1. (a) Sink volumetric flow $\geq$ sum of sources

$$\therefore \underline{k \geq 2}$$

(b) Complex potential: $F(z) = -\frac{km}{2\pi}\ln z + \frac{m}{2\pi}\left[\ln(z+d) + \ln(z-d)\right]$

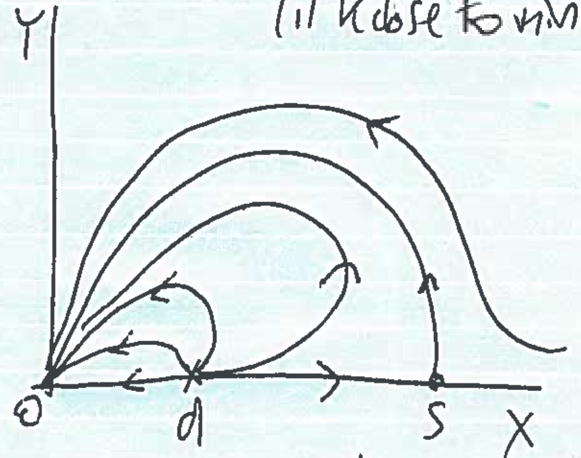
$$u - iv = \frac{dF}{dz} = \frac{m}{2\pi}\left[-\frac{k}{z} + \frac{1}{z+d} + \frac{1}{z-d}\right] = \frac{m}{2\pi}\left[-\frac{k}{z} + \frac{2z}{z^2-d^2}\right]$$

Stagnation point @ $u=v=0$ $\therefore$ $2z^2 = k(z^2-d^2)$

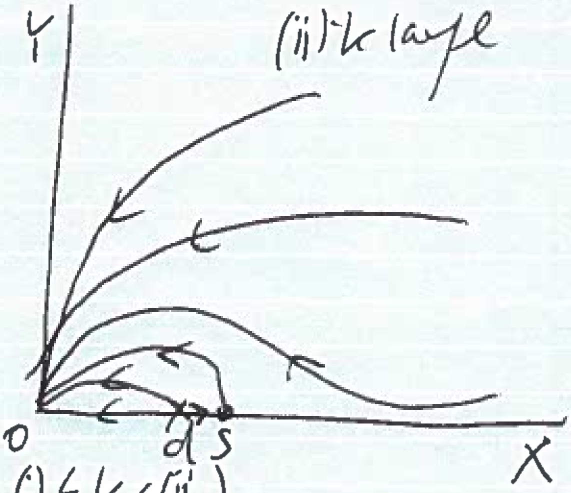
$$\Rightarrow \quad z^2 = \frac{kd^2}{k-2}$$

So for $k>2$, stagnation pt at $\underline{\pm\sqrt{\frac{k}{k-2}}\,d}$.

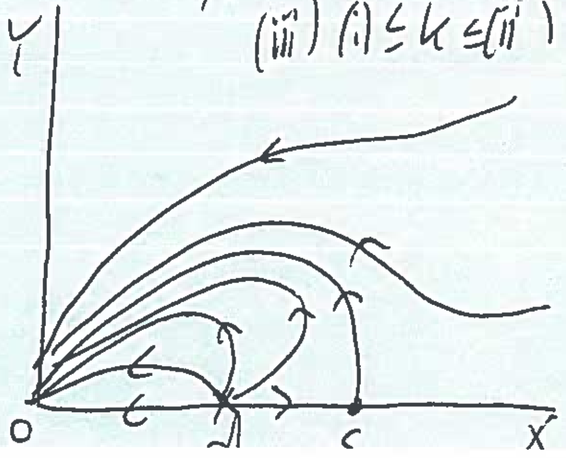
(c) (i) k close to min.



(ii) k large



(iii) (i) $\leq k \leq$ (ii)



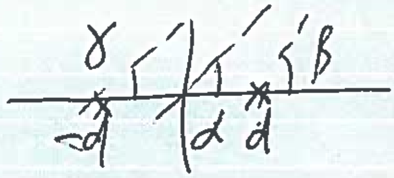
(d) (i) $F(z) = -\frac{km}{2\pi}(\ln r + i\alpha)$

$+ \frac{m}{2\pi}(\dots i\beta + i\gamma)$

$= \phi + i\psi$

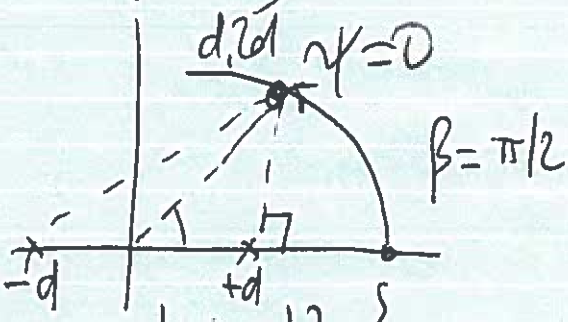
$z = re^{i\theta}$

$\ln z = \ln r + i\theta$



∴ stream function $\psi = -\frac{km}{2\pi}\alpha + \frac{m}{2\pi}(\beta+\gamma)$

Limiting case has stagnation streamline passing through $(d, 2d)$

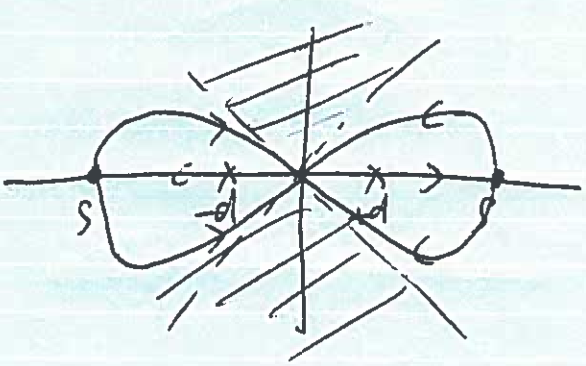


$\gamma = \frac{\pi}{4}$ $\alpha = \tan^{-1}2$

∴ $k\tan^{-1}2 = \frac{\pi}{2} + \frac{\pi}{4} = \frac{3\pi}{4}$

∴ $k = 2.13$

(ii)



