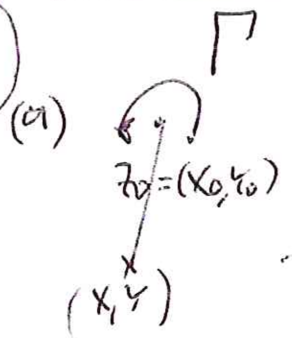
3A1: CRIB / WND Jan. 2015

17 pp...

① (a)



$F(z) = -\frac{i\Gamma}{2\pi}\log(z-z_0)$ $\quad u-iv=\frac{dF}{dz}$

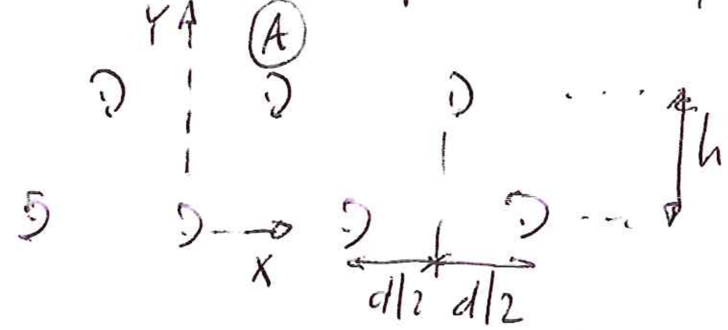
$\therefore u-iv = \frac{-i\Gamma}{2\pi(z-z_0)} = \frac{-i\Gamma}{2\pi}\cdot\frac{[()-i()]}{[(x-x_0)+i(y-y_0)][()-i()]}$

$\therefore \underline{u = -\frac{\Gamma}{2\pi}\cdot\frac{y-y_0}{(x-x_0)^2+(y-y_0)^2}}$; $\underline{v = +\frac{\Gamma}{2\pi}\cdot\frac{x-x_0}{(x-x_0)^2+(y-y_0)^2}}$

(b) (i) In a frame relative to vortex the force exerted on the vortex (from the Blasius relations ...) is $\underline{\rho U\Gamma}$

(ii) If a vortex experiences no force - it's stationary!

(c)



(i) The vortices move horizontally - and to the <u>left</u>.

(ii) For the upper row the u-velocity of, for example, vortex Ⓐ is:

$$u = \sum_{n=-\infty}^{+\infty} -\frac{\Gamma}{2\pi}\cdot\frac{h}{(\frac{d}{2}-nd)^2+h^2} = -\frac{\Gamma h}{2\pi d^2}\sum_{n=-\infty}^{+\infty}\frac{1}{(n-\frac{1}{2})^2+(\frac{h}{d})^2}$$

[use given...] (n = k) $= -\frac{\Gamma}{2d}\cdot\frac{\sinh(2\pi h/d)}{\cosh(2\pi h/d) - \underbrace{\cos(2\pi\frac{1}{2})}_{=1}}$

$\therefore$ row speed $= -\frac{\Gamma}{2d}\cdot\sinh(2\pi\frac{h}{d})/(\cosh(2\pi\frac{h}{d})-1)$

This will be a lower bound for the body speed (i.e. the body will be moving faster than its wake).

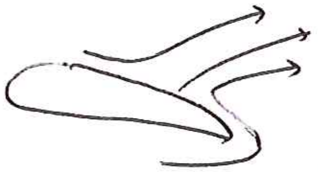
(iii) The wake centre-line velocity is likely to be closer to the body speed. To extract this replace h by h/2 and double (both rows...)

to $u = -\frac{\Gamma}{d} \cdot \frac{\sinh(\pi h/d)}{\cosh(\pi h/d) - 1}$

This is ~2x value in part (c.ii).

Q1. Examiner's Comment:
Nice mix of simple maths and real physics – very unpopular.

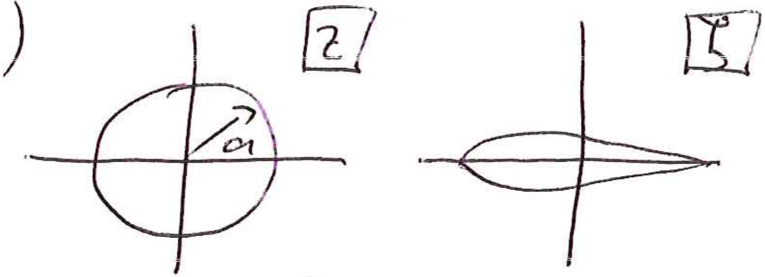
② (a)



very high curvature around TE leads to strong suction followed by diffusion which will cause b'layer to separate.

flow leaving the TE smoothly and this is what happens in practice ("Kutta condition").

(b)



symmetrical airfoil with chord on Re(ζ)

With $z = ae^{i\theta}$ expect LE at $\theta=\pi$ and TE at $\theta=0$

$$z=a \rightarrow \zeta=0$$

$$z=ae^{i\pi} = a(\underset{-1}{\cos\pi}+i\underset{0}{\sin\pi}) = -a$$

$$\therefore \zeta = \frac{(-a-a)^k}{(-a-a\varepsilon)^{k-1}} = \frac{(-a)^k 2^k}{(-a)^{k-1}(1+\varepsilon)^{k-1}} = -a\frac{2^k}{(1+\varepsilon)^{k-1}}$$

$\therefore$ airfoil chord $= \frac{2^k}{(1+\varepsilon)^{k-1}} a$

(c) In the z plane flow is:

$$F(z) = \underbrace{Uze^{-i\alpha} + U\frac{a^2}{z}e^{i\alpha}} - \underbrace{\frac{i\Gamma}{2\pi}\log z}$$

(i) free stream at angle of attack α plus doublet ⇒ cylinder + bound circulation

This can be related to the flow in the physical ζ-plane via the transformation and in particular via the velocities & $dz/d\zeta$.

