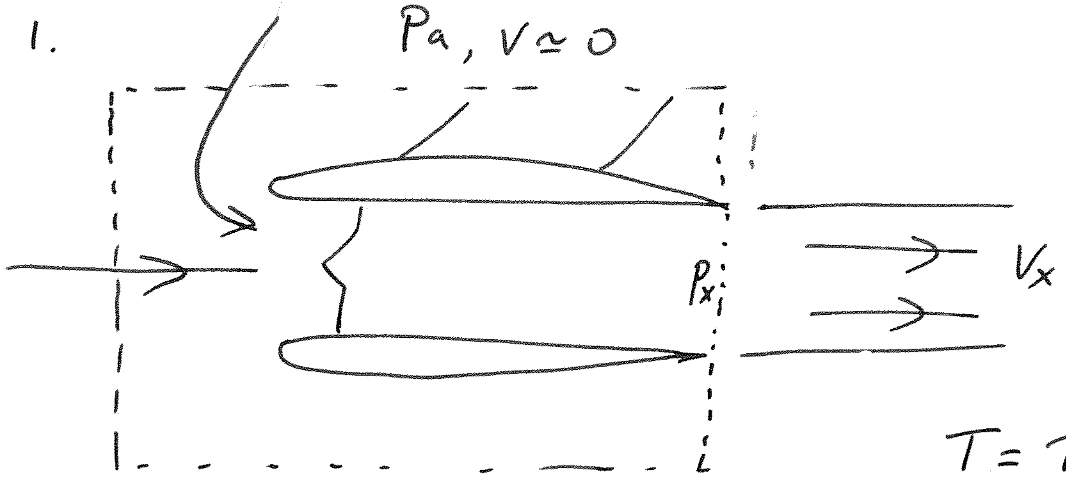
1.



T = Thrust

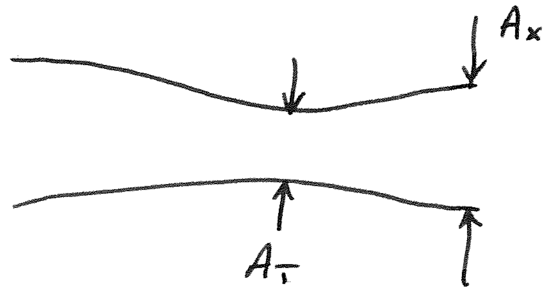
= force on air by engine

a) S.F.M.E. applied to C.V.

$$T + (p_a - p_x)A_x = \text{gain in momentum of air}$$
$$= \rho_x V_x A_x V_x$$

$$\therefore \quad T = (p_x + \rho_x V_x^2)A_x - A_x p_a$$
$$= F - A_x p_a \quad \text{where } F = p_x + \rho_x V_x^2$$

(b)



Isentropic flow throughout ⇒ no shocks downstream of nozzle ⇒ $p_x = p_a$

& $p_{0x} = p_{01} = 3.67$ bar

$\therefore$ At exit $\quad p_{0x} = p_x\left(1 + \frac{\gamma-1}{2}M_x^2\right)^{\gamma/\gamma-1}$

$\Rightarrow \underline{M_x = 1.5}$

$$\frac{\dot{m}\sqrt{c_p T_0}}{p_0 A_x} = f(1.5) = 1.0891 = \frac{\dot{m}\sqrt{c_p T_0}}{p_0 A_T}\cdot\frac{A_T}{A_x} = f(1)\frac{A_T}{A_x} = 1.281\frac{A_T}{A_x}$$

$$\therefore \frac{A_T}{A_x} = \cdot 8502 \quad \Rightarrow \quad A_x = 0.03695\ m^2$$

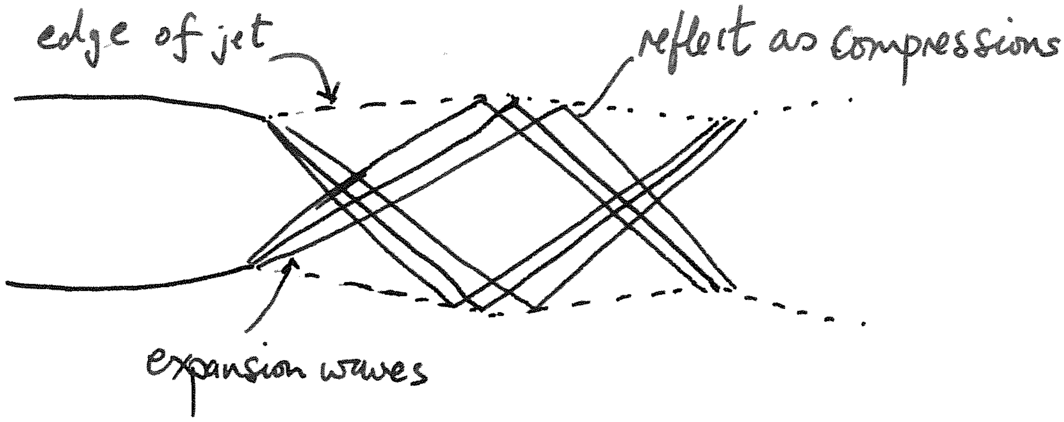
$$\text{Thrust} = \dot{m} V_x = \frac{\dot{m}\sqrt{c_p T_0}}{p_0 A} \cdot \frac{V_x}{\sqrt{c_p T_0}} p_0 A_x = 1.0891 \times .7878 \times 3.67\times10^5 \times .03695$$

$$= \underline{11635\ \text{N}} \quad (= 11600)$$

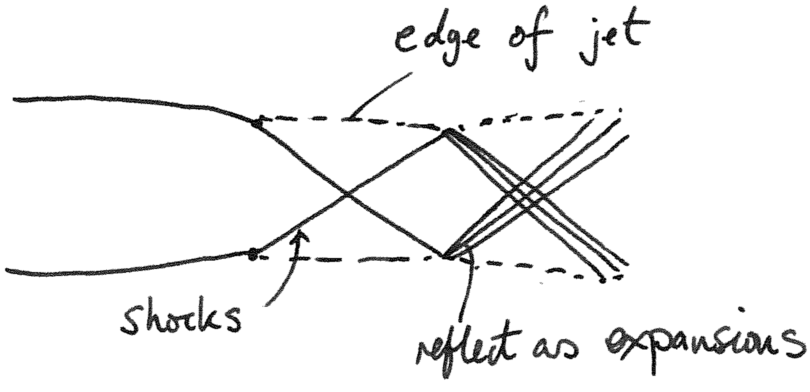
(c) Isentropic area ratio calculated above $= \dfrac{1}{.8502} = 1.176$

