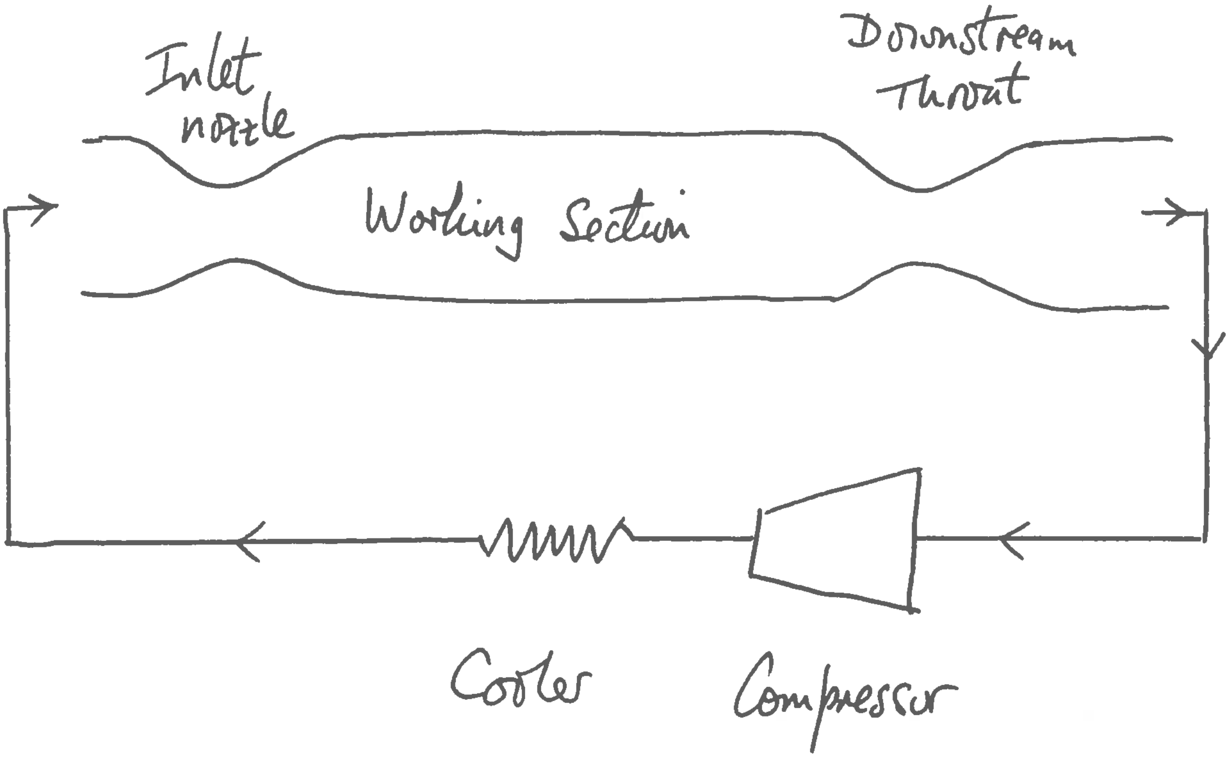
3A3 2019 FLUID MECHANICS II Version TPH5

1(a)



Isentropic operation is not possible due to viscous effects and the need to decelerate the flow which will involve a shock in a fixed geometry tunnel.

During starting, the inlet nozzle chokes and a shock wave forms in the working section. The downstream throat must remain unchoked so that the shock wave can be brought back through the working section to sit downstream of the downstream throat.

(b) $M = 2.5$ in working section $\Rightarrow \dfrac{\dot{m}\sqrt{c_p T_0}}{p_0 A} = .4858$

[Tables, $\gamma = 1.4$]

Nozzle throat is choked $\dfrac{\dot{m}\sqrt{c_p T_0}}{p_0 A_N^*} = 1.2810$

$\therefore$ Area of throat $A_N^* = \dfrac{.4858}{1.281} \times 0.2^2 = \underline{.01517\,m^2}$

(c) Downstream throat must remain unchoked with $M = 2.5$ shock in working section. Assuming isentropic flow other than across the shock:

$\dfrac{p_{0s}}{p_0} = .4990$ [Tables at $M = 2.5$]

At the downstream throat $\dfrac{\dot{m}\sqrt{c_p T_0}}{A_D^* \, p_{0s}} \le 1.2810$

Comparing the throats $\dfrac{A_D^*}{A_N^*} = \dfrac{p_0}{p_{0s}}$ at choke conditions

$\Rightarrow A_D^* \geqslant \dfrac{0.01517}{0.4990} = \underline{.03040\ m^2}$

d) Minimum power requires isentropic compressor.
Stagnation temperature unchanged through working section.
Stagnation pressure falls from 100 kPa to 49.9 kPa
No loss across heat exchanger.

Isentropic Compressor $\quad T_{oc} = 320\left(\frac{100}{49.9}\right)^{\frac{\gamma-1}{\gamma}} = 390.3\,K$

Compressor work $\quad \dot{m}\, C_p (T_{oc} - T_o)$

working section $\quad \frac{\dot{m}\sqrt{c_p T_o}}{p_o A} = .4858$

$$\therefore \quad \dot{m} = \frac{.4858 \times .2^2 \times 10^5}{\sqrt{1005 \times 320}} = 3.427\ kg/s$$

$$\therefore \text{Compressor work} = 3.427 \times 1005 \times (390.3 - 320)$$

$$= \underline{242\ kW}$$

e) Continuous operation: assume shock in downstream throat

$$\frac{\dot{m}\sqrt{c_p T_o}}{A_D^* p_o} = 1.281\,\frac{A_N^*}{A_D^*} = 1.2810 \times .4990 = .6392$$