(ii) The velocity field in the ζ-plane is

$$(u-iv)_\zeta = \frac{dF}{dz}\cdot\frac{dz}{d\zeta} = \frac{dF/dz}{d\zeta/dz}$$

$$\frac{d\zeta}{dz} = \frac{k(z-a)^k}{(z-a\epsilon)^{k-1}} - \frac{(k-1)(z-a)^k}{(z-a\epsilon)^k} = 0 \text{ @ } z=a \text{ the TE}$$

∴ [Kutta condition...] $$\left.\frac{dF}{dz}\right|_{z=a} = \underbrace{Ue^{-i\alpha} - Ue^{+i\alpha}}_{2iU\sin\alpha} - \frac{i\Gamma}{2\pi a} = 0$$

$$\therefore\ \underline{\Gamma = -4\pi Ua\sin\alpha}\ .$$

(d) The lift coefficient, $C_L = \frac{L}{\frac{1}{2}\rho U^2 c} = \frac{-\rho U\Gamma}{\frac{1}{2}\rho U^2 c}$

$$= \frac{\rho U 4\pi a\sin\alpha}{\frac{1}{2}\rho U^2\, 2^k a/(1+\epsilon)^{k-1}}$$

$$\therefore\ \underline{C_L = 2^{3-k}\pi(1+\epsilon)^{k-1}\sin\alpha}\ .$$

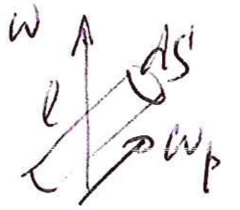
Q2. Examiner's Comment:
Popular and generally well done.

③

(a) (i) Kelvin's theorem: for an inviscid, constant density flow the circulation around a closed loop moving with the fluid is constant: $\frac{D\Gamma}{Dt}=0$.

(ii) $\Gamma = \oint \bar{u}\cdot d\bar{l} = \iint \bar{\omega}\cdot d\bar{S}$ by Stokes

So if Γ is constant so is the flux of vorticity

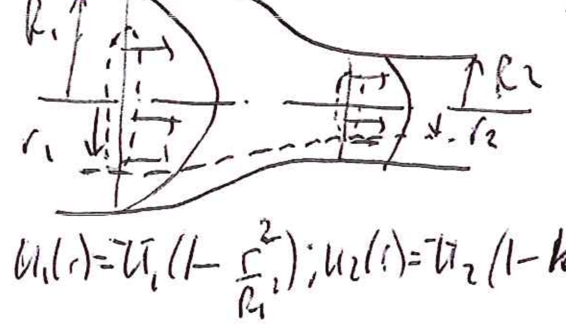


So with ω_p the component of vorticity parallel to the fluid filament $\omega_p dS \sim$ constant }

Also, by mass conservation $l\,dS \sim$ constant }

$\therefore$ $\underline{\omega_p \sim \text{const.}\, l}$.

(b)



Incompressible & inviscid contraction.

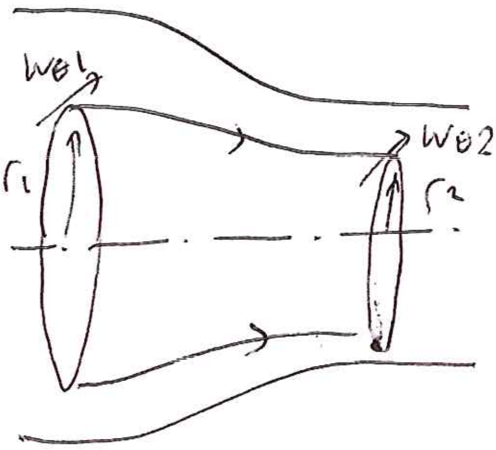
$u_1(r)=U_1\left(1-\frac{r^2}{R_1^2}\right)$; $u_2(r)=U_2\left(1-\frac{kr^2}{R_2^2}\right)$ axisymmetric.

(i) Vorticity, $\bar{\omega}=\omega_\theta \hat{e}_\theta = -\frac{du_z}{dr}\hat{e}_\theta$ from $\nabla\times\bar{u}$ in cylindrical polar co-ords

$\therefore$ $\omega_z, \omega_r = 0$ & $\omega_{\theta_1} = \frac{2U_1 r}{R_1^2}$ }

Similarly: $\underline{\omega_{\theta_2} = 2k\frac{U_2}{R_2^2} r}$ }

(ii)



Streamline from r_1 to r_2. From part a(ii)

$$\underset{\frac{2U_1}{R_1^2} r_1}{\frac{w_{\theta 1}}{T}} \Big/ 2\pi r_1 = \underset{\frac{2kU_2}{R_2^2} r_2}{\frac{w_{\theta 2}}{T}} \Big/ 2\pi r_2$$

$$\therefore \frac{U_1}{R_1^2} = k\frac{U_2}{R_2^2} \Rightarrow \frac{U_1}{U_2} = k\frac{R_1^2}{R_2^2} \quad \text{——} ①$$