$\therefore \dfrac{A_x}{A_T} = 1.1$ is an <u>underexpanded</u> jet

$\dfrac{A_x}{A_T} = 1.3$ is an <u>overexpanded</u> jet



$\dfrac{A_T}{A_x} = 1.1$



$\dfrac{A_T}{A_x} = 1.3$

$$\frac{A_x}{A_T} = 1.1 \Rightarrow \frac{\dot{m}\sqrt{c_p T_0}}{p_0 A_x} = \frac{\dot{m}\sqrt{c_p T_0}}{p_0 A_T}\frac{A_T}{A_x} = \frac{1.281}{1.1} = 1.165$$

$$\Rightarrow M_x = 1.37, \quad A_x = 0.03456\ \text{m}^2$$

$$\text{Thrust} = F - A_x p_a = \frac{F}{\dot{m}\sqrt{c_p T_0}} \, \frac{\dot{m}\sqrt{c_p T_0}}{p_0 A_x} A_x p_0 - A_x p_a$$

$$= 1.020 \times 1.165 \times 3.67 \times 10^5 \times .03456$$

$$- 1 \times 10^5 \times .03456$$

$$= \underline{11616 \text{ N}}$$ small reduction

$$\frac{A_x}{A_T} = 1.3 \Rightarrow \frac{\dot{m}\sqrt{c_p T_0}}{p_0 A_x} = \frac{1.281}{1.3} = .9854$$

$$\Rightarrow M_x = 1.66 \quad \text{and} \quad A_x = 0.4084 \text{ m}^2$$

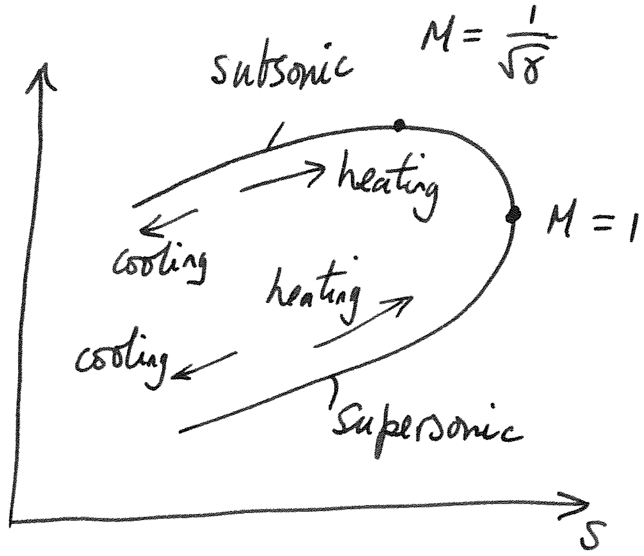
$$\text{Thrust} = F - A_x p_a = 1.0615 \times .9854 \times 3.67 \times 10^5 \times .04084$$

$$- 1 \times 10^5 \times 0.04084$$

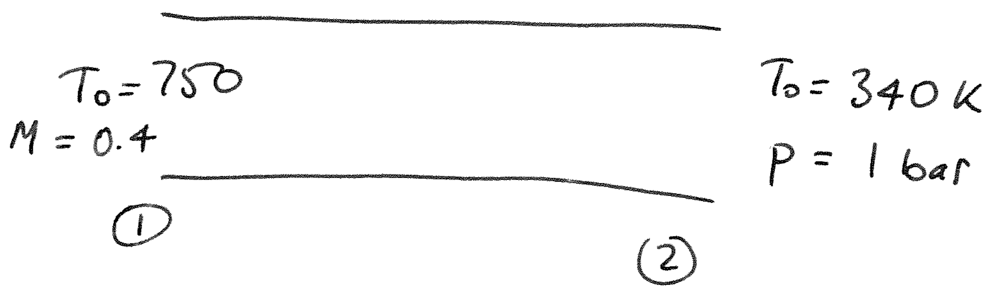
$$= \underline{11586 \text{ N}}$$ lager reduction due mainly to loss/entropy rise across shocks

2.

(a)



(b)



(i) Pipe Frictionless $\Rightarrow$ F = const

Also $\dot{m}$ = const , A = const

$$\therefore \quad \frac{F_2}{\sqrt{c_p T_{02}}} = \frac{F_1}{\sqrt{c_p T_{01}}} \cdot \sqrt{\frac{T_{01}}{T_{02}}} = 1.3608 \times \sqrt{\frac{750}{340}}$$

$$= 2.021 \Rightarrow \underline{M_2 = 0.24}$$

(ii) $\dfrac{\dot{m}\sqrt{c_p T_0}}{p\,A} = .5343 \Rightarrow \dot{m} = \dfrac{10^5 \times \pi \times .15^2}{\sqrt{1005 \times 340}} \times .5343$

$$\underline{= 6.461 \text{ kg/s}}$$

(iii) $\dot{Q} = \dot{m}\,\Delta h_0 = 6.461 \times 1005 \times (750 - 340) = 2662000$ W

$$\underline{= 2.662 \text{ MW}}$$

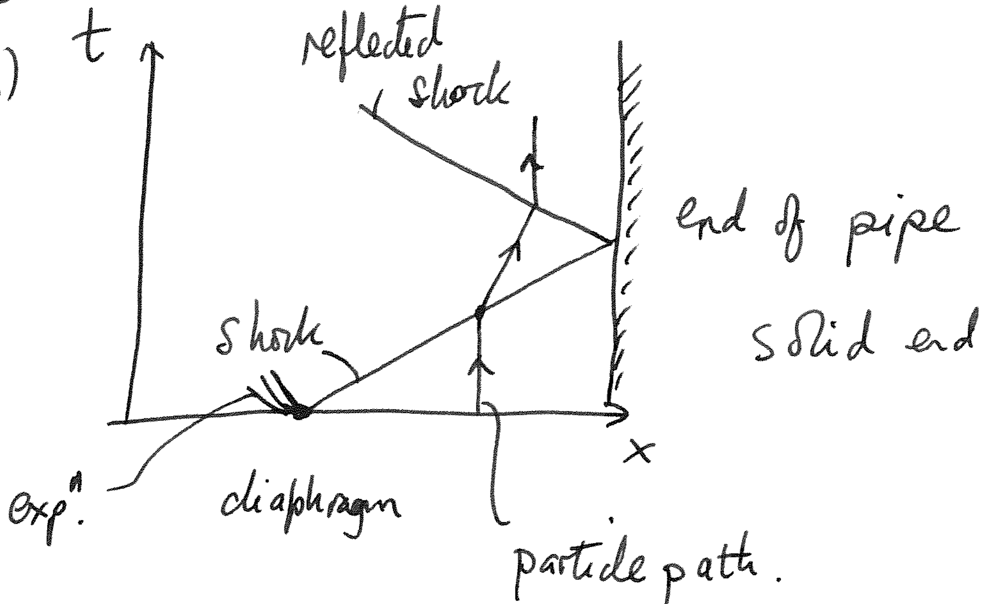
(iv) $\frac{V}{\sqrt{c_p T_0}} = .2490$ at inlet $\qquad \frac{V}{\sqrt{c_p T_0}} = .1509$ at exit