Corresponding to $M \simeq 2.20 \Rightarrow \frac{p_{os}}{p_o} = .6281$

Temp downstream of compressor $= T_{oc} = T_o\left(\frac{p_o}{p_{os}}\right)^{\frac{\gamma-1}{\gamma}}$

$$= 365.4\,K$$

Heat extracted $= 3.427 \times 1005 \times (365.4 - 320) = \underline{156\ kW}$

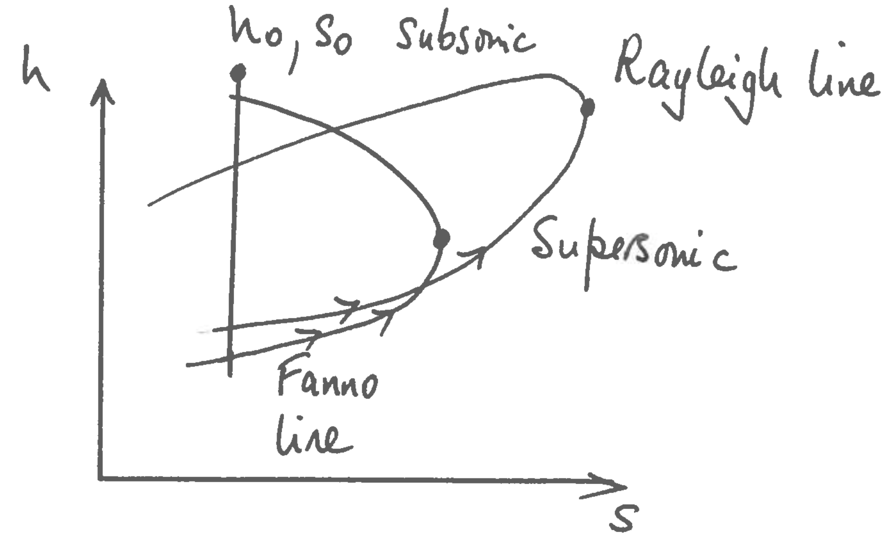
Examiner's Note

50 candidates did this question (out of 69)
Average mark 12.8/20.

Very well done by those who knew what they were doing. Layout description was good but not many candidates mentioned friction as a source of irreversibility. A common error was to say that M=1 at the second throat. Quite a few candidates had little clue about how to calculate the compressor work (and therefore the cooling necessary).

Less work is required if the heat exchanger precedes the compressor but the heat exchanger is then attempting to cool below the likely atmospheric temperature. 2 candidates used this arrangement which was accepted without penalty.

2 (a)



Velocity decreases with downstream distance

$\therefore$ Flow is supersonic in both cases

Case (i) Fanno flow : $\dot{m}$ constant

F falls (linearly if const c_f)

h_0 constant

s increases downstream

Case (ii) Rayleigh flow $\dot{m}$ constant

F constant

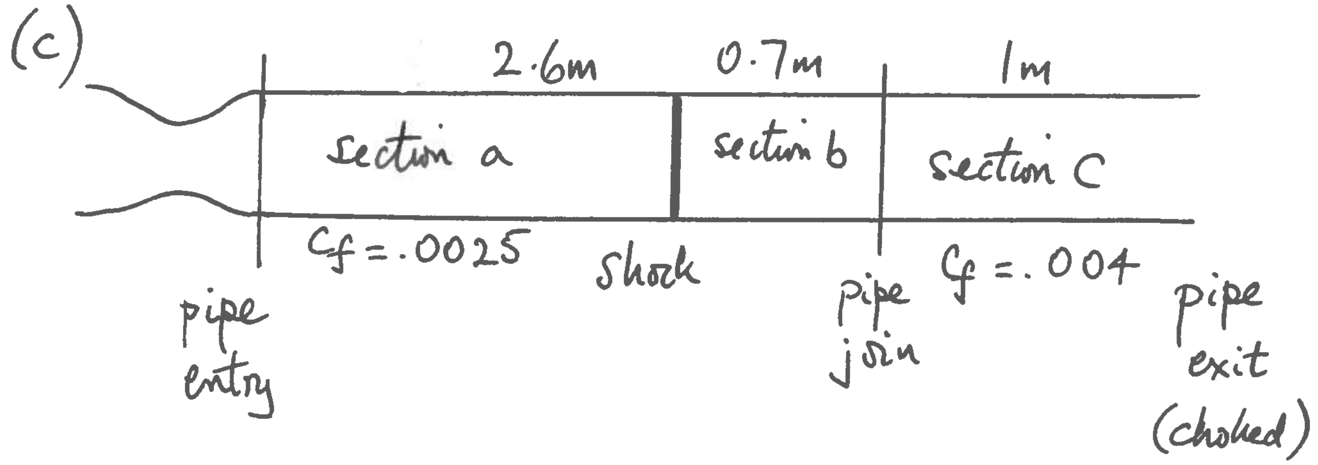
h_0 rises with heat addition

s increases with downstream distance

(b) Fanno flow $\frac{4 c_f L_{max}}{D} = \frac{4 \times 0.0025 \times 3.3}{.2} = .165$

Tables: $\underline{M = 1.58}$ at entry

(c)



In section c $\frac{4 c_f L}{D} = \frac{4 \times .004 \times 1}{0.2} = 0.08$

In section b $\frac{4 c_f L}{D} = \frac{4 \times 0.0025 \times 0.7}{0.2} = .0035$

Adding: total post-shock is 0.115 for b+c

Tables: $M = .7596$ post shock

$\Rightarrow$ Tables: $\underline{M = 1.355}$ pre shock

(d) At $M = 1.355$ (tables) $\frac{4 c_f L}{D} = .0838$

section a $\frac{4 c_f L}{D} = \frac{4 \times 0.0025 \times 2.6}{0.2} = 0.13$

Adding: total is 0.2138

Tables $\Rightarrow$ $\underline{M = 1.717}$ at entry

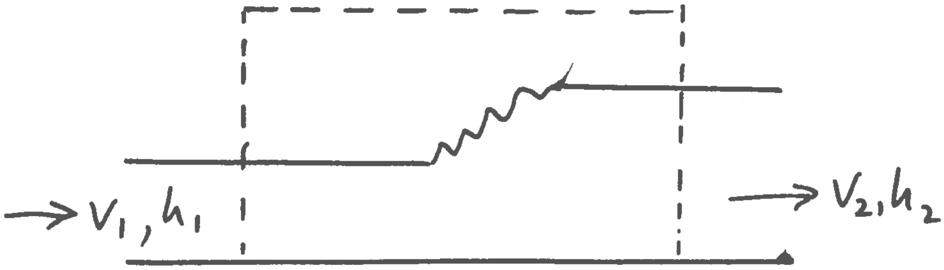
Examiner's Note

65 candidates did this question (out of 69).