Now use conservation of mass for that streamtube:

$$\int_0^{R_1} 2\pi r U_1\left(1-\frac{r^2}{R_1^2}\right)dr = \int_0^{R_2} 2\pi r U_2\left(1-k\frac{r^2}{R_2^2}\right)dr$$

①

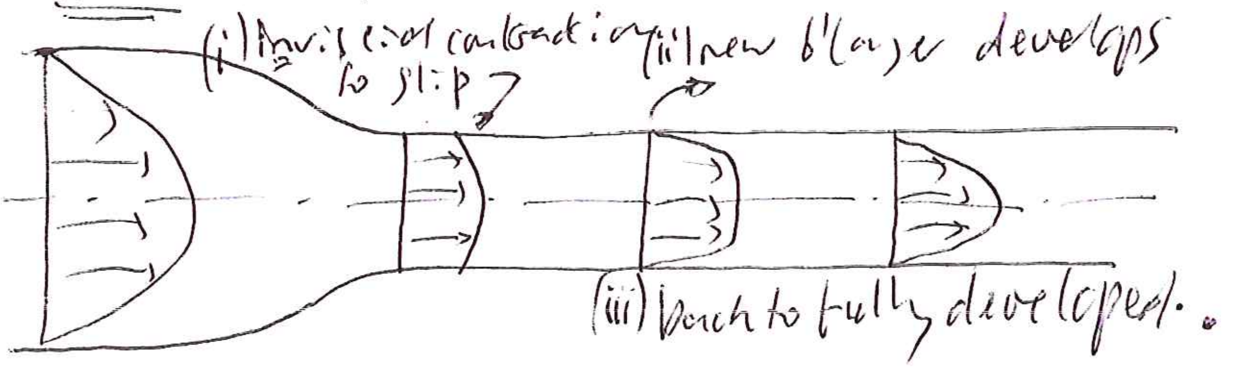
$$\therefore U_1\left[\frac{r^2}{2} - \frac{1}{4}\frac{r^4}{R_1^2}\right]_0^{R_1} = U_2\left[\frac{r^2}{2} - \frac{1}{4}\frac{r^4}{R_2^2}k\right]_0^{R_2}$$

$$U_1 R_1^2\left(1-\frac{1}{2}\right) = U_2 R_2^2\left(1-\frac{1}{2}k\right) \quad \text{——} ②$$

$$\therefore k\frac{R_1^4}{R_2^2}\cdot\frac{1}{2} = R_2^2 - \frac{1}{2}kR_2^2 \qquad \therefore \frac{k}{2}\left(\frac{R_1^4}{R_2^2}+R_2^2\right) = R_2^2$$

$$\therefore \underline{k = 2R_2^2/(R_1^4+R_2^4)} \;\&\; \underline{U_1/U_2 = \frac{2R_1^2R_2^2}{(R_1^4+R_2^4)}}$$

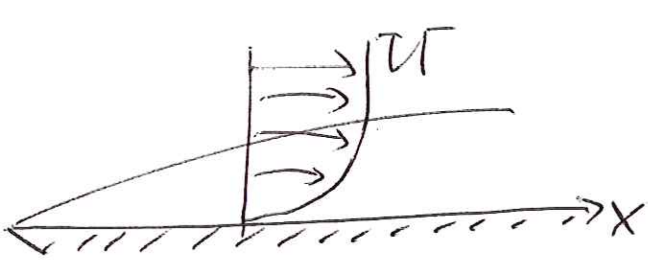
(c) Downstream $R_1>R_2$ so $k<1$ and walls viscous ...



Q3. Examiner's Comment:

Rather simple question based on physical understanding – with little maths. Popular – and done well by a few – but badly by many. It is very worrying when such basic physics, so central to our subject appears not to have got much traction

④



$\eta = y/\delta$

(a) $\frac{u}{U} = 2\left(\frac{y}{\delta}\right) - \left(\frac{y}{\delta}\right)^2$; $\theta = \int_0^\delta \left(1-\frac{u}{U}\right)\frac{u}{U}\,dy = \int_0^1 (1-2\eta+\eta^2)(2\eta-\eta^2)\,d\eta$

$$\theta/\delta = \int_0^1 (2\eta - \eta^2 - 4\eta^2 + 2\eta^3 + 2\eta^3 - \eta^4)\,d\eta$$

$$= \left[\eta^2 - \frac{5}{3}\eta^3 + \eta^4 - \frac{1}{5}\eta^5\right]_0^1$$

$$= 1 - \frac{5}{3} + 1 - \frac{1}{5} = \frac{30-25-3}{15} = \frac{2}{15}$$

$$\therefore \underline{\theta = \frac{2}{15}\delta}$$

Momentum integral equation: $\frac{d\theta}{dx} + \cancel{(H+2)\frac{\theta}{U}\frac{dU}{dx}} = \frac{C_f}{2}$

[DATA SHEET]

