. the student could creatively use the slide above

The benefits that are delivered:

- what benefits do customers and users derive from the product?
- The particular product attributes: perhaps the easiest way to segment a market is to compare product attributes. Doesn't relate to the customers and is often the weakest approach.
- The characteristics of the consumer: this can be split to include demographics, and psychographics. Demographics relates to aspects such as social class, age, house size, sex etc. Psychographics relates to the user's attitudes and beliefs, what they feel, their lifestyle etc.
- The product use: describing ways in which a product is used. There might be strong customer loyalty, products might be used rarely or frequently, it could describe purchase behaviour (e.g. Distress purchase, seasonal patterns or regular upgrades etc).

The maps shown here are examples rather than being the only answers. Other equally valid axes include speed; radical or traditional configuration.-
'Price' is never preferable as a dimension.
The good answer would draw from these categories and attempt to segment the market

Acceptable maps are 3 marks (one point each) and a good explanation of the axes and what the maps were trying to achieve would give the rest

30 b
There are 4 sources of funding:

Grant = a 'gift' from an organisation such as Innovate UK or NESTA (National Endowment for Science, Technology and the Arts). The type of projects that are eligible for funding through this route may be very restricted.

Debt = borrowing money from a bank or specialist finance organisation. You can only borrow money if you can convince the bank that you can repay the money, plus interest, exactly when they want it. This is usually very hard for a new company to do.

Equity = selling part of the ownership (shares) of your business in return for cash. The assumption is that whoever buys part of your business will wish to sell this part of the business to someone else in the future at a higher price. The only reason for doing this is if the investor can be convinced that:
(a) the business is really going to grow (and quite fast) and that
(b) there will be someone else willing and able to buy their share in the company at a later date

Sales= money from selling something. If a company hasn't yet got a product to sell, there still may be ways of getting revenue. For example, selling expertise (consultancy) in your specialist area may be one way of bringing money in to fund the development of a product

a quick way to answer the second point in this question is to draw the picture below, without making the distinction of the individual sources. Written descriptions are also ok. A key point is that, there is overlap, debt are only good when the market opportunities are really obvious to banks and the risk is low. Grants are instead less interesting at the end.
30 c
Sustaining innovation = improving the performance of existing products whilst keeping as a key reference the current customers performance
E.g. = improving capacity of mobile phones batteries, size of displays, etc.

Disruptive innovation = pursuing innovation which might be less performant against current key performances used for current product, but has an additional benefit which some other/new customers segments might find attractive (e.g. 3d printed meat)

30 d
The students should be able to list these categories of challenges. And explain examples of how these are important from each of the partners

	Large	Small
Size difference and proximity		
Strategy and business models		
Sector and organisational 'clockspeed'		
Strategy and business models		
Resources and funding		
Partnering capability		

Size difference and proximity – There are some basic practical issues around bringing together a company with a few employees with one having tens of thousands. Who should you talk to in the larger company? How to deal with the fact that people frequently change roles within larger firms? Also, if the two companies are located at opposite sides of the world, this can lead to some real management problems. One quote from a start-up that had many problems working with a larger company reveals how this situation can feel: "We felt like s a small speedboat trying to dock with a supertanker".

Strategy and business models – Both sides of any partnership will have their own strategy and business models. Partnerships are formed when there is mutual strategic need. But strategies and business models tend to be dynamic. What happens when one partner's strategy changes?

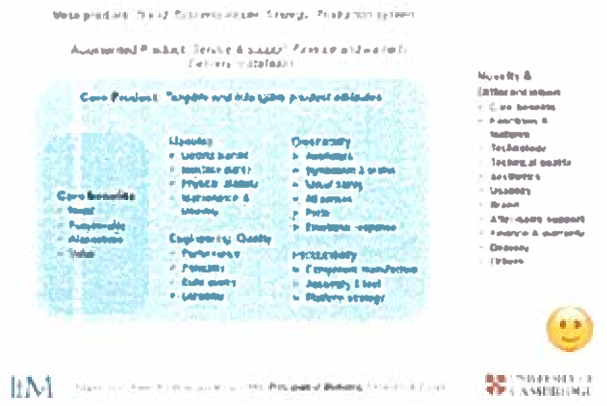
Sector and organisational 'clockspeed' – Companies and industries have particular 'clockspeeds'. For example, in business to business selling, the time between making first contact with a customer and receiving payment may be either a few days to several months (for aerospace, it may be years). If one company is used to operating at the short deal cycle end and the other at the long deal cycle end, this can cause practical cash flow problems.

Resources and funding – Partnerships take a lot of time to make them work. For a small company with very few people, the proportion of time that they are devoting to making the partnership work can be very significant, and may lead to insufficient time being available for other management tasks. For the bigger firm, they may be able to devote substantial resource to making the partnership work. For the smaller firm, the partnership may be come so time consuming and distracting that it becomes harmful to the overall success of the business.

Partnering capability – The ability to work with other firms has become a key skill for many high tech firms. The level of skill is often linked to amount of experience the companies have at managing partnerships, and whether lessons learned are converted into management practices. For a start-up, a good question to ask of a big firm is 'Have you ever worked with a company small as ours?

A good student might also refer to complexity in the relationship (e.g. other organisations involved) and trust establishment.