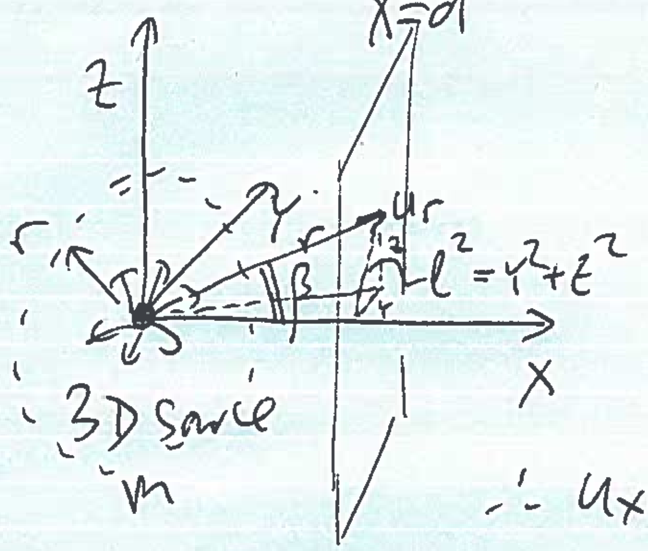
Near the origin $\gamma \sim 0$, $\beta \sim \pi$

∴ $\alpha = \frac{m/2}{km/2\pi} = \frac{\pi}{k} = 1.48$

∴ "wedges" have: semi-angle $\frac{\pi}{2} - \alpha = 0.095$ rad

or: included angle 0.189 rad

2, (a)



radial velocity,

$u_r = \frac{m}{4\pi r^2}$ 3D !

(i) $\therefore u_x = u_r \cos\beta$

$= u_r \frac{d}{r}$

$\therefore u_x = \frac{m}{4\pi} \cdot \frac{d}{r^3}$ but $r^2 = d^2 + l^2$

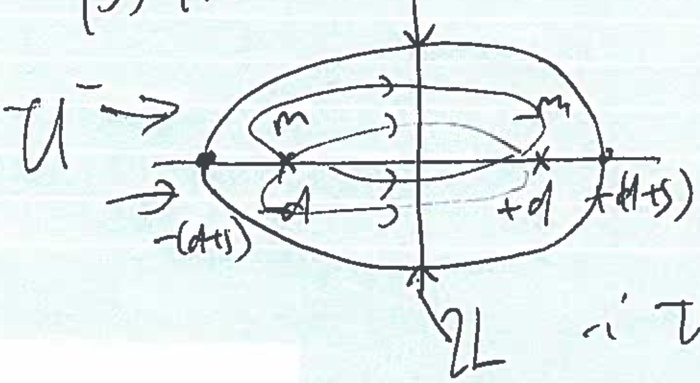
$\therefore u_x = \frac{m}{4\pi} \cdot \frac{d}{(d^2+l^2)^{3/2}}$ where $l^2 = y^2 + z^2$

(ii) $Q(L) = \int_0^L u_x 2\pi l \, dl = \frac{md}{2} \int_0^L \frac{l}{(d^2+l^2)^{3/2}} dl$

$= \frac{md}{2} \left[-(d^2+l^2)^{-1/2} \right]_0^L$

$\therefore Q(L) = \frac{1}{2} m \left[1 - \frac{d}{\sqrt{d^2+L^2}} \right]$

(b) (i)



At the upstream stagnation point, $x = -(d+s)$

$U - \frac{m}{4\pi s^2} + \frac{m}{4\pi (2d+s)^2} = 0$

$\therefore U = \frac{m}{4\pi} \left[\frac{(2d+s)^2 - s^2}{s^2 (2d+s)^2} \right]$.

$$\therefore \; U = \frac{m}{4\pi}\left[\frac{4d^2 + 4ds}{s^2(2d+s)^2}\right] = \frac{m}{\pi}\left[\frac{d}{s^2}\cdot\frac{(d+s)}{(2d+s)^2}\right]$$

$$\therefore \; \frac{m}{\pi d^2 U} = \left(\frac{s}{d}\right)^2 \frac{(2+s/d)^2}{(1+s/d)}$$

(ii) The max. cross section is at $x = 0$ and the total volume flow $= m$, so:

$$m = \underbrace{2\,Q(L)}_{\text{source, sink}} + \underbrace{U\pi L^2}_{\text{free stream}}$$

$$\therefore \; m = m\left[1 - \frac{d}{\sqrt{d^2+L^2}}\right] + \pi L^2 U \qquad \div\, d^2 U$$

$$\therefore \; \frac{m}{\pi d^2 U}\left\{\cancel{1} - \cancel{1} + \frac{d}{\sqrt{d^2+L^2}}\right\} = \left(\frac{L}{d}\right)^2$$

$$\therefore \; \frac{m}{\pi d^2 U} = \left(\frac{L}{d}\right)^2 \sqrt{1 + \left(\frac{L}{d}\right)^2}$$

(c) (i) $\left(\frac{L}{d}\right) \ll 1 \Rightarrow \frac{m}{\pi d^2 U} \sim \left(\frac{L}{d}\right)^2 \quad \therefore \; \underline{\frac{L}{d} \sim \sqrt{\frac{m}{\pi d^2 U}}}$.