$\frac{C_f}{2} = \frac{\tau_w}{\rho U^2} = \frac{\mu}{\rho U^2}\frac{du}{dy}\Big|_{y=0} = \frac{\mu}{\rho U^2}\frac{2U}{\delta} = \frac{2\nu}{U\delta}$ $\quad (2/\delta - 2y/\delta^2)$

0
zero pressure gradient

$$\therefore \frac{d\theta}{dx} = \frac{2\nu}{U\delta} = \frac{2\nu\, 2}{U\, 15\,\theta} \quad \therefore \theta\frac{d\theta}{dx} = \frac{4\nu}{15U}$$

$$\therefore \frac{\theta^2}{2} = \frac{4\nu}{15U}x + A \quad ; \theta = 0 @ x = 0 \Rightarrow A = 0$$

$$\therefore \underline{\theta/x = 1\Big/\sqrt{\frac{15}{4}\frac{Ux}{\nu}}} \qquad \sqrt{\frac{15}{4}} = 1.94 = 1/\underline{0.52}$$

(b) Now (!!!) $u/U = \frac{3}{2}\eta - \frac{1}{2}\eta^3$ where $\eta = y/\delta$

$$\theta = \int_0^1 \left(1 - \frac{3}{2}\eta + \frac{1}{2}\eta^3\right)\left(\frac{3}{2}\eta - \frac{1}{2}\eta^3\right)d\eta$$

$$\theta/\delta = \int \left(\frac{3}{2}\eta - \frac{1}{2}\eta^3 - \frac{9}{4}\eta^2 + \frac{3}{4}\eta^4 + \frac{3}{4}\eta^4 - \frac{1}{4}\eta^6\right)d\eta$$

$$\therefore\ \theta = \left[\cancel{3/4} - \underset{\checkmark}{\frac{1}{8}} - \cancel{9/12} + \underset{\checkmark}{\frac{3}{20}} + \underset{\checkmark}{\frac{3}{20}} - \frac{1}{28}\right]\delta$$

$$= \frac{1}{280}(-35 + 84 - 10)\,\delta$$

$$\therefore\ \underline{\theta = \frac{39}{280}\,\delta}$$

As before, momentum integral equation: $\frac{d\theta}{dX} = \frac{3\nu}{2\delta U}$

$$\therefore\ \theta\frac{d\theta}{dX} = \frac{3\nu\, 39}{2U\,280}$$

$$\therefore\ \underline{\theta/x = 1/\sqrt{\frac{560}{117}\cdot\frac{U}{\nu X}}}$$

$\sqrt{\frac{560}{117}} = 2.19 = 1/\underline{\underline{0.457}}$

(c) $\theta = \int_0^\infty \left(1 - \frac{u}{U}\right)\frac{u}{U}\,dY = \int_0^\infty \left(1 - \underbrace{f'(\eta)\big)f'(\eta)}_{\text{given}}\,d\textcircled{Y} ; \underbrace{\eta = Y\sqrt{\frac{U}{\nu X}}}_{\text{given}}$

$$= \sqrt{\frac{\nu X}{U}}\int_0^\infty (1 - f')f'\,d\eta = \sqrt{\frac{\nu X}{U}}\int_0^\infty \left(f' - (f')^2\right)d\eta$$

Now: $(f')^2 = \underbrace{(ff')'}_{(f')^2 + ff''} - \underbrace{ff''}_{f''' + ff'' = 0 \text{ Blasius}} = (ff')' + f''' = (ff' + f'')'$

$$\therefore\ \theta\sqrt{\frac{U}{\nu X}} = \int_0^\infty (f - ff' - f'')^{\textcircled{1}}\,d\eta = \left[f(1 - f') - f''\right]_0^\infty$$

$$= \underbrace{f(\infty)(1 - f'(\infty))}_{0} - \underbrace{f''(\infty)}_{0} - \underbrace{f(0)}_{0}(1 - \underbrace{f'(0)}_{0}) - f''(0))$$

$$\therefore\ \theta/X = f''(0)\Big/\sqrt{\frac{UX}{\nu}}$$

; $\underline{f''(0) = 0.4696}$ (d)

(d) So the exact solution from (c): $\theta/x = 0.47/\sqrt{Re_x}$

from (a): $\theta/x = 0.52/\sqrt{Re_x}$

from (b): $\theta/x = 0.46/\sqrt{Re_x}$

∴ relative errors (a) $\left|\frac{0.52-0.47}{0.47}\right| = 10\%$

(b) $\left|\frac{0.46-0.47}{0.47}\right| = 2\%$

(e) B'layer equation: $u\frac{du}{dx} + v\frac{du}{dy} = -\frac{1}{\rho}\frac{dp}{dx} + \frac{d\tau}{dy}$ $\mu\frac{du}{dy}$

At wall $u=v=0$ plus U = constant so $\frac{dp}{dx}=0$

∴ $\left.\frac{d^2u}{dy^2}\right|_{y=0} = 0$ ("second compatibility condition").

Profile from (a): $\left.\frac{d^2u}{dy^2}\right|_{y=0} = -\frac{2U}{\delta^2} \neq 0$

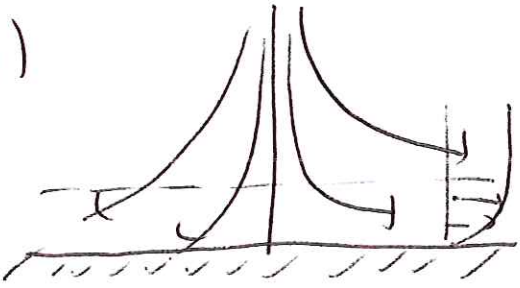
(b) $\left.\frac{d^2u}{dy^2}\right|_{y=0} = 0$

∴ the profile from part (b) is "consistent".