$\Rightarrow \quad V_{in} = .2490 \times \sqrt{1005 \times 750} \qquad V_{ex} = .1509 \times \sqrt{1005 \times 340}$

$= 216.2 \text{ m/s} \qquad = 88.21 \text{ m/s}$

3.

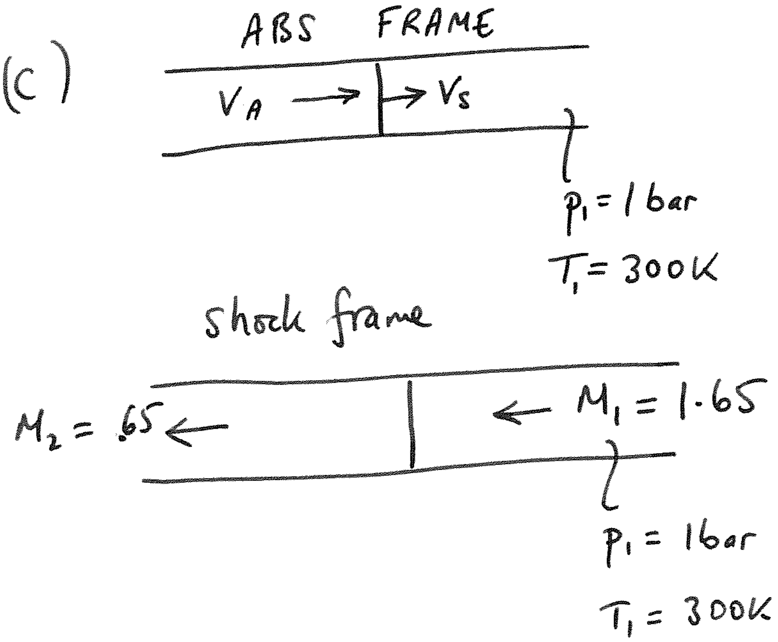
(a)



solid end ⇒ shock reflects as shock

(b) $\frac{p_s}{p} = 3 \Rightarrow \underline{M_{s_{up}} = 1.650}$ from tables $M_{s_{DOW}} = .65$

(c)



$a_1 = \sqrt{\gamma R T_1} = 347.2 \Rightarrow V_1 = 1.65 a_1 = 572.9 = V_s$

T_0^{Rel} = const across shock $\Rightarrow T_2 (1 + \frac{\gamma - 1}{2} .65^2) = 300 (1 + \frac{\gamma - 1}{2} 1.65^2)$
$= 463.4$

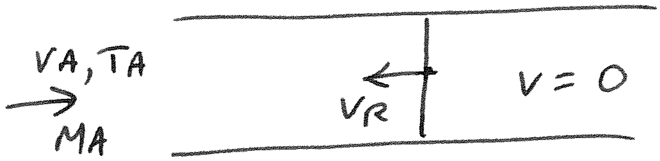
$\Rightarrow \underline{T_2 = 427.2}$

$\Rightarrow V_2 = .65 \times \sqrt{1.4 \times 287.1 \times 427.2}$
$= 269.3$ m/s

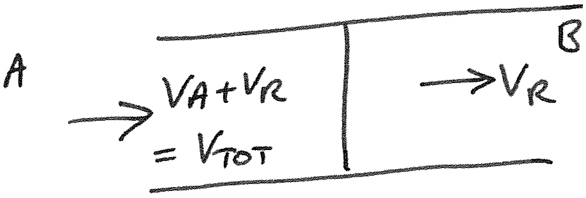
$V_A = V_s - V_2 = 572.9 - 269.3$
$= 303.6$

$\therefore \underline{T_A = 427.2\ \text{K} \quad V_A = 303.6\ \text{ms}^{-1}}$ $\underline{a_A = 414.4\ \text{ms}^{-1}}$

(d) ABS FRAME



Shock Frame



Databook $\Rightarrow$

$$\frac{\rho_A}{\rho_B} = \frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_{TOT}^2}$$

Mass conservation $\Rightarrow$ $\rho_A V_{TOT} = \rho_B (V_{TOT} - V_A)$

$$\therefore \quad 1 - \frac{V_A}{V_{TOT}} = \frac{\gamma-1}{\gamma+1} + \frac{2}{(\gamma+1)M_{TOT}^2} \qquad \& \quad \frac{V_A}{V_{TOT}} = \frac{M_A}{M_{TOT}}$$

$$\therefore \quad -\frac{M_A}{M_{TOT}} = \frac{2}{\gamma+1} + \frac{2}{(\gamma+1)M_{TOT}^2}$$

$$\Rightarrow \quad M_{TOT}^2 - \frac{\gamma+1}{2} M_A M_{TOT} - 1 = 0$$

or $M_{TOT}^2 - .879\, M_{TOT} - 1 = 0$ taking $M_A = \dfrac{303.6}{414.4}$

$$\Rightarrow \quad M_{TOT} = \frac{.879 + \sqrt{.879^2 + 4}}{2} = 1.53$$

$$\therefore \quad \frac{T_B}{T_A} = 1.34 \quad \Rightarrow \quad \underline{T_B = 572\ K}$$