(ii) $\therefore \; \frac{m}{\pi d^2 U} \ll 1$ also, so $\therefore \; s/d \ll 1$ as well.

$$\therefore \; \frac{L}{d+s} = \frac{L/d}{(1+s/d)} \sim \frac{L}{d} \simeq \sqrt{\frac{m}{\pi d^2 U}} \text{ also}$$

(iii) $$c_p = \frac{p - p_\infty}{\frac{1}{2}\rho U^2} = \frac{\cancel{p_0} + \frac{1}{2}\rho u^2 - \cancel{p_\infty} - \frac{1}{2}\rho U^2}{\frac{1}{2}\rho U^2} = 1 - \left(\frac{u}{U}\right)^2$$

Max. suction at max. speed which will be at max. diameter of body – and, helpfully, axial!

So, from (a)(i) max. $u = U + 2 \times \frac{m}{4\pi} \cdot \frac{d}{(d^2+L^2)^{3/2}}$

$$\therefore \frac{u}{U} = 1 + \underbrace{\frac{m}{2\pi U} \cdot \frac{1}{d^2}}_{\left(\frac{L}{d+s}\right)^2} \cdot \frac{1}{\left(1 + \underbrace{\left(\frac{L}{d}\right)^2}_{\ll 1}\right)^{3/2}}$$

$$\therefore c_p = 1 - \left(\frac{u}{U}\right)^2 = 1 - \left[1 + \frac{1}{2}\left(\frac{L}{d+s}\right)^2\right]^2$$

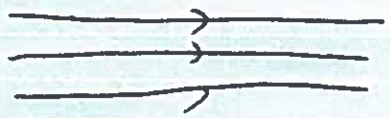
$$\sim 1 - \left(1 + \left(\frac{L}{d+s}\right)^2 + \dots\right)$$

$$\therefore c_{p\,max} \sim \left(\frac{L}{d+s}\right)^2$$

3. (a)

$$\underbrace{\frac{D\bar{\omega}}{Dt}}_{\text{Convection of vorticity.}} = \underbrace{\bar{\omega}\cdot\nabla\bar{u}}_{\text{Vortex stretching and tilting (3D only).}} + \underbrace{\nu\nabla^2\bar{\omega}}_{\text{Viscous diffusion of vorticity.}}$$

(b)(i)



$\frac{du}{dx} + \frac{dv}{dy} = 0$; $v=0$ ∴ $\frac{du}{dx} = 0$

2D u, $v=0$

∴ u independent of x

(but not necessarily constant, may vary in y).

(ii) In 2D w is a scalar, $\omega = \frac{du}{dy} - \cancel{\frac{dv}{dx}}$ ($v=0$) ∴ $\bar{\omega}\cdot\nabla\bar{u} = 0$

$$\therefore \frac{d\omega}{dt} + \cancel{u\frac{d\omega}{dx}} + \cancel{v\frac{d\omega}{dy}} = \nu\nabla^2\omega$$

$\frac{d\omega}{dx} = \cancel{\frac{d}{dx}\left(\frac{du}{dy}\right)}$ 0 as $u \neq fn(x)$; $v=0$; $\nabla^2\omega = \cancel{\frac{d^2\omega}{dx^2}} + \frac{d^2\omega}{dy^2}$ 0 as $d\omega/dx = 0$

$$\Rightarrow \underline{\frac{d\omega}{dt} = \nu\frac{d^2\omega}{dy^2}}.$$

(iii) Given a candidate solution: $w = \frac{A}{\sqrt{\nu t}}e^{-y^2/4\nu t}$

Substitute into (ii)

$$LHS = \frac{A}{\sqrt{\nu}}\cdot e^{-y^2/4\nu t}\cdot -\frac{1}{2}\frac{1}{t^{3/2}} + \frac{A}{\sqrt{\nu t}}e^{-y^2/4\nu t}\cdot -\frac{y^2}{4\nu}\cdot\frac{-1}{t^2}.$$

$$\therefore \ LHS = \frac{A}{\sqrt{\nu t}} e^{-y^2/4\nu t}\left[\frac{-1}{2t} + \frac{y^2}{4\nu t^2}\right] = \frac{A}{\sqrt{\nu t}} e^{-y^2/4\nu t}\left[\frac{y^2 - 2\nu t}{4\nu t}\right]$$

Whereas:

$$RHS = \nu \frac{d}{dy}\left[\frac{A}{\sqrt{\nu t}} e^{-\frac{y^2}{4\nu t}} \cdot -\frac{2y}{4\nu t}\right] = \frac{-1}{2}\frac{\nu A}{(\nu t)^{3/2}}\frac{d}{dy}\left[e^{-y^2/4\nu t}\, y\right]$$