Q4. Examiner's Comment:

Very popular question and generally well done except for the last part based on a similarity solution – which seems to lead most candidates into an algebraic black hole…

⑤. (a)



$$\left.\begin{array}{l} u = Bx \\ v = -By \end{array}\right\}$$

$\psi = Bx f(y) ; f(0)=0$.

$$u = \frac{\partial\psi}{\partial y} = Bx f' ; \quad v = -\frac{\partial \psi}{\partial x} = -Bf$$

$$\left.\begin{array}{l} \text{when } y=0,\ u(0)=0 \Rightarrow f'(0)=0 \\ \text{when } y\to\infty,\ u\to Bx \Rightarrow f'(\infty)=1 \end{array}\right\}.$$

(b) $u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu\nabla^2 v$: y-component of NS eq^{n}...

↳ $\frac{\partial v}{\partial x}=0 ; \frac{\partial^2 v}{\partial x^2}=0 ; \frac{\partial v}{\partial y} = -Bf' ; \frac{\partial^2 v}{\partial y^2} = -Bf''$

∴ $\frac{1}{\rho}\frac{\partial p}{\partial y} = -\nu Bf'' - B^2 ff'$ only a function of y

∴ $\underline{\partial^2 p/\partial x\partial y = 0}$.

(c) $u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$: x-component of NS eq^{n}!

↳ $\frac{\partial u}{\partial x} = Bf' ; \frac{\partial u}{\partial y} = Bxf'' ; \frac{\partial^2 u}{\partial x^2} = 0 ; \frac{\partial^2 u}{\partial y^2} = Bxf'''$.

∴ $Bxf'Bf' - Bf\,Bxf'' = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu Bxf'''$

$$\therefore \underbrace{\frac{1}{\rho}\frac{1}{x}\frac{\partial p}{\partial x}}_{fn(x)} = \underbrace{B^2 ff'' + \nu Bf''' - B^2(f')^2}_{fn(y)} = \text{constant}$$

Separation of variable(s):

$$f''' + \frac{\beta}{\nu}(ff'' - f'^2) = \text{constant}$$

At $Y \to \infty$, $f''(\infty) = f'''(\infty) = 0$ & $f'(\infty) = 1$ ∴ $\underline{\text{constant} = -\frac{\beta}{\nu}}$.

Hence, the differential equation for f is:

$$\underline{f''' + \frac{\beta}{\nu}(ff'' - f'^2) = -\beta/\nu \quad \text{with } f(0) = f'(0) = 0;\ f'(\infty) = 1}$$

(d) The length scale $g = \sqrt{\nu/\beta}$

and the velocity scale $U = \sqrt{\nu\beta}$.

(e) $\psi = x\beta f(Y) = x F(\eta)\sqrt{\beta\nu}$

$f(Y) = F(\eta)\sqrt{\frac{\nu}{\beta}}$; $\eta = Y\sqrt{\frac{\beta}{\nu}}$; $\frac{d\eta}{dY} = \sqrt{\frac{\beta}{\nu}}$

∴ $\frac{df}{dY} = \frac{dF}{d\eta}\cdot\frac{d\eta}{dY}\sqrt{\frac{\nu}{\beta}} = F'(\eta)$

$\frac{d^2f}{dY^2} = F''(\eta)\frac{d\eta}{dY} = F''(\eta)\sqrt{\frac{\beta}{\nu}}$

$\frac{d^3f}{dY^3} = F'''(\eta)\,\beta/\nu$

Substitute into the ode. for f from part (c):

$$F'''\cancel{\frac{\beta}{\nu}} + \cancel{\frac{\beta}{\nu}}\left(F\cancel{\sqrt{\frac{\nu}{\beta}}}\cdot F''\cancel{\sqrt{\frac{\beta}{\nu}}} - F'^2\right) = -\cancel{\frac{\beta}{\nu}}$$

$$\therefore \quad \underline{F''' + FF'' - F'^2 + 1 = 0}$$

with $F(0) = F'(0) = 0$ & $F'(\infty) = 1$.

Q5. Examiner's Comment:

Not popular, and not well done. Limited physical grasp and excessive algebra hampered all but the best candidates.

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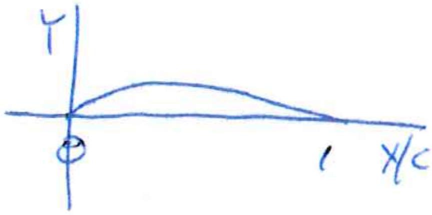
⑥



Camber line:

$$Y_c = h\,\frac{X}{c}\left(1-\frac{X}{c}\right)\left(\frac{X}{c}+a\right) \quad \left\{\begin{array}{l}\frac{X}{c}=0 \quad Y_c = 0\\ \phantom{\frac{X}{c}}=1 \quad \ 0\end{array}\right.$$

DATA BOOK: camber slope $-2\frac{dY_c}{dX} = g_0 + \sum_{n=1}^{\infty} g_n \cos n\theta$

with $\frac{X}{c} = \frac{1}{2}(1+\cos\theta)$

and $C_L = \pi\left(g_0 + \frac{1}{2}g_1\right)$; $C_m = \frac{C_L}{4} + \frac{\pi}{8}(g_1+g_2)$.