Average mark 16.4/20

Probably too straightforward a question. Well done by nearly every candidate. Those that did have trouble got themselves confused by badly misreading tables and "fat-finger syndrome" with calculators.

3 (a)



Continuity: $h_1 V_1 = h_2 V_2$

momentum (SFME): $\frac{1}{2}\rho g h_1^2 - \frac{1}{2}\rho g h_2^2 = \rho V_2^2 h_2 - \rho V_1^2 h_1$

Combine: $\frac{1}{2} g (h_1^2 - h_2^2) = V_2 V_1 h_1 - V_1 V_2 h_2$

$\Rightarrow \quad V_1 V_2 = \frac{1}{2} g (h_1 + h_2)$

Continuity $\Rightarrow \quad V_1^2 \frac{h_1}{h_2} = \frac{1}{2} g (h_1 + h_2)$

$\Rightarrow \quad Fr_1^2 = \frac{V_1^2}{g h_1} = \frac{1}{2} \frac{h_2}{h_1} \left(\frac{h_2}{h_1} + 1 \right)$

(b) From $V_1 V_2 = \frac{1}{2} g (h_1 + h_2)$ and $V_1 = \frac{V_2 h_2}{h_1}$

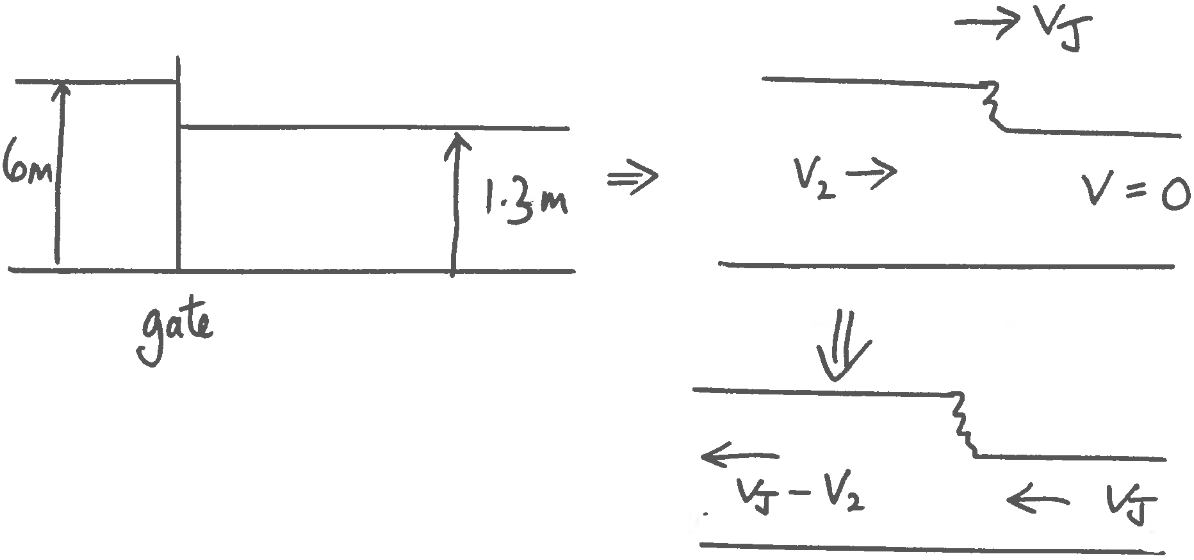
we have $V_2^2 \frac{h_2}{h_1} = \frac{1}{2} g (h_1 + h_2)$

i.e. $Fr_2^2 = \frac{V_2^2}{g h_2} = \frac{1}{2} \frac{h_1}{h_2} \left(\frac{h_1}{h_2} + 1 \right)$

Since $\frac{h_2}{h_1} > 1 \Rightarrow Fr_1^2 > \frac{1}{2} \cdot 1 \cdot (1+1)$ i.e. $Fr_1^2 > 1$ supercritical

& $Fr_2^2 < \frac{1}{2} \cdot 1 \cdot (1+1)$ i.e. $Fr_2^2 < 1$ subcritical

(c)



$$V_J(V_J - V_2) = \frac{1}{2} g(h_1 + h_2) \quad \text{and} \quad \frac{V_J^2}{g h_1} = \frac{1}{2}\frac{h_2}{h_1}\left(\frac{h_2}{h_1} + 1\right)$$