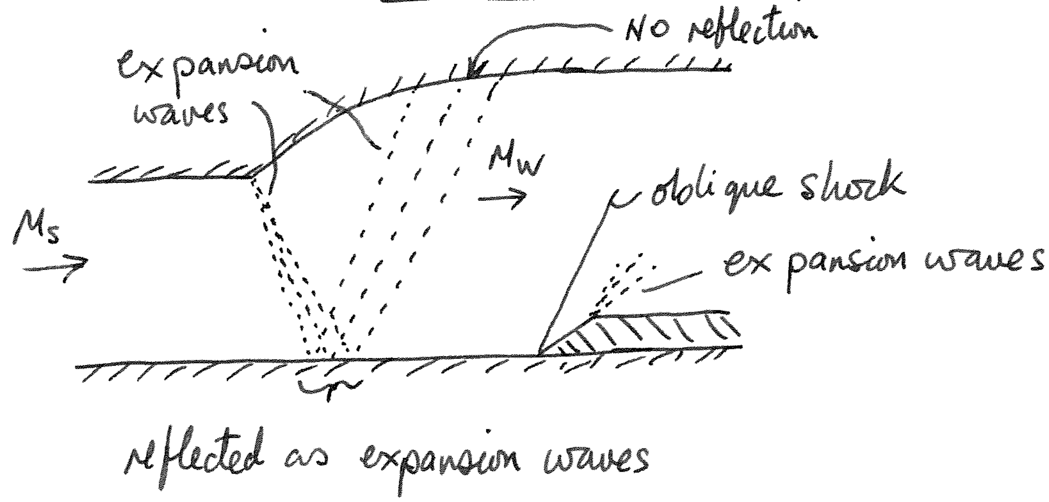
4 (a) $$\frac{\dot m\sqrt{c_pT_0}}{p_0A_w} = \frac{\dot m\sqrt{c_pT_0}}{p_0A_s}\cdot\frac{A_s}{A_w} \qquad M_s = 1.082 \qquad \frac{A_w}{A_s} = 1.0738$$

Tables M	$\frac{\dot m\sqrt{c_pT_0}}{p_0A}$
1.08	1.2745
1.09	1.2728

$\Rightarrow$ for $M_s = 1.082$ $\quad \frac{\dot m\sqrt{c_pT_0}}{p_0A_s} = 1.2742$

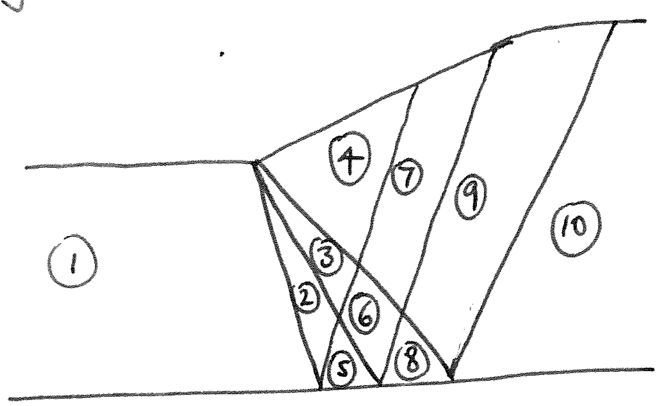
In working section, $\frac{\dot m\sqrt{c_pT_0}}{p_0A_w} = 1.1866 \underset{\text{(tables)}}{\Rightarrow} \underline{M_W = 1.330}$

(b)



(c)
$$\left.\begin{aligned} M_W &= 1.330 \Rightarrow \nu = 7^\circ \\ M_s &= 1.082 \Rightarrow \nu = 1^\circ \end{aligned}\right\} \text{total change} = 6^\circ$$

using 1° discretisation



Region	θ°	ν°	M
1	0	1	1.082
2	1	2	1.133
3	2	3	1.177
4	3	4	1.218
5	0	3	1.177
6	1	4	1.218
7	2	5	1.257
8	0	5	1.257
9	1	6	1.294
10	0	7	1.330

See next sheet for details

Tables

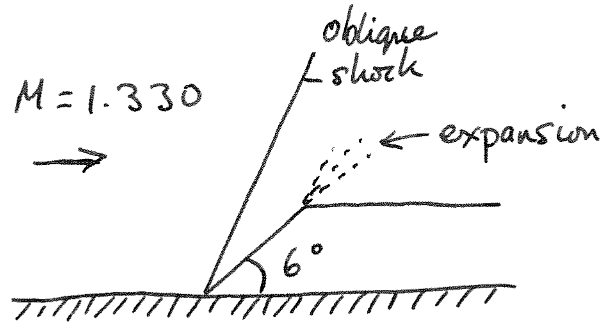
M	ν	
1.13	1.94	$\Rightarrow \nu = 2$ has $M = 1.133$
1.14	2.16	
1.17	2.84	$\Rightarrow \nu = 3$ has $M = 1.177$
1.18	3.07	
1.21	3.81	$\Rightarrow \nu = 4$ has $M = 1.218$
1.22	4.06	
1.25	4.83	$\Rightarrow \nu = 5$ has $M = 1.257$
1.26	5.09	
1.29	5.90	$\Rightarrow \nu = 6$ has $M = 1.294$
1.30	6.17	

(d) For $M = 1.330$ $\quad \theta_{max} = 7.494°$

Tables

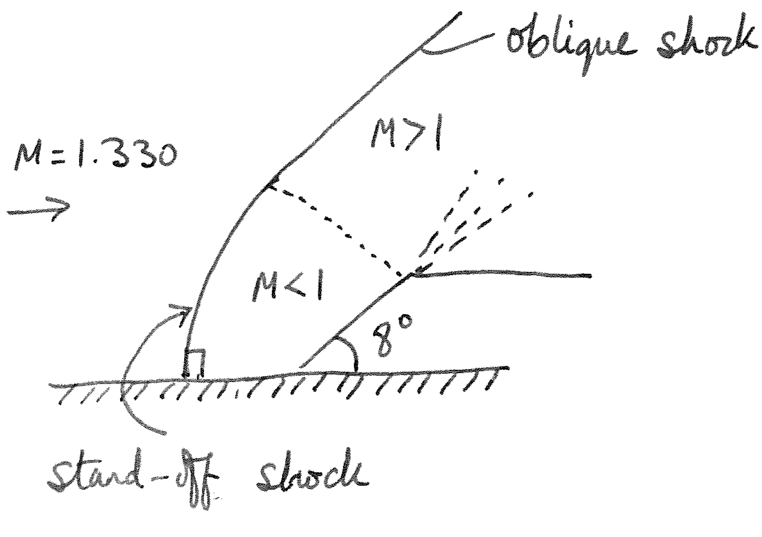
M	θ_{max}	
1.30	6.662	$\Rightarrow M = 1.33$ has $\theta_{max} = 7.494$
1.35	8.048	

(i)