(a)

$$\frac{Y_c}{h} = \frac{X}{c}\left(\frac{X}{c} + a - \frac{X^2}{c^2} - \frac{X}{c}a\right) = -\frac{X^3}{c^3} + (1-a)\frac{X^2}{c^2} + a\frac{X}{c}$$

$$\therefore \frac{1}{h}\frac{dY_c}{dX} = -\frac{3X^2}{c^3} + 2(1-a)\frac{X}{c^2} + \frac{a}{c} \qquad \text{Sub: } \frac{X}{c} = \frac{1}{2}(1+\cos\theta)$$

$$= -\frac{3}{c}\left(\frac{1+\cos\theta}{2}\right)^2 + \frac{\cancel{2}(1-a)}{c}\left(\frac{1+\cos\theta}{\cancel{2}}\right) + \frac{a}{c}$$

$$= \frac{a}{c} + \frac{1-a}{c} + \cos\theta\cdot\frac{1-a}{c} - \frac{3}{4c}\left(1 + 2\cos\theta + \underbrace{\cos^2\theta}_{\frac{1}{2}(1+\cos2\theta)}\right)$$

$$= \left[\frac{1}{c} - \underset{-\frac{3}{8c}}{\frac{3}{4c}}\right] + \cos\theta\left[\frac{1-a}{c} - \frac{3}{2c}\right] + \cos2\theta\left[-\frac{3}{8c}\right]$$

$$\therefore \frac{dY_c}{dX} = \frac{h}{c}\left\{\overset{g_0}{\left[\frac{1}{8}\right]} + \cos\theta\overset{g_1}{\left[-\left(a+\frac{1}{2}\right)\right]} + \cos2\theta\overset{g_2}{\left[-\frac{3}{8}\right]}\right\}$$

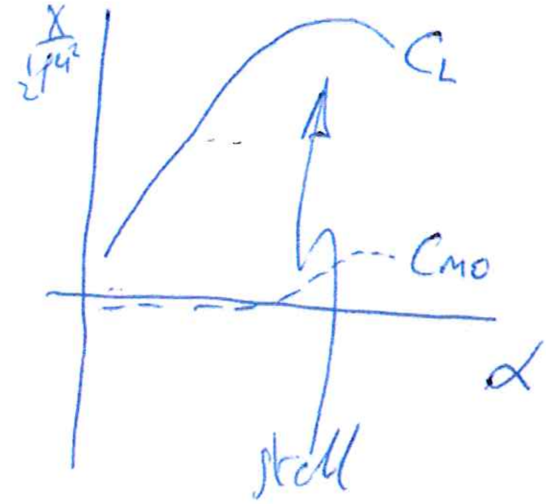
(−2×) **Hence**: $g_0 = \frac{1}{4}*$; $g_1 = (2a+1)*$; $g_2 = +\frac{3}{4}*$; all $* \frac{h}{c}$.

$$\therefore \quad C_L = \pi\frac{h}{c}\left(a+\frac{3}{4}\right); \quad C_m = \frac{C_L}{4} + \frac{\pi}{8}\frac{h}{c}\left(2a+\frac{7}{4}\right).$$

(b) Writing $C_m = C_L/4 + \underbrace{C_{m0}}_{\frac{\pi}{8}(g_1+g_2)}$

suggests that the lift acts at the quarter chord point — and that we expect C_{m0} to be a constant.

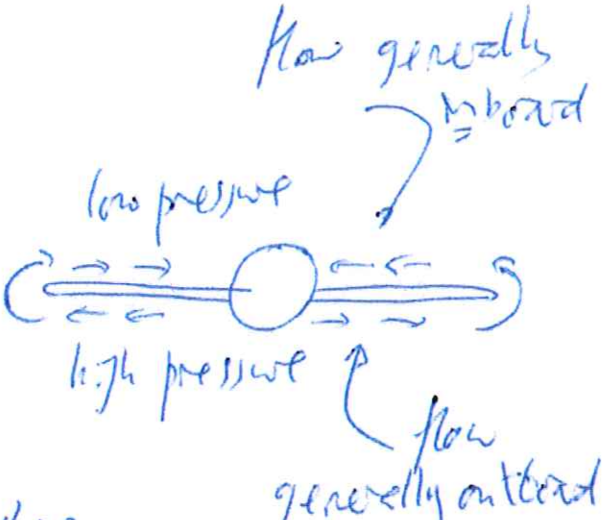
This is borne out by experimental data (see right) as long as viscous effects are not significant (as we expect).