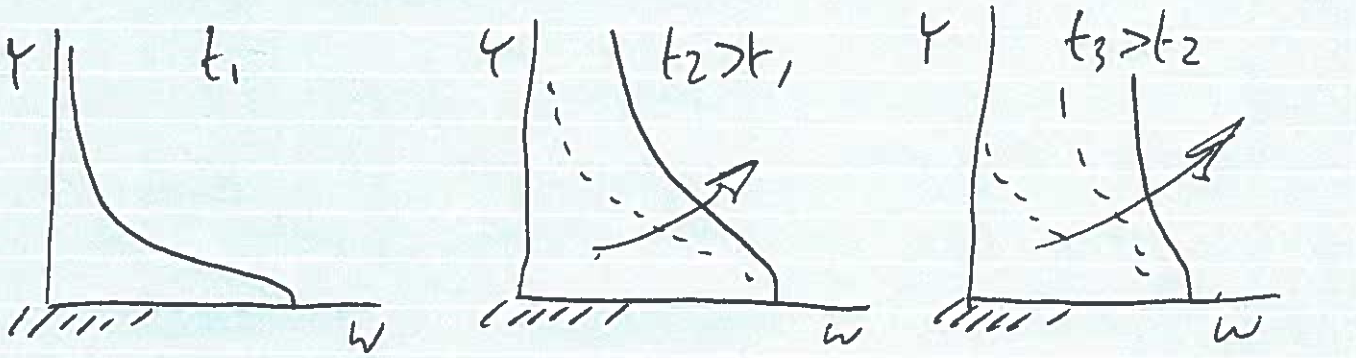
$\frac{dw}{dy}$

$$= \frac{-1}{2}\frac{\nu A}{(\nu t)^{3/2}}\left[e^{-y^2/4\nu t} + y e^{-y^2/4\nu t} \cdot -\frac{2y}{4\nu t}\right]$$

$\frac{1}{2}\frac{A}{\sqrt{\nu t}}\cdot\frac{1}{t}$

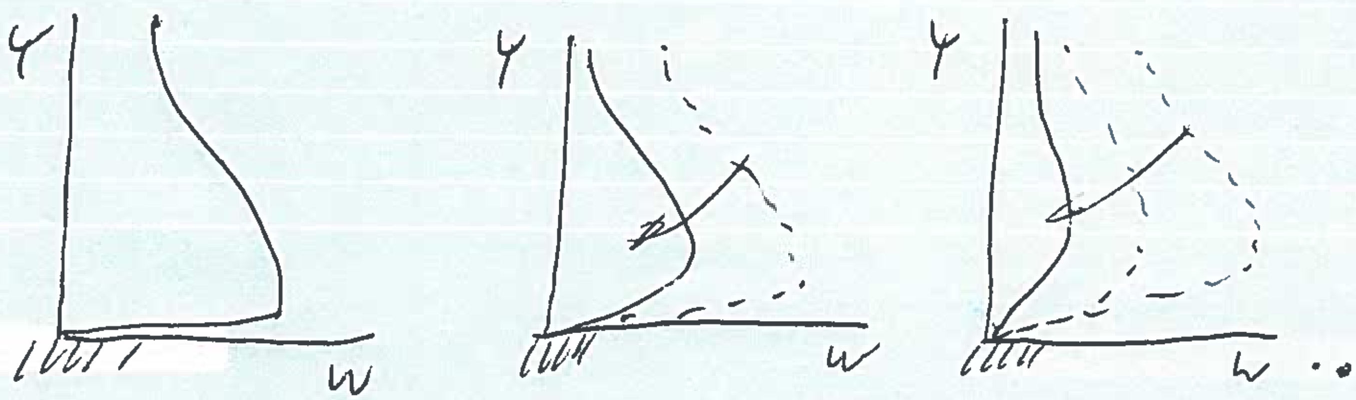
$$= \frac{A}{\sqrt{\nu t}} e^{-y^2/4\nu t}\left[-\frac{1}{2t} + \frac{y^2}{4\nu t}\right] = \frac{A}{\sqrt{\nu t}} e^{-y^2/4\nu t}\left[\frac{y^2 - 2\nu t}{4\nu t}\right]$$

$\therefore$ RHS = LHS confirming candidate solution.

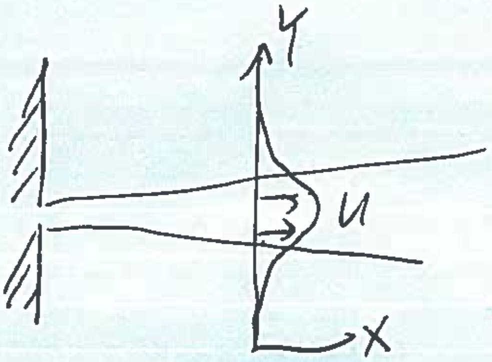


(c) (i) If now no shear at $y=0$ then $w = 0$.

(ii) The effect will diffuse away from the wall on similar time scales to (b)(iii) above



4. (a)



B'layer equations with zero pressure gradient

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial x^2}$$

with $u(x,\infty)=0$, $\frac{\partial u}{\partial y}(x,0)=0$ } b/c's.

(b) There are no pressure gradients nor external forces so conservation of momentum means

$$J = \int_{-\infty}^{+\infty} \rho u^2 \, dy = \text{constant}$$

$\therefore$ J is independent of x.

(c) Given the streamfunction $\psi = \nu^{1/2} x^{1/3} f(\eta)$

where $\eta = \frac{y}{3\nu^{1/2}x^{2/3}}$

$$\therefore u = \frac{\partial \psi}{\partial y} = \nu^{1/2}x^{1/3} f' \cdot \frac{1}{3\nu^{1/2}x^{2/3}} = \frac{f'}{3x^{1/3}} \qquad \left(f' = \frac{df}{d\eta}\right)$$

$$v = -\frac{\partial \psi}{\partial x} = -\nu^{1/2}\frac{1}{3x^{2/3}} f - \nu^{1/2}x^{1/3} f' \frac{\partial \eta}{\partial x}$$

$$\frac{\partial u}{\partial x} = \frac{f'}{3} \cdot -\frac{1}{3}\frac{1}{x^{4/3}} + \frac{1}{3x^{1/3}} f'' \frac{\partial \eta}{\partial x}$$

$$\frac{\partial u}{\partial y} = \frac{f''}{3x^{1/3}} \cdot \frac{1}{3\nu^{1/2}x^{2/3}} = \frac{f''}{9x\nu^{1/2}} \; ; \; \frac{\partial^2 u}{\partial y^2} = \frac{f'''}{27\nu x^{5/3}}.$$