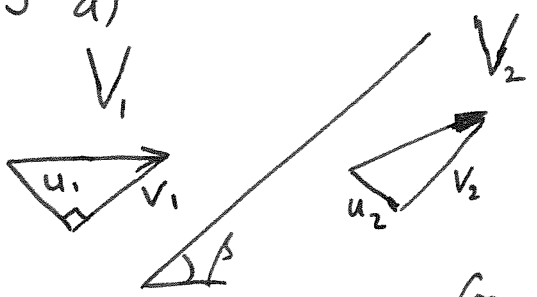
$6° < \theta_{max}$

(ii)



$8° > \theta_{max}$

5 a)



Cons of mass across shock

$\Rightarrow \rho_1 u_1 = \rho_2 u_2 = \dot{m}$

Cons of tangential momentum $\dot{m} v_1 = \dot{m} v_2$

i.e. $v_1 = v_2$

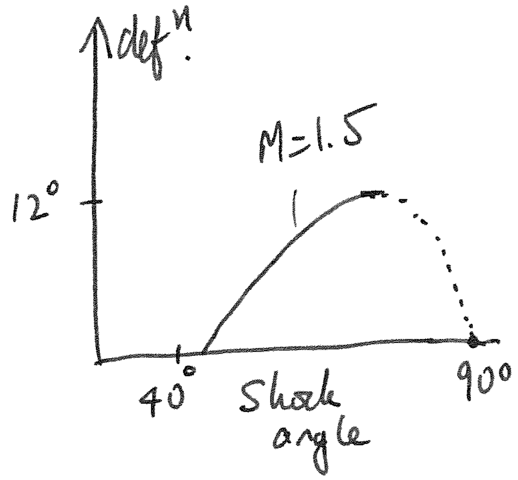
Viewed from frame moving // shock at speed $v_1 = v_2$

Shock is normal with mach no upstream $= \dfrac{u_1}{a_1} = M_1 \sin\beta$

$\therefore \dfrac{\rho_2}{\rho_1}$ = normal shock value with Mach no $M_1 \sin\beta$

$$= \frac{(\gamma+1)\, M_1^2 \sin^2\beta}{2\left(1 + \frac{\gamma-1}{2} M_1^2 \sin^2\beta\right)}$$

(b) From tables $\beta = 52.6°$ or $79.7°$



(i) – Corresponds to $M^{\text{Downstream}}_{\text{shock}} > 1$ with little entropy rise – weak shock

& ..... like "normal" shock – $M^{\text{Downstream}}_{\text{shock}} < 1$ (most of the time)

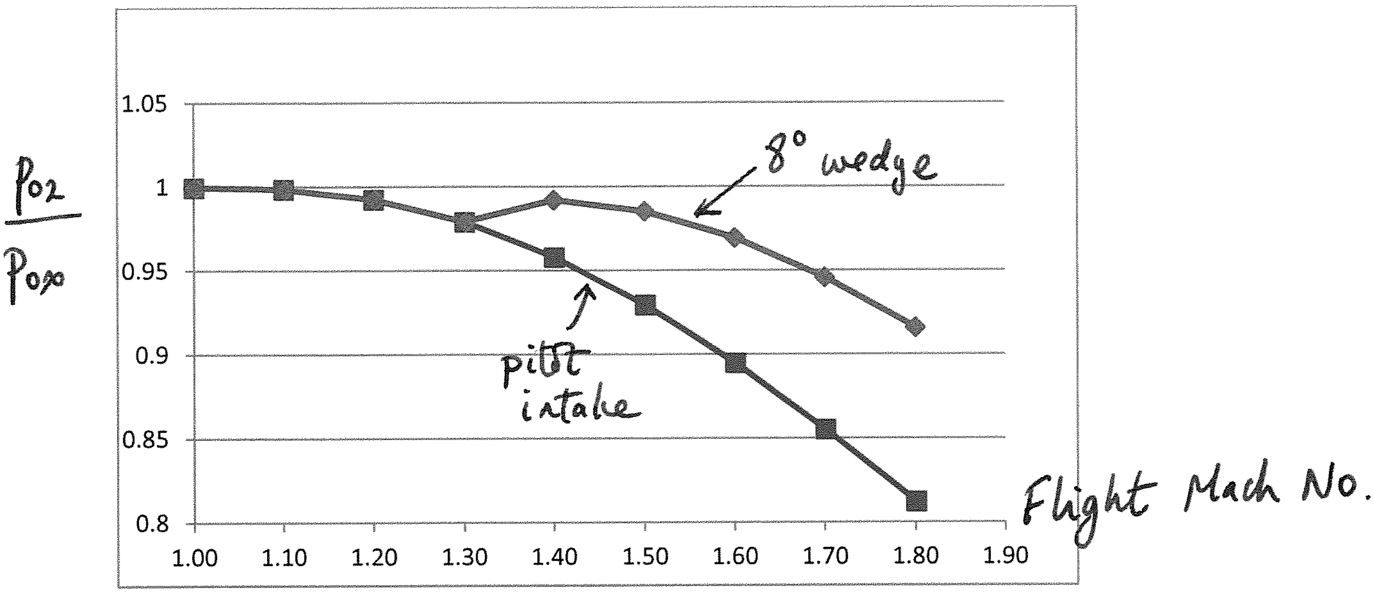
strong shock

(ii) weak shocks are usually the form observed.

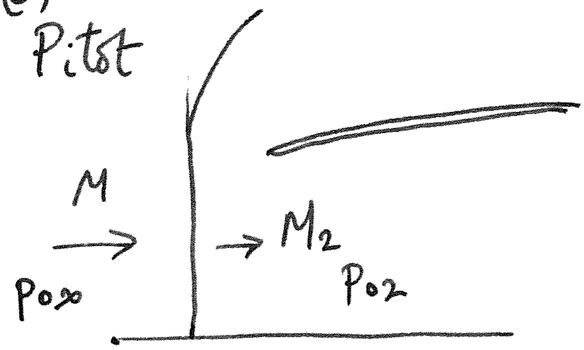
strong shocks occur when shock is, for example, "standing off" a body.

$M>1 \rightarrow$ ("strong shocks" $M<1$ <

Big p_s rise – disruptive of flow – needs high back pressure.