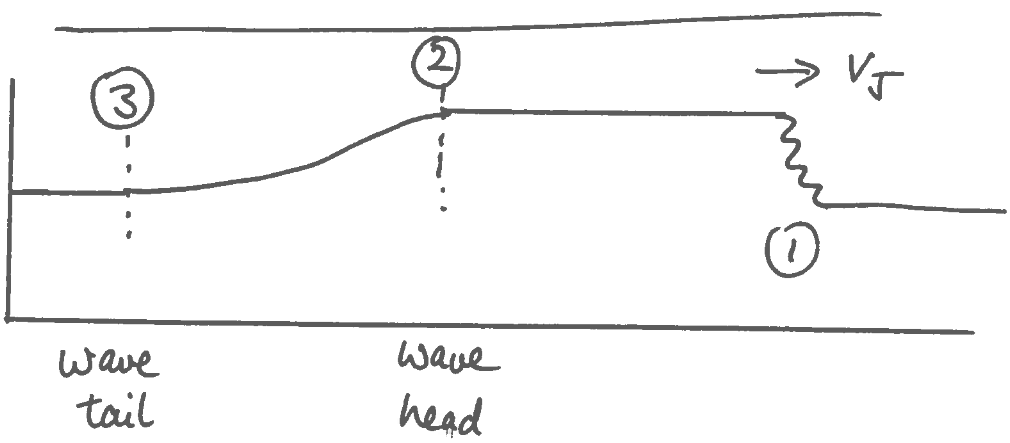
$$\Rightarrow \quad V_J^2 = \frac{1}{2} 9.81 \times 1.6 \left(\frac{1.6}{1.3} + 1\right) = 17.51$$

$$\therefore \quad V_J = 4.184 \text{ m/s}$$

$$V_J - V_2 = \frac{1}{2} g \frac{(h_1 + h_2)}{V_J} = \frac{1}{2} \frac{9.81 \times (1.6 + 1.3)}{4.184} = 3.400 \text{ m/s}$$

$$\therefore \quad V_2 = 4.184 - 3.4 = .784 \text{ m/s}$$

(d)



Right running waves $\quad V_3 - 2c_3 = V_2 - 2c_2$

Since $V_3 = 0$ (closed end) $\quad c_3 = c_2 - \frac{V_2}{2}$

$$c_2 = \sqrt{g h_2} = \sqrt{9.81 \times 1.6} = 3.962 \text{ m/s}$$

$$c_3 = 3.962 - 0.784/2 = 3.57 \text{ m/s}$$

V_H

Speed of wave head in "lab coords" $= V_2 + C_2 = \underline{4.746 \text{ m/s}}$

" " " tail " " $= V_3 + C_3 = \underline{3.57 \text{ m/s}}$

Time taken for waves to catch up is t where

$$(t + 40) V_J = t V_H$$

$$\Rightarrow \quad t = \frac{40 \times 4.184}{4.746 - 4.184} = \underline{298 \text{ s}}$$

Examiner's Note

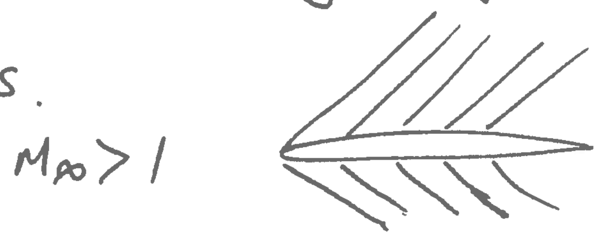
Attempts 51 Mean 14.4/20

Quite well done. Hydraulic jumps were fine. The only part not done well was the last part, with many candidates either not attempting it or treating the waves as hydraulic jumps.

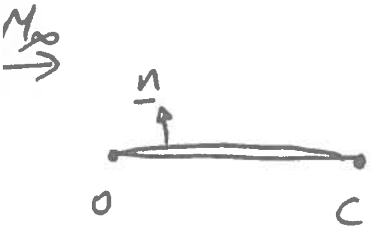
4.

(a) When $M_\infty < 1$ equation is elliptic (= Laplace like) which means that local pressure disturbances can influence the whole flowfield and, in particular, exert an upstream influence.

When $M_\infty > 1$ equation is hyperbolic (= wave like) and local disturbances can only be felt along characteristic directions.



(b) If the aerofoil surfaces are given by $y = \tau f\left(\frac{x}{c}\right)$ boundary condition on the aerofoil is



$$\underline{V} \cdot \underline{n} = 0 \Rightarrow \left(-\frac{\tau}{c} f'\left(\frac{x}{c}\right), 1\right) \cdot (U_\infty + u, v) = 0$$

or $v = -\frac{\tau}{c} f'\left(\frac{x}{c}\right) U_\infty$ to first order on $y = 0$

i.e. $\frac{\partial \varphi}{\partial y} = -\frac{\tau}{c} f'\left(\frac{x}{c}\right) U_\infty$ on $y = \tau f\left(\frac{x}{c}\right)$ $0 < x < c$.

Define $\xi = x/c$ $\quad \eta = \frac{y\sqrt{1-M_\infty^2}}{c} \Rightarrow \frac{\partial}{\partial x} = \frac{1}{c}\frac{\partial}{\partial \xi}$

& $\frac{\partial}{\partial y} = \frac{\sqrt{1-M_\infty^2}}{c} \frac{\partial}{\partial \eta}$ $\quad \Rightarrow \underline{\frac{\partial^2 \varphi}{\partial \xi^2} + \frac{\partial^2 \varphi}{\partial \eta^2} = 0}$ Laplace eq.^{n}

If we use $\tilde{\varphi} = \dfrac{\sqrt{1-M_\infty^2}}{U_\infty \tau}\varphi$ i.e. $\varphi = \dfrac{\tau U_\infty}{\sqrt{1-M_\infty^2}}$

then $\dfrac{\partial^2 \tilde{\varphi}}{\partial \xi^2} + \dfrac{\partial^2 \tilde{\varphi}}{\partial \eta^2} = 0$ and $\dfrac{\partial \tilde{\varphi}}{\partial \eta} = -f'(\xi)$

on $\eta = 0 \quad 0 < \xi < 1$

(c) To this (linearised) level of approximation

$$p - p_\infty = -\rho_\infty U_\infty \frac{\partial \varphi}{\partial x} \quad \text{(Euler eq}^{\text{n}}\text{ x-component)}$$

so that $$C_p = \frac{p - p_\infty}{\rho_\infty U_\infty^2/2} = -\frac{2}{U_\infty}\frac{\partial \varphi}{\partial x} = -\frac{2}{U_\infty}\frac{\tau U_\infty}{c\sqrt{1-M_\infty^2}}\frac{\partial \tilde{\varphi}}{\partial \eta}$$

$$= -\frac{2\tau}{c\sqrt{1-M_\infty^2}}\frac{\partial \tilde{\varphi}}{\partial \eta}.$$

For $M_\infty = 0$ (incompressible case) $C_{p0} = -\dfrac{2\tau}{c}\dfrac{\partial \tilde{\varphi}}{\partial \eta}$

$\Rightarrow \; C_p = \dfrac{C_{p0}}{\sqrt{1-M_\infty^2}}$ at the same $\xi \; (= \frac{x}{c})$

Integrating over both surface to find the lift

$C_L = \dfrac{C_{L0}}{\sqrt{1-M_\infty^2}}$ or taking difference between surfaces $C_L = \dfrac{C_{L0}}{\sqrt{1-M_\infty^2}}$.