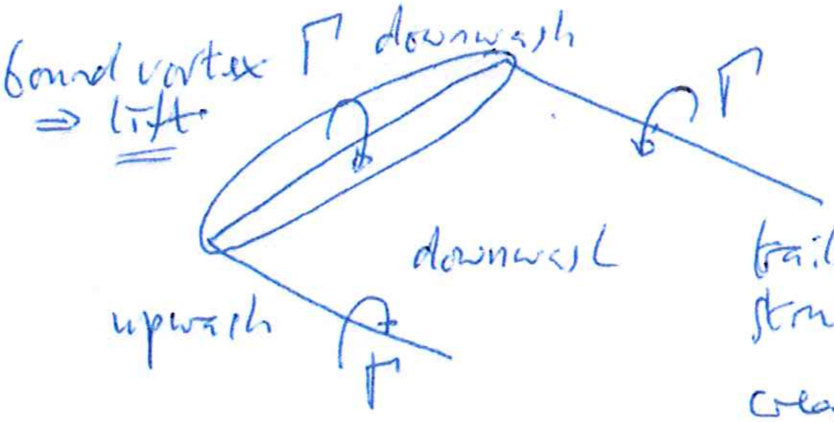
(c) Once stall occurs then the pitching moment will be positive or negative depending on whether the stall starts from the TE (responding to increasing suction side loading) or from the LE (responding to increased streamline curvature and growing suction peak followed by stronger diffusion)..

Q6. Examiner's Comment: Very popular and generally well done.

7 (a)



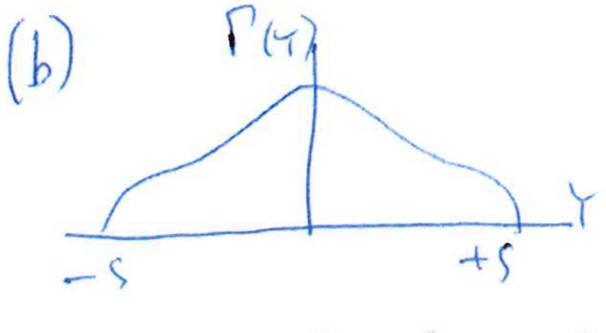
flow from lower to higher pressure around wing tip ⇒ trailed vortex



trailed vortical structures ⇒ drag created by the dissipation of the secondary KE.

[all "Γ" by Kelvin]

(b)



$\Gamma(Y=-s\cos\theta)=Us\sum_{n\,odd}G_n\sin n\theta$

DATA BOOK

Downwash angle, $\alpha_d(Y)=\frac{1}{4\pi U}\int_{-s}^{+s}\frac{d\Gamma}{dY}\cdot\frac{d\eta}{Y-\eta}$

with $\eta=-s\cos\phi$ & $Y=-s\cos\theta$ and using the Fourier series for Γ ... $\alpha_d=\frac{1}{4\pi}\sum nG_n\underbrace{\int_0^\pi\frac{\cos n\phi}{\cos\phi-\cos\theta}d\phi}$

DATA BOOK: Glauert Integral $\pi\sin n\theta/\sin\theta$

Hence, $\alpha_d(y) = \frac{1}{4}\sum_{n\,\text{odd}} n G_n \frac{\sin n\theta}{\sin\theta}$

(c) Next, lift and drag.

$y = -s\cos\theta$

Lift, $C_L = \frac{\rho U \int_{-s}^{+s} \Gamma(y)\,dy}{\frac{1}{2}\rho U^2 S} = \frac{2}{US}\int_0^\pi \left(Us\sum_{n\,\text{odd}} G_n \sin n\theta\right) s\sin\theta\,d\theta$

$= \frac{2s^2}{S}\int_0^\pi G_1 \sin^2\theta\,d\theta = \frac{\pi AR}{4} G_1$ ——— ①

$AR = 4s/S$

Induced Drag, $C_{D_i} = \frac{\rho \int_{-s}^{+s} \Gamma\, \overset{\overset{\alpha_d/U}{=}}{w_d}\,dy}{\frac{1}{2}\rho U^2 S} = \frac{2}{US}\int_0^\pi \underset{\Gamma}{\left(Us\sum G_n \sin n\theta\right)} \underset{\alpha_d}{\left(\frac{1}{4}\sum m G_m \frac{\sin m\theta}{\sin\theta}\right)} s\sin\theta\,d\theta$

!!!

$= \frac{s^2}{2S}\sum\sum m G_m G_n \underbrace{\int_0^\pi \sin n\theta \sin m\theta\,d\theta}$

$= 1$ if $m = n$
0 if $m \neq n$ } "orthogonal"

Sub ① ↗

$\therefore C_{D_i} = \frac{\pi AR}{16}\sum_{n\,\text{odd}} n G_n^2 = (1+\delta)\frac{C_L^2}{\pi AR}$

$\delta = 3\left(\frac{G_3}{G_1}\right)^2 + 5\left(\frac{G_5}{G_1}\right)^2 + \cdots$

Hence an elliptically loaded wing ($G_{3,5,\ldots} = 0$) has minimum induced drag.

Confirmed via DATA BOOK.

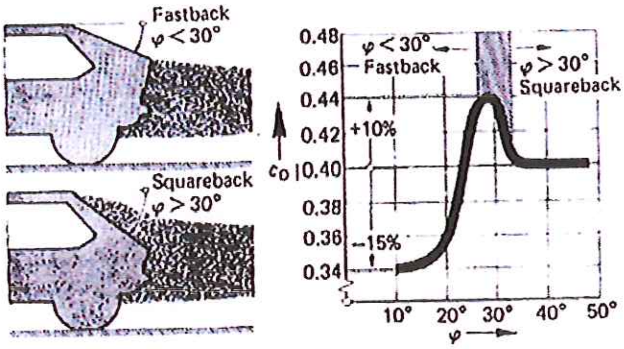
Q7. Examiner's Comment:
Rather unpopular for such a standard question; generally good and complete answers.