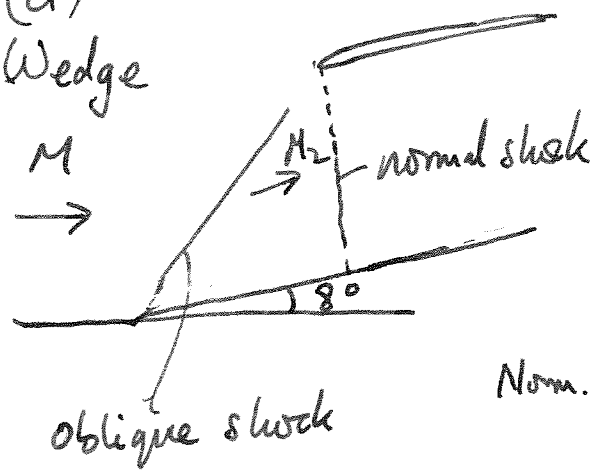
(c)
Pitot



Intake recovery has shock loss for normal shock at aircraft Mach No.

M	1	1.2	1.3	1.4	1.5	1.6	1.7	1.8
$\frac{p_{o2}}{p_{o\infty}}$	1	.9928	.979	.958	.930	.895	.856	.812

(d)
Wedge



Assuming oblique shock loss negligible

M	1.4	1.5	1.6	1.7	1.8
M_2	1.074	1.208	1.32	1.423	1.523
Norm. Shock (ratio)	$\simeq 1$	.992	.976	.952	.9225
Oblique Shock	.9924	.9936	.9938	.9935	.9931
Recovery	.992	.986	.970	.946	.916

N.B For 8° wedge need $M > 1.4$ for $\theta_{max} > 8°$
i.e. below $M \leq 1.3$ have normal shock as for pitot

e) At $M=1.8$

Pitot $\Rightarrow$ normal shock @ $M=1.80$ $\quad \frac{P_s}{P} = \underline{3.613}$

Wedge has

oblique shock 8° turning $\quad \frac{P_2}{P_1} = 1.504$

followed by normal shock $M=1.523$ $\quad \frac{P_s}{P} = 2.539$

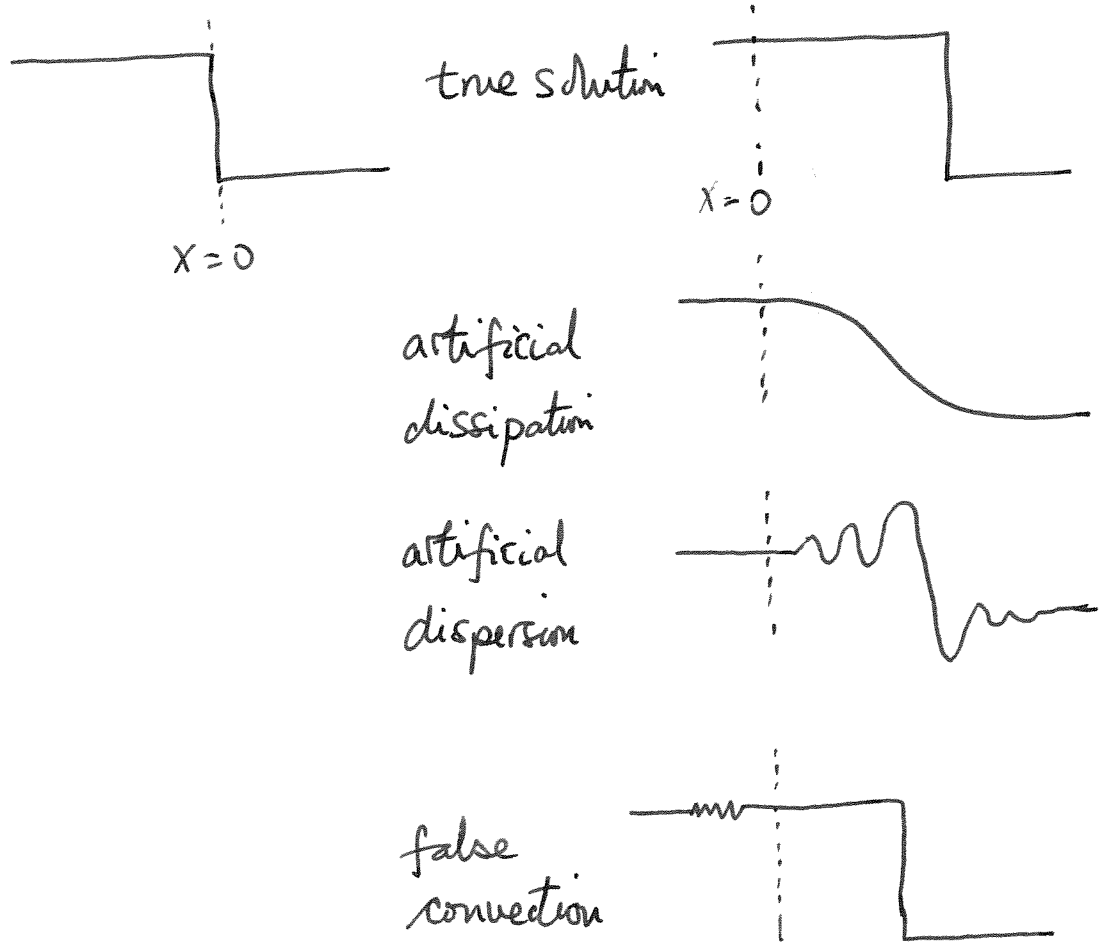
$\Rightarrow$ overall static pressure ratio $\underline{3.819}$

$\Rightarrow$ wedge case has 5.7% increased pressure rise.

---

6 (a) $\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0$ means that the true solution should convect without change of form. Truncation errors of even orders act like viscosity and smooth gradients out (artificial dissipation). Odd order truncation errors lead to different wavelengths travelling at different speeds (artificial dispersion). False convection refers to errors propagating in the upstream direction.

e.g. Solution $t = 0$ Solution later t



(b) $$u_i^{n+1} = u_i^n + \Delta t \frac{\partial u}{\partial t} + \frac{\Delta t^2}{2}\frac{\partial^2 u}{\partial t^2} + O(\Delta t^3)$$

If $\frac{\partial u}{\partial t} = -A\frac{\partial u}{\partial x} \Rightarrow$
$$\frac{\partial^2 u}{\partial t^2} = \frac{\partial}{\partial t}\left(-A\frac{\partial u}{\partial x}\right) = -A\frac{\partial}{\partial x}\frac{\partial u}{\partial t}$$
$$= -A\frac{\partial}{\partial x}\left(-A\frac{\partial u}{\partial x}\right) = A^2\frac{\partial^2 u}{\partial x^2}$$

$$\therefore \quad u_i^{n+1} = u_i^n - A\Delta t\frac{\partial u}{\partial x} + \frac{A^2\Delta t^2}{2}\frac{\partial^2 u}{\partial x^2} + O(\Delta t^3) \qquad (1)$$

artificial dissipation if ignored.