31 - a



The exercise involves using the dimensions of the fig above to describe key design features of the product of their choice. All the elements below need to be considered for a basic answer

CORE BENEFITS

All products (hopefully) provide some real & identifiable benefits or value. This has been called the product 'value proposition'. it is important that this value proposition has been carefully considered.

What is it that the customers/stakeholders are really buying? Why is it exciting to them? What must the product do, and how is it different to the competition.

- ACTUAL PRODUCT

: In order to deliver the core benefits, the product must have a collection of 'actual' properties:
TANGIBLE ATTRIBUTES:
Quantifiable and often measurable. The kind of attributes which it is easy to put on the product specification - 0-60 in 5 seconds. It is easy to compare different products against their tangible qualities - it is an objective assessment. Important attributes will be related to the engineering qualities, and possibly design for manufacture.
Some aspects of usability are highly tangible.
INTANGIBLE ATTRIBUTES:
More difficult & subjective to judge. Aspects such as aesthetics will be perceived differently by different people. But, it is these elements which often inform our opinion about a product. A Porsche needs to look quick if people are to buy it as performance car. Product attributes that are important here tend to be things like the appeal, experience, and desirability

- META PRODUCT

: Finally, the success of a product is highly tied into the overall business model and product economics. Increasingly, car companies are selling cars under an extended lease programme. Rolls Royce sell 'power by the hour', mobile phones are sold as loss leaders and the real money is to be made through the customer contract. The ipod is a great example of a business model supported product, with availability of musical media online being critical to the product's overall success. A new business model might be sufficient to make an old product highly successful.
There are obviously many potential elements of the design mix. Shown here are those which are typically considered most important to competition.

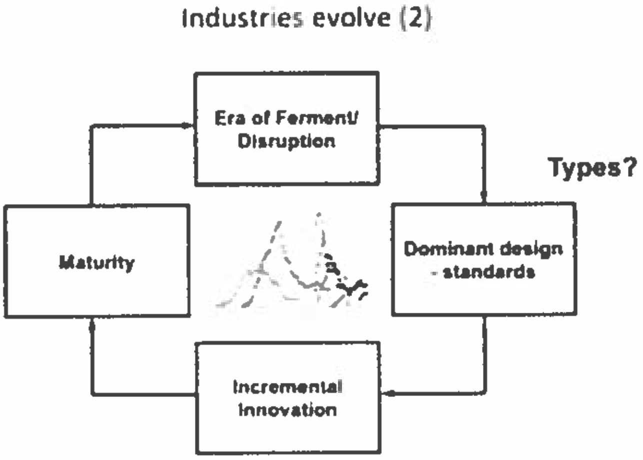
Useful
– appropriate functionality & performance
Aesthetics
– looks right for the intended audience
Technical quality
– well engineered, robust, durable, long life
Usability
– ergonomics, inclusive design, simple to use and understand, service and maintenance
Services
– warranty, brand etc
Producible & profitable
– can be made and sold at a profit, appropriate business model

DIFFERENTIATION:
Well designed products tend to perform well for all of the above criteria. But, excellent products are clearly differentiated from the competition in at least one area. The very best products are not just clearly differentiated, but are also novel in one or more aspect

The good answer would

- Define the characteristics of the product
- Express the customer needs in customers' terms
- Is not limiting the designer at the same time it Provides targets and sets constraints
- Provides unambiguous details of what the product does
- Could be evaluated
- An excellent answer would also indicate that there might have been more than one product specifications which might have evolved

31 b



The student needs to explain the figure above.
Era of ferment / disruption: From a small number of experiments emerge numerous competing approaches to addressing a particular opportunity. E.g., PC platforms, powered road transport, etc.

•

Dominant design / standards: The market settles down onto one or two standard / dominant designs. Once this happens, more customers will have the confidence to buy and producers of peripherals will have the confidence to enter the market.

•

Incremental innovation: Once there is some agreement in the market as to what is wanted, companies will focus effort in improving cost, reliability, efficiency, etc, rather than developing completely new products.

•

Maturity: Once the product has reached the limits of cost effective improvements, the market may slow down until another disruption occurs and the cycle starts over.

A good answer would also connect the phases with the number of companies in the market explaining that at the start the activities of a few pioneer companies show the potential of the opportunity and there is then a rapid increase in the number of companies trying to address that opportunity. Once the market matures, the number of companies drops as many are merged together or fail.

31 c

VCs will be interested to understand

- what is the need for this innovation in the market and
- what is the proponents view of how the innovation will be responding to this need and what will it be (e.g. product or service).
- The team's expertise and capabilities both in the technology and in the market (Management team's capabilities)
- The proponent's operations (business operations)
- The financial projections (how much and when will they make money)
- How can the business convince the buyers/customers to part with money for this product/service (Marketing strategy)
- What are the resources in house and what is needed
- The exit opportunities

31 d

These two types of manufacturing systems are used for high volumes manufacturing and relatively low to low variability. In general, the level of investment required to start a production system – and hence the risk (that the money invested won't be returned through sales) – increases as the volume increases.

The batch production could be used to retain some manufacturing flexibility and hence allows variety/change to be introduced more cheaply, it can be used while the product becomes established in the market.

As sales increase, and the design can be fixed, and the manufacturing can move to a more dedicated production system and produce higher volumes at lower costs. This type of production requires very high investments (dedicated equipment, reliability, coordinated response to errors) and hence it needs to be considered also in the context of how long the production line will be used for (how long will the product survive?).