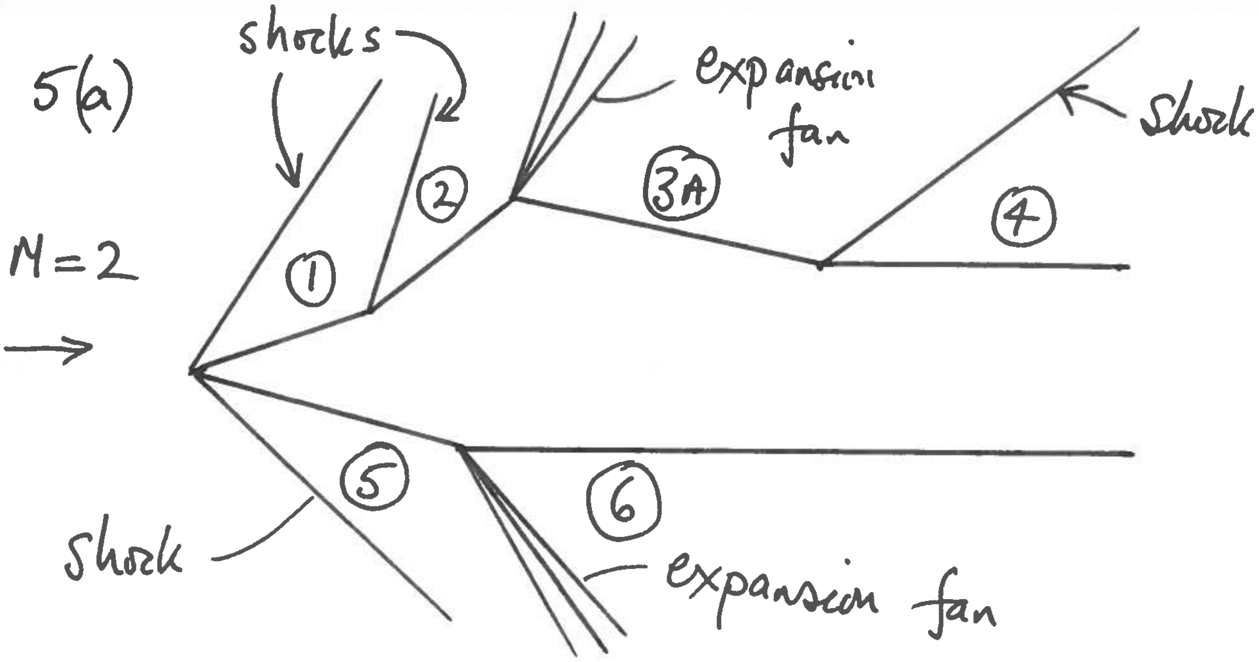
(d) Drag comes from form drag & viscous drag in Subsonic flow and neither of these are handled by the potential flow assumption. C_D will be zero within this framework.

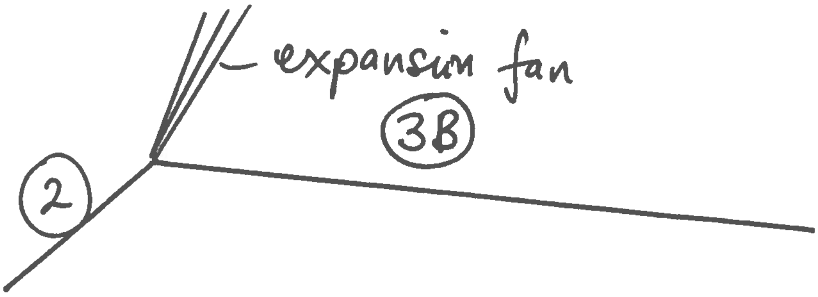
(e) Different numerical spatial schemes need to be used in steady transonic flow for the subsonic and supersonic regions to reflect the physics described in part (a). Failure to do so usually results in numerical instability. The subsonic and supersonic regions are not known until the solution is obtained. One method that works is to use an unsteady formulation and then iterate until the solution settles down.

<u>Examiner's Note</u> Attempts 14 Mean 13.4/20

Not a popular question. Very well done by most who attempted it seriously. Only "howler" was three candidates thought transonic flow has elliptic & hyperbolic regions (which is correct) then "elliptic + hyperbolic" = parabolic (which most certainly is not)

5(a)





Regions 1, 2, 5 & 6 are the same for A & B

(b) Region 1 Oblique shock from $M=2$ through $16°$

Tables $\frac{p_1}{p_\infty} = 2.3076$ $M_1 = 1.403$

Region 2 Oblique shock from M_1 through $8°$

$\frac{p_2}{p_1} = 1.5247$ $M_2 = 1.079$ (interpolating 1.40 – 1.45)

2 → 3A $\nu + \theta = \text{const}$

M_2	ν_2	θ_2	θ_3	ν_3	M_3	p_3/p_2
1.079	.95	24	−16	40.95	2.580	.1075

$$\left\{ \frac{p_3}{p_2} = \left(\frac{1+\frac{\gamma-1}{2}M_2^2}{1+\frac{\gamma-1}{2}M_3^2} \right)^{\frac{\gamma}{\gamma-1}} \text{ since } p_0 \text{ const across expansion} \right\}$$