∴ assembling: $\frac{f'}{3x^{1/3}}\left[-\frac{f'}{9x^{4/3}} + \frac{1}{3x^{1/3}} f'' \frac{d\eta}{dx}\right]$ $\qquad u\frac{du}{dx}$

$$+ \left[-\frac{\nu^{1/2}}{3x^{2/3}} f - \nu^{1/2} x^{1/3} f' \frac{d\eta}{dx}\right] \frac{f''}{9x\nu^{1/2}} \qquad v\frac{du}{dy}$$

$$= \nu f''' / 27\nu x^{5/3} \qquad \nu\frac{d^2u}{dy^2}$$

∴ $-\frac{f'^2}{\cancel{27}x^{5/3}} + \cancel{\frac{f'f''\,d\eta/dx}{9x^{2/3}}} - \frac{ff''}{\cancel{27x^{5/3}}} - \cancel{\frac{f'f''\,d\eta/dx}{9x^{2/3}}} = \frac{\cancel{\nu}f'''}{\cancel{27\nu x^{5/3}}}$

∴ $\underline{f'^2 + ff'' + f''' = 0}$ $\quad \left.\begin{array}{l} f(0)=f''(0)=0 \\ f'(\infty)=0 \end{array}\right\}$ bcs.

(d) $0 = f'^2 + ff'' + f''' = (f'' + ff')'$!

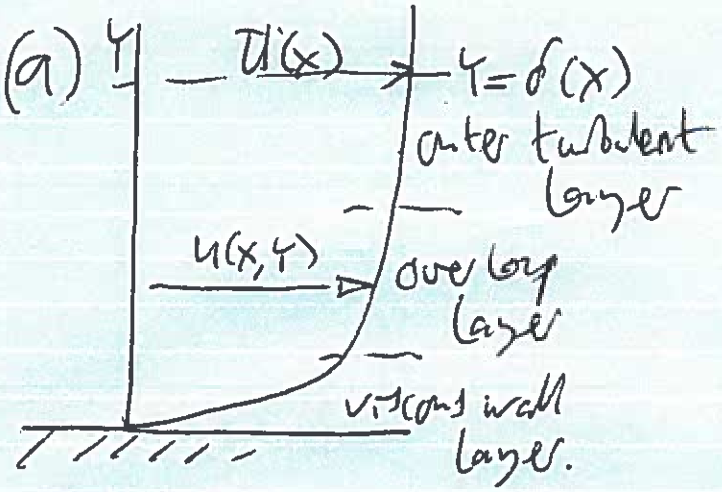
∴ $f'' + ff' = 0$ as $f(0) = f''(0) = 0$

↑ constant of integration

Again, $0 = (f'' + ff') = (f' + \frac{1}{2}f^2)'$!

$$\Rightarrow \quad \underline{f' + \frac{1}{2}f^2 = \text{constant}}.$$

5. (a)



Friction velocity
$u^* = \sqrt{\tau_w/\rho}$

Away from the outer layer, near the wall, the flow is not dependent on U or δ so

$$u = f(y, u^*, \nu)$$

ie. $\frac{u}{u^*} = F\left(\frac{y u^*}{\nu}\right)$ the "law of the wall"

(b) In the overlap layer the viscosity ceases to play a role so we can say the mean gradient

$$\frac{du}{dy} = f(u^*, y)$$

Dimensional arguments give: $\frac{du}{dy} = \frac{u^*}{\kappa y}$ where κ is the von Karman const.

Hence integrating: $\frac{u}{u^*} = \frac{1}{\kappa}\ln\left(\frac{u^* y}{\nu}\right) + B$

[OR: Prandtl mixing length: $\mu_T = \rho(\kappa y)^2\left|\frac{du}{dy}\right|$ near wall

$\tau = \text{const.} = \tau_w = \mu_T\frac{du}{dy} \Rightarrow \tau_w = \rho(\kappa y)^2\left(\frac{du}{dy}\right)^2$

ie. $\sqrt{\tau_w/\rho} = u^* = \kappa y\frac{du}{dy}$; integrate $\frac{u}{u^*} = \frac{1}{\kappa}\ln y + A$.]

(c) Very close to the wall in the viscous sub layer the velocity profile is linear & laminar

$$\therefore \tau_w = \mu \frac{du}{dY} = \mu \frac{u}{Y}$$

$$\therefore \rho u^{*2} = \mu \frac{u}{Y} \Rightarrow \frac{u}{u^*} = \frac{u^* Y}{\nu} \quad [u^+ = y^+]$$

(d) The average velocity in the circular pipe is: $V = \frac{1}{\pi R^2}\int_0^R u(r)\, 2\pi r\, dr$

$$\therefore V = \frac{2}{R^2}\int_0^R \left(\frac{u^*}{k}\ln\frac{(R-r)u^*}{\nu} + B\right) r\,dr \qquad Y = (R-r)u^*/\nu$$

$$= \frac{2u^*}{R^2}\int_{Ru^*/\nu}^{0}\left(\frac{1}{k}\ln Y + B\right)\left(R - \frac{\nu}{u^*}Y\right)\cdot -\frac{\nu}{u^*}dY$$

$$= \frac{2\nu}{R^2}\int_0^{Ru^*/\nu}\left(\frac{R}{k}\ln Y + BR - \frac{\nu}{ku^*}Y\ln Y - \frac{B\nu Y}{u^*}\right)dY$$

$$= \frac{2\nu}{R^2}\left[\frac{R}{k}Y\ln Y - \frac{R}{k}Y + BRY - \frac{\nu}{ku^*}\frac{Y^2}{2}\ln Y + \frac{\nu}{ku^*}\frac{Y^2}{4} - \frac{1}{2}\frac{B\nu Y^2}{u^*}\right]_0^{Ru^*/\nu}$$