(8).

Solution

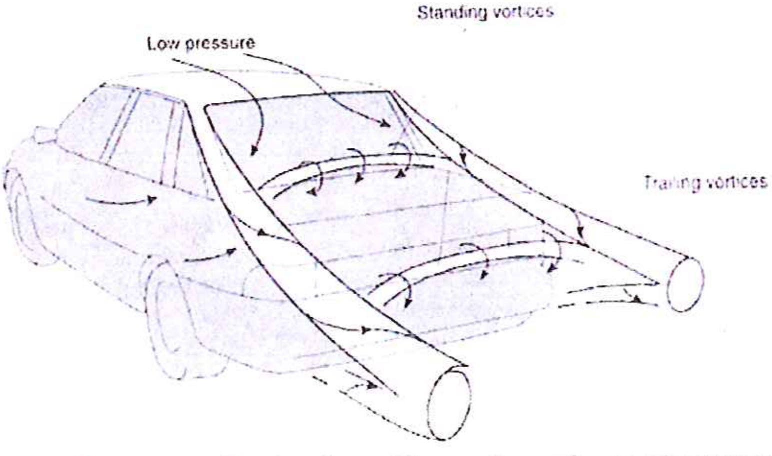
a) The angle α defines the slope of the rear windscreen (φ in the sketch below).

b) The two flow regimes are 'fastback' (attached flow along the windscreen) and 'squareback' (flow separates at the roof/windscreen junction).



The squareback shape has a larger wake and thus a larger drag coefficient. The drag minimum is typically in the region of 10°-20°.

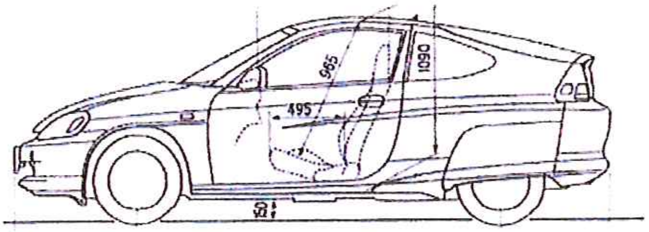
c) In regime 2, as the angle increases a pair of counter-rotating vortices form above the rear windscreen. These vortices increase in strength as the angle increases until the flow separates at the roof. The additional energy in the vortices incurs additional drag.



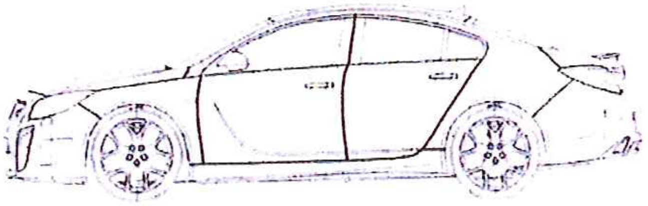
d) Small cars need to be short. Thus, a long sloping rear would significantly reduce the internal volume and therefore the most common solution is the 'squareback' design with a steep rear windscreen (and flow separation at the roof/windscreen junction). This maximises the available volume (for rear passengers and boot space).

Large cars can afford the additional length incurred by a gently sloped rear windscreen. Thus larger cars are often designed either as:

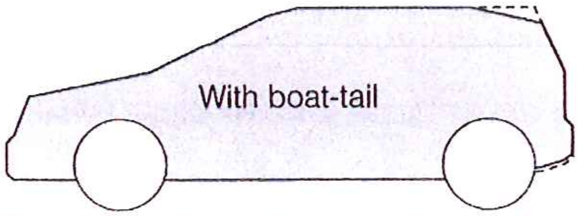
1) a 'fastback' with a slope of around 20° like the Toyota Prius (cd=0.25):



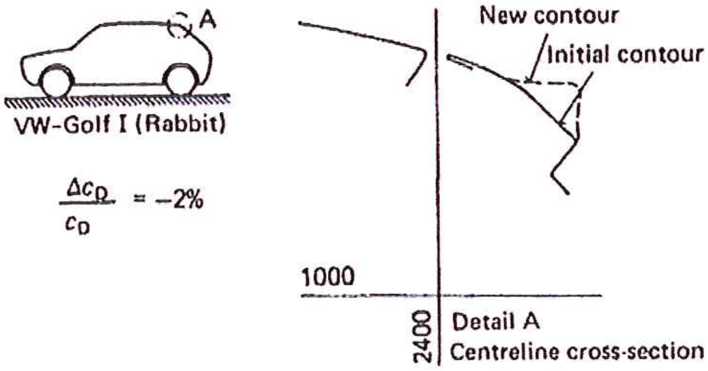
2) or a typical 'saloon' type shape with a gently sloped rear windscreen (again around 20°) followed by an additional boot, as seen on the Vauxhall Insignia (Cd=0.26-0.27):



e) Hatchback shapes can incorporate a small amount of boat-tailing in the roof line:



The addition of a small rear spoiler can also reduce drag slightly:



Q8. Examiner's Comment:

Most popular question – luckily, as it pushed up the average marks to the target zones. Often poorly structured written answers and generally nasty, poorly labelled sketches.

—END—