② → ③B $\nu + \theta = \text{const}$

M_2	ν_2	θ_2	θ_3	ν_3	M_3	P_3/P_2
1.079	.95	24	−6	30.95	2.170	.2039

Region ⑤ Oblique shock from $M=2$ through $12°$

$$\frac{P_5}{P_\infty} = 1.8884$$

Region	P/P_∞	$\dfrac{P-P_\infty}{P_\infty}$
1	2.308	1.308
2	3.560	2.560
3_A	.3827	−.617
3_B	.7259	−.274
5	1.888	0.888

Pressure drag ratio

$$= \frac{(P_1 - P_\infty)A_1^\perp + (P_2 - P_\infty)A_2^\perp + (P_5 - P_\infty)A_5^\perp - (P_{3A} - P_\infty)A_{3A}^\perp}{(P_1 - P_\infty)A_1^\perp + (P_2 - P_\infty)A_2^\perp + (P_5 - P_\infty)A_5^\perp - (P_{3B} - P_\infty)A_{3B}^\perp}$$

$$= \frac{1.308 \times 1.5 \sin 16° + 2.560 \times 1.5 \sin 24° + .888 \times 2 \sin 12° + .617 \times 3 \sin 16°}{\text{"} \quad + \quad \text{"} \quad + \quad \text{"} \quad + .274 \times 7.9 \sin 6°}$$

$$= \underline{\underline{1.105}}$$

(c) At 10,000m

$$\frac{p_\infty}{p_{SL}} = .2615 \qquad p_{SL} = 101.325 \text{ kPa}$$

$$\therefore F = (p_\infty - p_{3B}) A_{door} = .274 \times 3 \times .2615 \times 101.325 \text{ kN}$$

$$= 21.8 \text{ kN}$$

(d) Apart from enabling vertical take off/landing:

Canopy A has 180° visibility and is easier for incorporation of ejector seat because of the increased area. The flow is delivered to the tailplane/rest of aircraft parallel to flight direction.

Canopy B has less pressure drag on airframe but delivered flow will need further turning towards flight direction, implying a further shock.

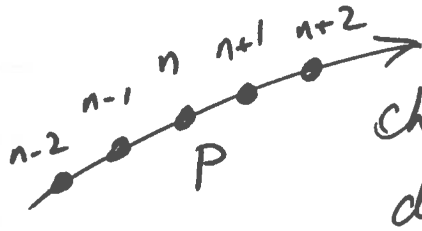
Examiner's Note 52 attempts Mean 14.0

All candidates had a clear idea of the shock and expansion systems implied by the two designs. The rest of the question was generally well done but quite a few candidates laboured over the finding of static pressure ratios across shocks; tending to work via stagnation pressure ratios and Mach numbers and then getting bogged down and losing heart. Static pressure ratios are also part of the tables.

Most candidates defined "pressure drag" as $\int p\, dA_\perp$ rather than $\int (p - p_\infty)\, dA_\perp$. Since this definition is also widely used, it was accepted as correct (i.e. not penalised). The pressure drag ratio with this definition is 1.085.

For closed surfaces, the p_∞ cancels, In this case the forebody is not a closed surface, so $\int p\, dA_\perp$ will be non-zero even when the aircraft is at rest, Hence the use of $\int (p - p_\infty)\, dA_\perp$ which is a true measure of the drag force.

6(a) For hyperbolic equations information travels along characteristic directions. Upwind differences are one-sided differences which use only nodes in the "upstream" sense.



characteristic direction

Derivatives at P use a stencil which only involves $u_n, u_{n-1}, u_{n-2}, \ldots$ and not $u_{n+1}, u_{n+2}, \ldots$

(b)

$$u_i^{n+1} = u_i^n + \Delta t \frac{\partial u}{\partial t} + \frac{\Delta t^2}{2!}\frac{\partial^2 u}{\partial t^2} + O(\Delta t^3)$$

$$u_{i-1}^n = u_i^n - \Delta x \frac{\partial u}{\partial x} + \frac{\Delta x^2}{2!}\frac{\partial^2 u}{\partial x^2} + O(\Delta x^3)$$

$$u_i^{n+1} = u_i^n - c\left(u_i^n - u_{i-1}^n\right)$$

$$\Rightarrow \quad \Delta t \frac{\partial u}{\partial t} + \frac{\Delta t^2}{2!}\frac{\partial^2 u}{\partial t^2} + O(\Delta t^3) = -\frac{A\Delta t}{\Delta x}\left(\Delta x \frac{\partial u}{\partial x} - \frac{\Delta x^2}{2!}\frac{\partial^2 u}{\partial x^2} + O(\Delta x^3)\right)$$

Equivalent differential equation

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = A\frac{\Delta x}{2}\frac{\partial^2 u}{\partial x^2} - \frac{\Delta t}{2}\frac{\partial^2 u}{\partial t^2}$$

Now $\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = 0 \Rightarrow \frac{\partial^2 u}{\partial t^2} = -\frac{\partial}{\partial t}\left(A\frac{\partial u}{\partial x}\right)$

$$\Rightarrow \frac{\partial^2 u}{\partial t^2} = -A\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial t}\right) = -A\frac{\partial}{\partial x}\left(-A\frac{\partial u}{\partial x}\right)$$