Approximating $$\frac{\partial u}{\partial x} = \frac{u_{i+1} - u_{i-1}}{2\Delta x} + O(\Delta x^2)$$ since central difference

[or $$u_{i+1} = u_i + \Delta x\frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \cdots$$

$$u_{i-1} = u_i - \Delta x\frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \cdots$$

$$\therefore \quad \frac{u_{i+1} - u_{i-1}}{2\Delta x} = \frac{\partial u}{\partial x} + \frac{\Delta x^2}{6}\frac{\partial^3 u}{\partial x^3} + \cdots \quad]$$

and $$\frac{u_{i+1} - 2u_i + u_{i-1}}{\Delta x^2} = \frac{\partial^2 u}{\partial x^2} + O(\Delta x^2)$$

Hence from (1) $$u_i^{n+1} - u_i^n + \frac{c}{2}(u_{i+1} - u_{i-1}) - \frac{c^2}{2}(u_{i+1} - 2u_i + u_i)$$
$$= O(\Delta t^3, \Delta t\,\Delta x^2)$$ i.e. scheme free from dissipation

(c) If error at time n is a "sawtooth".

Error at time $n+1$ satisfies

$$\varepsilon^{n+1} = \varepsilon - \frac{c}{2}\left[(-\varepsilon) - (-\varepsilon)\right] + \frac{c^2}{2}(-\varepsilon - 2\varepsilon - \varepsilon)$$

$$= \varepsilon(1 - 2c^2)$$

$\therefore$ For stability, need

$$\left|\frac{\varepsilon^{n+1}}{\varepsilon}\right| \leq 1 \Rightarrow -1 \leq 1 - 2c^2 \leq 1$$

$$\Rightarrow \quad 2c^2 \leq 2 \quad \text{or} \quad c \leq 1$$

7 a) (i) $\dfrac{\partial u}{\partial x} \simeq \dfrac{u_i - u_{i-1}}{\Delta x} = \dfrac{3^2 - 2.9^2}{.1} = 5.9$

(ii) $\dfrac{\partial u}{\partial x} = \dfrac{u_{i+1} - u_{i-1}}{2\Delta x} = \dfrac{3.1^2 - 2.9^2}{2 \times .1} = 6.0$

$$u_{i+1} = u_i + \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} + \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + O(\Delta x^4)$$

$$u_{i-1} = u_i - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta x^3}{6}\frac{\partial^3 u}{\partial x^3} + \cdots$$

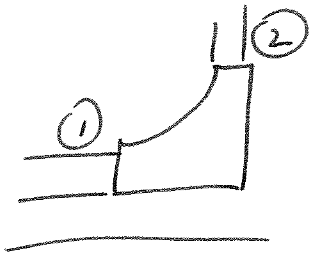
$\Rightarrow \dfrac{u_{i+1} - u_{i-1}}{2\Delta x} = \dfrac{\partial u}{\partial x} + \dfrac{\Delta x^2}{6}\dfrac{\partial^3 u}{\partial x^3}$ i.e. 2nd order accurate.

(iii) Not for this application. $u = x^2 \Rightarrow \dfrac{\partial u}{\partial x} = 2x \quad \dfrac{\partial^2 u}{\partial x^2} = 2$

$\dfrac{\partial^3 u}{\partial x^3} = 0$ and all higher $\dfrac{\partial^n u}{\partial x^n} = 0$

i.e. 2nd order accuracy is exact when applied to quadratics.

b) (i) $\eta_{is} = \dfrac{T_{02}^{is} - T_{01}}{T_{02} - T_{01}}$



$\Rightarrow \eta_c = \dfrac{\left(\dfrac{T_{02}}{T_{01}}\right)^{is} - 1}{\dfrac{T_{02}}{T_{01}} - 1}$ & $\dfrac{p_{02}}{p_{01}} = \left(\dfrac{T_{02}^{is}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$ $\psi = \dfrac{c_p \Delta T_0}{U^2}$

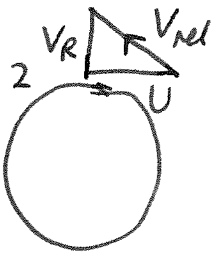
$\therefore \eta_c \dfrac{c_p (T_{02} - T_{01})}{U^2} \cdot \dfrac{U^2}{c_p T_{01}} = \eta_c \dfrac{\psi U^2}{c_p T_{01}} = \left(\dfrac{p_{02}}{p_{01}}\right)^{\frac{\gamma-1}{\gamma}} - 1$

$\Rightarrow \dfrac{p_{02}}{p_{01}} = \left(1 + \eta \psi \dfrac{U^2}{c_p T_{01}}\right)^{\frac{\gamma}{\gamma-1}}$

(ii) Expect ψ to vary relatively slowly with speed along compressor op. line $\Rightarrow$ major influence on $\frac{p_{02}}{p_{01}}$ is $\frac{U^2}{c_p T_{01}}$ which is directly influenced by speed

(iii)

At exit



Radial blades & flow follows blades $\Rightarrow$ $V_{\theta_2}^{rel} = 0$ i.e. $V_{\theta_2} = U_2$