Given: $\left[\int \ln y\, dy = y\ln y - y \;:\; \int y\ln y\, dy = \frac{1}{2}y^2\ln y - \frac{1}{4}y^2\right]$

$$= \frac{2\nu}{R^2}\left[\frac{1}{k}\frac{R^2 u^*}{\nu}\ln\left(\frac{Ru^*}{\nu}\right) - \frac{1}{2k}\frac{\nu}{u^*}\frac{R^2 u^{*2}}{\nu^2}\ln\left(\frac{Ru^*}{\nu}\right) + A\right.$$

$$\therefore \frac{V}{u^*} = \frac{1}{k}\ln\left(\frac{Ru^*}{\nu}\right) + B - \frac{3}{2k}$$

all the rest

Note this equals earlier "B".

(e) Darcy factor $f = \frac{8\tau_w}{\rho V^2} = 8\left(\frac{u^*}{V}\right)^2$

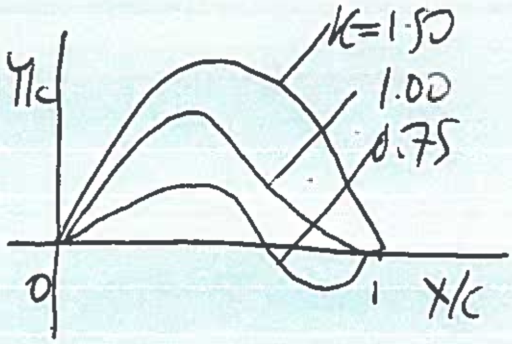
$$u_{max} = u(r=0) = u^*\left(\frac{1}{\kappa}\ln\left(\frac{R u^*}{\nu}\right) + B\right)$$

and recall $V = u^*\left(\frac{1}{\kappa}\ln\left(\frac{R u^*}{\nu}\right) + B - \frac{3}{2\kappa}\right)$

$$\therefore \frac{V}{u_{max}} = \frac{V/u^*}{u_{max}/u^*} = \frac{V/u^*}{V/u^* + \frac{3}{2\kappa}} = \frac{\sqrt{8/f}}{\sqrt{8/f} + \frac{3}{2\kappa}}$$

$$\therefore \frac{V}{u_{max}} = \frac{1}{1 + \frac{3}{2\kappa}\sqrt{f/8}}$$

6. (a)



Camber line:

$$Y_c = \frac{h}{k}\cdot\frac{x}{c}\left(1-\frac{x}{c}\right)\left(k-\frac{x}{c}\right)$$

$$= \frac{h}{k}\left[k\frac{x}{c} - \left(\frac{x}{c}\right)^2 - k\left(\frac{x}{c}\right)^2 + \left(\frac{x}{c}\right)^3\right]$$

$$\therefore\ -2\frac{dY_c}{dx} = -\frac{2h}{k}\left[\frac{k}{c} - \frac{2}{c}(1+k)\left(\frac{x}{c}\right) + \frac{3}{c}\left(\frac{x}{c}\right)^2\right]$$

Transform: $\frac{x}{c} = \frac{1}{2}(1+\cos\theta)$

$$\therefore\ -2\frac{dY_c}{dx} = \frac{2h}{kc}\left[-k + \cancel{2}(1+k)\frac{1}{\cancel{2}}(1+\cos\theta) - 3\left(\frac{1+\cos\theta}{2}\right)^2\right]$$

$$\therefore -2\frac{dY_c}{dx} = \frac{2h}{kc}\left[\cancel{-k} + 1 + \cos\theta + \cancel{k} + k\cos\theta - \frac{3}{4}\left(1+\underbrace{\cos^2\theta}_{\frac{1}{2}(1+\cos 2\theta)} + 2\cos\theta\right)\right]$$

$$= \frac{2h}{kc}\left[\frac{1}{4} + \left(k-\frac{1}{2}\right)\cos\theta - \frac{3}{4}\cdot\frac{1}{2}(1+\cos 2\theta)\right]$$

$$= \frac{2h}{kc}\left[-\frac{1}{8} + \underbrace{\left(k-\frac{1}{2}\right)}_{\frac{1}{2}(2k-1) = \frac{k}{2}\left(2-\frac{1}{k}\right)}\cos\theta - \frac{3}{8}\cos 2\theta\right]$$

$$= \sum g_n \cos n\theta$$

So by inspection: $g_0 = -\frac{1}{4}\cdot\frac{h}{kc}$; $g_1 = +\frac{h}{c}\left(2-\frac{1}{k}\right)$; $g_2 = -\frac{3h}{4kc}$

[This can also be done via classical Fourier analysis but is very much more tedious ...]?.