$$= A^2\frac{\partial^2 u}{\partial x^2}$$

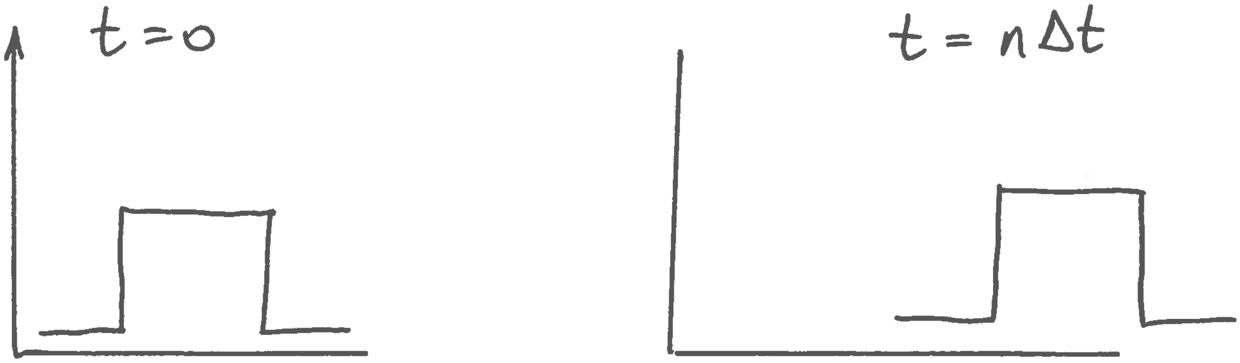
$\therefore$ Equiv Diff Eq^n is

$$\frac{\partial u}{\partial t} + A\frac{\partial u}{\partial x} = \left(\frac{A\Delta}{2} - A^2\frac{\Delta t}{2}\right)\frac{\partial^2 u}{\partial x^2}$$

$$= \alpha\frac{\partial^2 u}{\partial x^2}$$

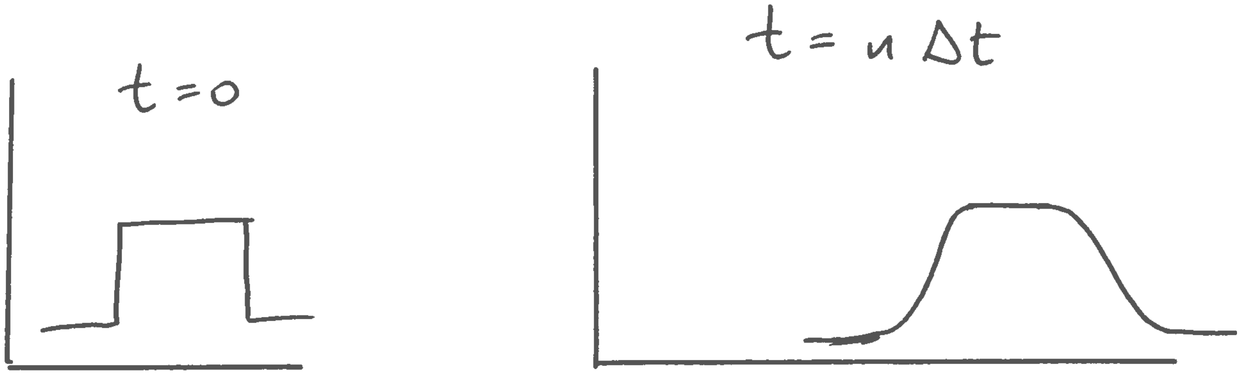
where $\alpha = \underbrace{A\frac{\Delta x}{2} - A^2\frac{\Delta t}{2}}_{\text{1st order accurate}} = \frac{c - c^2}{2}\frac{\Delta x^2}{\Delta t}$

(c) (i) $c = 1 \Rightarrow \alpha = 0$ no diffusion



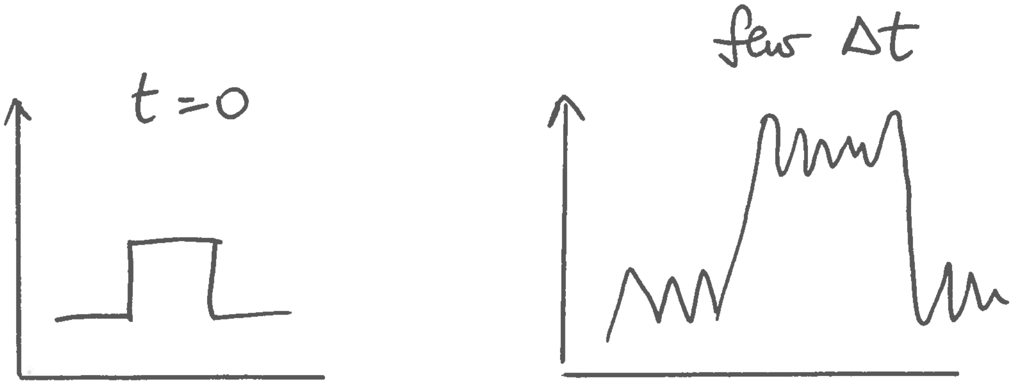
convection without distortion.

(ii) $c = \frac{1}{2}$ $\qquad \alpha = \text{maximum}$



Diffusion of sharp gradients

(iii) $c = 1.5$ $\qquad \alpha$ negative $\Rightarrow$ unstable



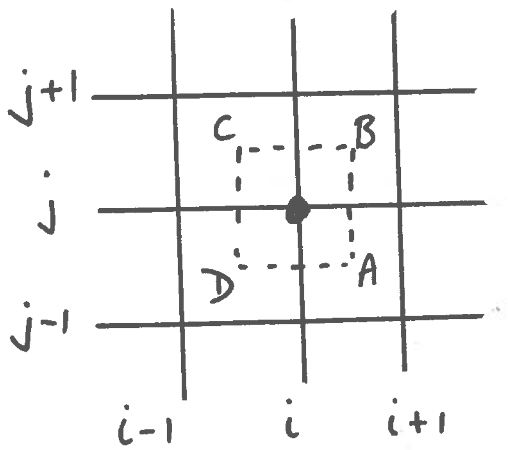
unstable

Examiner's Note

Attempts 57 $\qquad$ Mean 15.3/20

All candidates knew exactly what to do and did it well. Only weak part was the solution sketching.