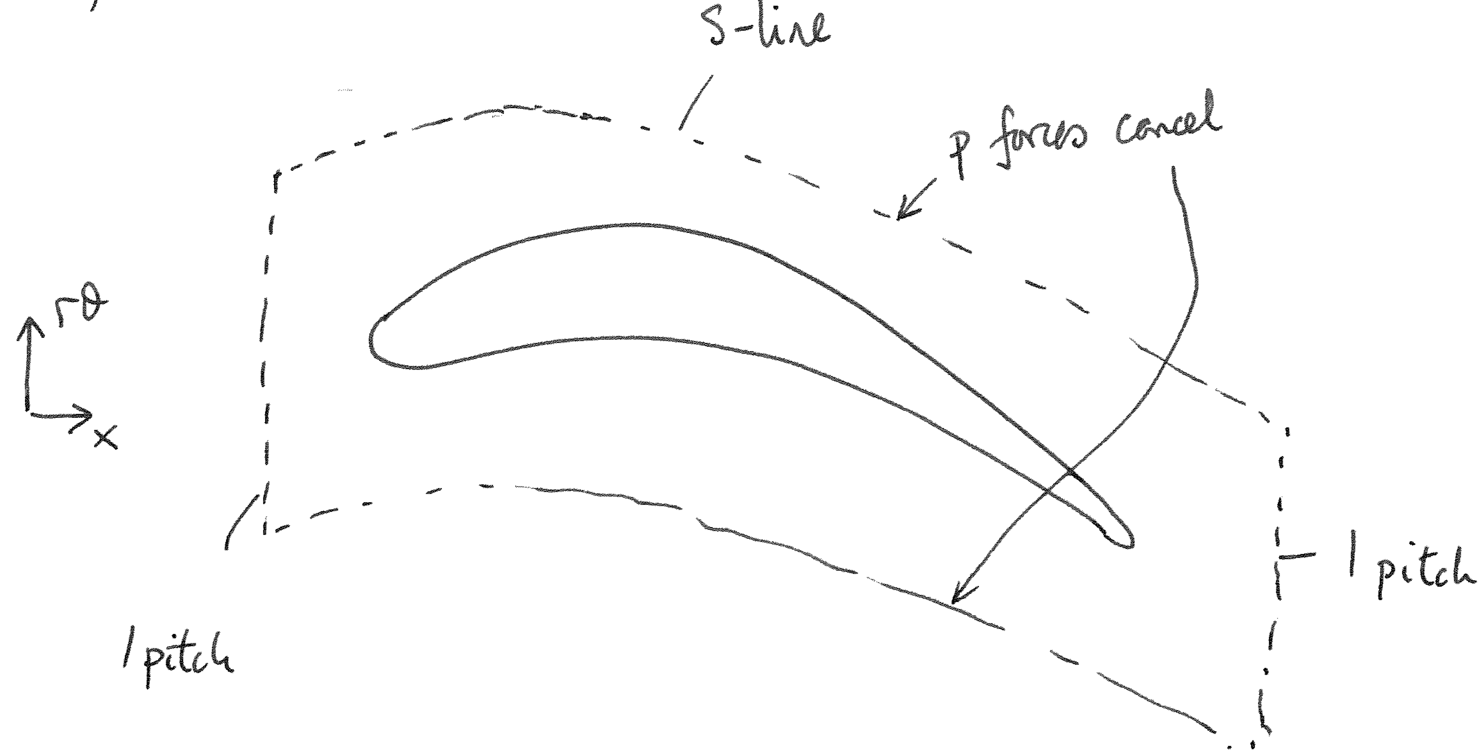
Euler equation $\quad \Delta h_0 = \Delta(U v_\theta) = U_2 V_{\theta_2} - \underset{0}{\underbrace{U_1 v_{\theta_1}}}$

$$\therefore \quad \frac{\Delta h_0}{U^2} = 1$$

Hence $$\frac{p_{02}}{p_{01}} = \left(1 + \frac{.85 \times 1 \times 300^2}{1005 \times 300}\right)^{\frac{\gamma}{\gamma-1}}$$

$$= 2.21$$

8 a)



Assume flow is 2-D (constant stream tube height) & constant r

Force on fluid in C.V. = rate at which momentum flows out

$$F_x + p_1 A_x - p_2 A_x = \dot{m}(V_{x_2} - V_{x_1})$$

A_x = pitch × thickness of s.tube

F_x = force on fluid by blade $= (p_2 - p_1) A_x + \dot{m}(V_{x_2} - V_{x_1})$

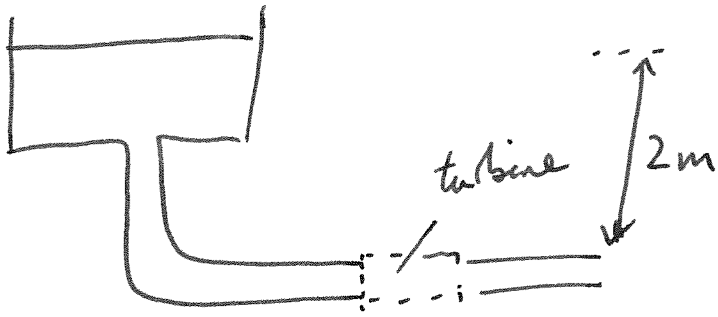
$$F_\theta = \dot{m}(V_{\theta_2} - V_{\theta_1})$$

$\dot{m}$ = mass flow through streamtube.

(iv) indept of whether blade moving since

$$V_{\theta_2}^{rel} - V_{\theta_1}^{rel} = V_{\theta_2} - V_{\theta_1}$$

(b)



(i) $\Delta h_0 = \dfrac{\Delta p_0}{\rho} = \dfrac{\rho g h}{\rho} = 2 \times 9.81 = 19.62 \ m^2 s^{-2}$

(ii) $\psi = 1.2 \Rightarrow U = \sqrt{\dfrac{\Delta h_0}{\psi}} = \sqrt{\dfrac{19.62}{1.2}} = 4.04 \ m s^{-1}$

(iii) $\varphi = 0.5 \Rightarrow V_x = 2.02 \ m s^{-1}$

(iv) $\dot{m} = \rho V_x \pi (r_t^2 - r_h^2) = 10^3 \times 2.02 \times \pi (.1^2 - .06^2)$

$= 40.6 \ kg \, s^{-1}$

(v) Power $= \dot{m} \Delta h_0 = 40.6 \times 19.62$

$= 797 \ W$

(c) $\Delta h_0 = \Delta (U V_\theta)$ & $V_{\theta_2} = 0$

$\Rightarrow V_{\theta_1} = 4.86 \ m s^{-1}$

$\therefore F_\theta = \dfrac{\dot{m} V_\theta}{10}$ per blade $= 19.7 \ N$

$$p_2 - p_1 = p_{02} - p_{01} - \rho \frac{V_{x_2}^2}{2} - \rho \frac{V_{\theta_2}^2}{2} + \rho \frac{V_{x_1}^2}{2} + \rho \frac{V_{\theta_1}^2}{2}$$

Flow incompressible $\Rightarrow V_{x_2} = V_{x_1}$, Turbine design $\Rightarrow V_{\theta_2} = 0$

$$\therefore \quad p_2 - p_1 = -\rho \times 19.62 + \rho \frac{V_{\theta_1}^2}{2}$$

$$= 10^3 \left[-19.62 + \frac{4.86^2}{2} \right] = -7810 \text{ Pa}$$

$$\therefore \quad F_x = -7810 \times \frac{\pi (r_t^2 - r_h^2)}{10} = -15.7 \text{ N}$$

This is force on fluid   Force on blade $= 15.7\text{N} \rightarrow$

$+ \ 19.7\text{N} \uparrow$

$\therefore$ Total Force $= 25.2$ N

T.P.H.