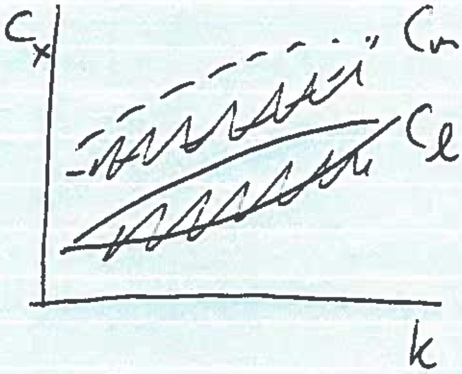
(b) Thin airfoil theory: $C_l = 2\pi\alpha + \pi(g_0 + \frac{1}{2}g_1)$

$$C_{m_0} = \frac{\pi}{8}(g_1 + g_2)$$

$$\therefore C_l = 2\pi\alpha + \pi\left[-\frac{1}{4}\left(\frac{h}{kc}\right) + \frac{1}{2}\frac{h}{kc}(2k-1)\right] = 2\pi\alpha + \frac{\pi h}{kc}\left(k - \frac{3}{4}\right)$$

$$C_l = \left(2\pi\alpha + \frac{\pi h}{c}\right) - \frac{3\pi}{4c} \cdot \frac{h}{k}$$

& $C_{m_0} = \frac{\pi}{8} \cdot \frac{h}{kc}\left[(2k-1) - \frac{3}{4}\right]$ $\therefore C_{m_0} = -\frac{7}{32}\frac{\pi h}{c}\frac{1}{k} + \frac{\pi}{4} \cdot \frac{h}{c}$



Therefore both C_l & C_{m_0} increase as k increases

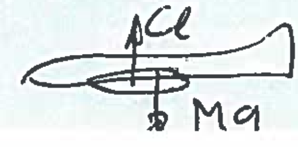
$$\frac{dC_l}{dk} = + \frac{3}{4} \cdot \frac{\pi h}{c k^2} ; \quad \frac{dC_{m_0}}{dk} = + \frac{7}{32} \frac{\pi h}{c k^2}$$

This is expected as the lift & moment are very sensitive to the camber slope near the trailing edge.

(c) The centre of pressure, x_c is: $\frac{x_c}{c} = \frac{1}{4} + \frac{C_{m_0}}{C_l}$

$$\therefore \frac{x_c}{c} = \frac{1}{4} + \frac{2k - 7/4}{8k-6}$$

The centre of pressure is undefined as $k \to 3/4$ as the associated lift coefficient also $\to 0$.
The centre of pressure is used to trim the aircraft wrt. pitching



7. (a) Datasheet: $\alpha_d(y) = \frac{1}{4\pi U}\int_{-s}^{+s} \frac{d\Gamma}{d\eta} \cdot \frac{1}{y-\eta}\, d\eta$

here $\Gamma = \Gamma_0\sqrt{1-\left(\frac{y}{s}\right)^2} = \Gamma_0\sqrt{1-\cos^2\theta}$ $\quad y = -s\cos\theta$

$= \Gamma_0 \sin\theta \quad \eta = -s\cos\phi$

$$\therefore \alpha_d() = \frac{1}{4\pi U}\int_0^{\pi} \Big(\underbrace{\Gamma_0 \cos\theta \, . \frac{d\phi}{d\eta}}_{d\Gamma/d\eta}\Big) \frac{1}{s\cos\phi - s\cos\theta}\, d\eta$$

$$= \frac{\Gamma_0}{4\pi U s}\underbrace{\int_0^{\pi} \frac{\cos\phi \, d\phi}{\cos\theta - \cos\phi}}_{\text{Glauert – see Datasheet}} = \underline{\underline{\frac{\Gamma_0}{4Us}}}$$

(b) Wing lift $L = \rho U\int_{-s}^{+s}\Gamma(y)\,dy = \rho U\int_0^{\pi}\underbrace{\Gamma_0\sin\theta}_{\Gamma} . \underbrace{s\sin\theta\, d\theta}_{dy}$

$$= \frac{\pi}{2}\rho U \Gamma_0 s$$

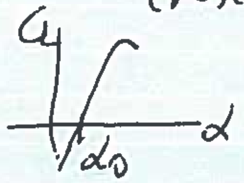
$$\therefore \Gamma_0 = \frac{120{,}000}{\frac{\pi}{2}\times 1.2\times 100\times 3.18\,(?)} = \underline{\underline{200 \ m^2 s^{-1}}}$$

(c) Lifting line: $\frac{\Gamma(y)}{\pi U c(y)} = \alpha - \underline{\underline{\alpha_0(y)}} - \underline{\alpha_d(y)}$

$\alpha_d(y) = \Gamma_0/4Us$ constant

AoA

local zero lift angle

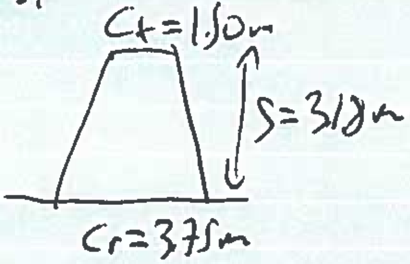


α is set such that $\alpha_0 = 0$ at the tip $y = s$

At $y = s$, $\Gamma = 0$ (no lift at tip) $\therefore \alpha = \alpha_d = \frac{200}{4\times100\times3.18} = \underline{\underline{9.0^{\circ}}}$

(0.157 rad)

$$\therefore \frac{\Gamma(y)}{\pi U c(y)} = -\alpha_0(y) \text{ as } \alpha = \alpha_d$$

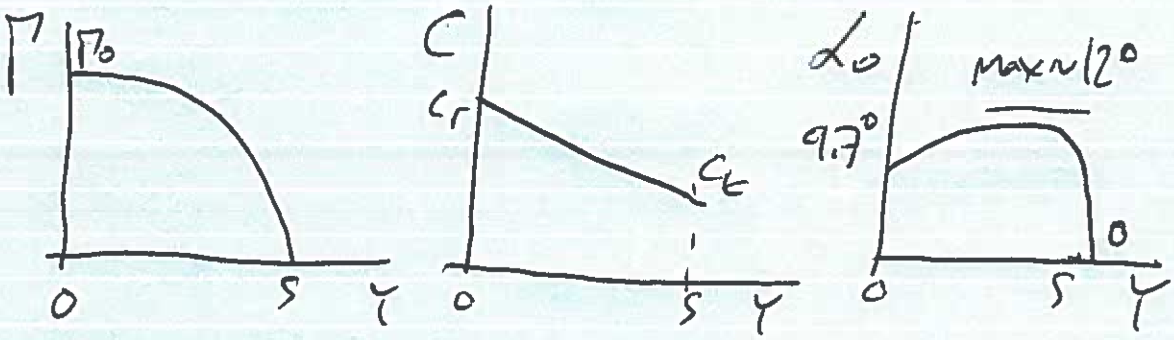


$$\therefore \alpha_0(y) = \frac{\Gamma_0\sqrt{1-(y/s)^2}}{\pi U c_r(1-0.6y/s)}$$

$$\frac{c_t}{c_r} = \frac{1.50}{3.75} = 0.4$$

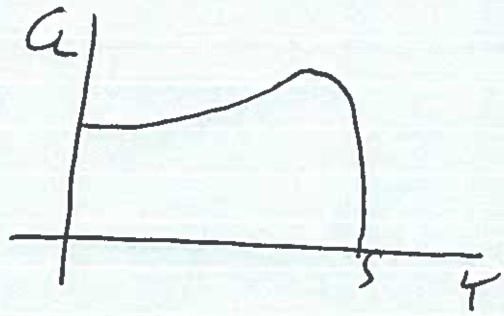
$$\therefore \alpha_0(\text{root}, y=0) = \frac{200}{\pi \times 100 \times 3.75} = 9.7^\circ$$

(0.170 rad)

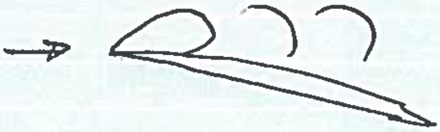


(d) If instead the wing has uniform camber and no twist then α_0 will be constant along the wing.

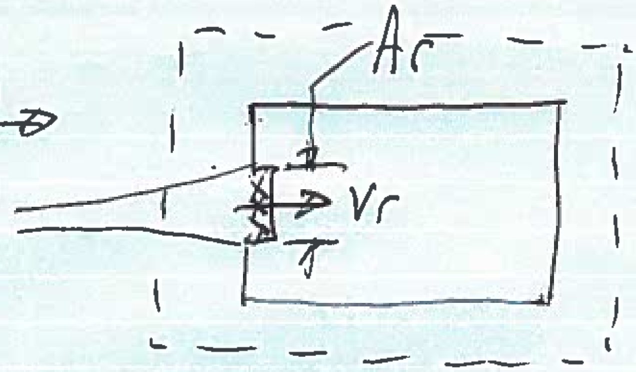
This will then change the spanwise loading as shown — no longer elliptic



This makes the wing vulnerable to tip stall — combined with the very thin LE (so LE stall which is abrupt) makes for a very dangerous airplane!



8. (a) V_∞ p_∞



Assuming the air entering the radiator leaves with negligible velocity,

$$F = \dot{m}_r(0 - V_\infty) \quad \therefore \underline{DRAG = \rho A_r V_r V_\infty}$$

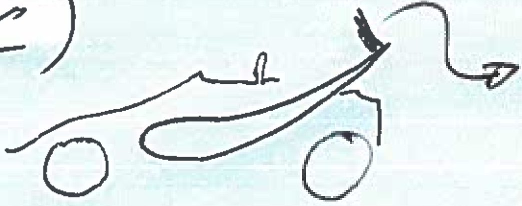
(b) The additional drag associated with the radiator expressed as a coefficient is

$$\Delta C_{Dr} = \frac{\rho A_r V_r V_\infty}{\frac{1}{2}\rho V_\infty^2 \underset{\text{car ref. area}}{A}} = 2\frac{V_r}{V_\infty}\cdot\frac{A_r}{A} = \frac{2\dot{m}_r}{\rho V_\infty A}$$

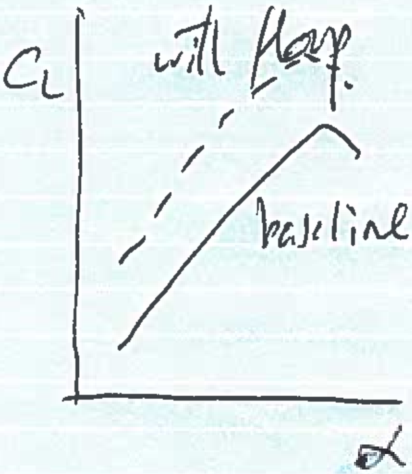
$$\therefore \Delta C_{Dr} = \frac{2\times 1}{1.2\times 30\times 2.5} = 0.022$$

A typical passenger car has $C_D \sim 0.3$ so radiator drag is about 7%; modern cars have rather small radiators to minimize this. The radiator is located near the front stagnation area and low to the ground to try to minimize underbody drag.

(c)



A Gurney flap is a flat plate fixed on the pressure side at the TE of an airfoil which increases the lift of the airfoil (at the expense of extra drag) by increasing the effective camber.



Due to its simplicity, and ease of adjustment, it is commonly used on racing cars to increase the downforce — and, hence, the cornering speed. It's particularly handy in F1 as the size of the wing can still fit inside the Regulation box.

WND
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