7 (a) (i)



$$\nabla^2 \varphi = 0$$

Taking ABCD as the control volume

$$\int_{ABCD} \nabla^2 \varphi \, dV = 0$$

$$\Rightarrow \int_{\substack{\text{around} \\ ABCD}} \frac{\partial \varphi}{\partial n} \, dS = 0$$

On AB $\quad \dfrac{\partial \varphi}{\partial n} = \dfrac{\varphi_{i+1\,j} - \varphi_{ij}}{\Delta x} \Rightarrow \displaystyle\int_{AB} \frac{\partial \varphi}{\partial n} dS = \frac{\varphi_{i+1\,j} - \varphi_{ij}}{\Delta x} \Delta y$

Similarly for other sides

$$\int \frac{\partial \varphi}{\partial n} dS = 0 \Rightarrow 0 = \frac{\varphi_{i+1\,j} - \varphi_{ij}}{\Delta x} \Delta y + \frac{\varphi_{i\,j+1} - \varphi_{ij}}{\Delta y} \Delta x$$

$$+ \frac{\varphi_{i-1\,j} - \varphi_{ij}}{\Delta x} \Delta y + \frac{\varphi_{i\,j-1} - \varphi_{ij}}{\Delta y} \Delta x$$

$$\Rightarrow \underline{\frac{\varphi_{i+1\,j} - 2\varphi_{ij} + \varphi_{i-1\,j}}{\Delta x^2} + \frac{\varphi_{i\,j+1} - 2\varphi_{ij} + \varphi_{i\,j-1}}{\Delta y^2} = 0}$$

(ii) $\varphi_{i\pm1\,j} = \varphi_{ij} \pm \Delta x \dfrac{\partial\varphi}{\partial x} + \dfrac{\Delta x^2}{2!}\dfrac{\partial^2\varphi}{\partial x^2} \pm \dfrac{\Delta x^3}{3!}\dfrac{\partial^3\varphi}{\partial x^3} + O(\Delta x^4)$

$$\therefore\ \varphi_{i+1\,j} - 2\varphi_{ij} + \varphi_{i-1\,j} = \Delta x^2 \frac{\partial^2\varphi}{\partial x^2} + O(\Delta x^4)$$

ie $$\frac{\varphi_{i+1\,j} - 2\varphi_{ij} + \varphi_{i-1\,j}}{\Delta x^2} = \frac{\partial^2\varphi}{\partial x^2} + O(\Delta x^2)$$

$\Rightarrow$ <u>2nd order accurate</u>

Similarly for y-derivatives

(b) (i) Euler's Work Eq^n:

$$h_{02} - h_{01} = U_2 V_2 - U_1 V_1$$

$U = \Omega r$
$V = \theta$-component of vel

$$\Rightarrow h_{02} - r_2\Omega V_2 = h_{01} - r_1 \Omega V_1$$

ie $$h_0 - \Omega r V = \text{const.}$$

Now $h_0^{rel} = h + \frac{1}{2}(u^2 + V_{rel}^2 + w^2)$ $\qquad V_{rel} = V - \Omega r$

$$= h + \tfrac{1}{2}(u^2 + V^2 - 2\Omega r V + \Omega^2 r^2 + w^2)$$

$$= h + \tfrac{1}{2}(u^2 + V^2 + w^2) + \tfrac{1}{2}\Omega^2 r^2 - \Omega r V$$

$$\Rightarrow \quad \underline{h_0^{rel} - \tfrac{1}{2}\Omega^2 r^2 = h_0 - \Omega r V = \text{const}}$$

(ii) 700 rpm $\Rightarrow$ $\Omega = 73 \cdot 3$ rad/s

$r_A = .1\,m \quad p_{0A}^{rel} = 1$ bar $\quad r_B = .2\,m \quad p_{0B}^{rel} = 2$ bar

$$\therefore \; h_0^{rel} - \frac{\Omega^2 r^2}{2} = \text{const} \Rightarrow \quad h_{0_2}^{rel} - h_{0_1}^{rel} = \tfrac{1}{2}\Omega^2(r_2^2 - r_1^2)$$

$$\Rightarrow \; h_{0_2}^{rel} - h_{0_1}^{rel} = \tfrac{1}{2}(73 \cdot 3)^2(.2^2 - .1^2) = 80.6 \text{ J kg}^{-1}$$

$$(h_{02}^{Rel} - h_{01}^{Rel})_{IS} = \frac{p_{02}^{rel} - p_{01}^{Rel}}{\rho} = \frac{2\times10^5 - 1\times10^5}{10^3}$$

$$= 100 \text{ J kg}^{-1}$$

Thus $|\Delta h_0^{rel}| < |\Delta h_{0IS}^{rel}| \Rightarrow$ <u>turbine</u>

Examiner's Note

Attempts 26 Average 8.6/20

Not a popular question. Most candidates missed the point of part (a) and leapt straight into Taylor's Theorem and finite differences. Part (b) was found to be tricky.

$dh = \frac{dp}{\rho} + T\,ds$ is the key. $ds > 0$ for real device (following flow).

8 (a) 10,000 rpm $\Rightarrow \Omega = 1047.2$ rad/s

$\bar{r} = .3\,m \Rightarrow \bar{U} = \Omega\bar{r} = 314.2$ m/s

$$\psi = \frac{\Delta h_0}{\bar{U}^2} = 1.4 \Rightarrow \Delta h_0 = 138200 \text{ J/kg}$$

$$\text{Power} = \dot{m}\,\Delta h_0 = 50 \times 138200 = \underline{\underline{6.91\,MW}}$$

$$T_{03} = T_{01} - \frac{\Delta h_0}{c_p} = 1200 - \frac{138200}{1149} = 1080\,K$$

$$\eta_{tt} = \frac{\Delta T_0}{\Delta T_0^{ISEN}} = \frac{T_{01} - T_{03}}{T_{01} - T_{03}^{ISEN}}$$

$$\Rightarrow T_{03}^{ISEN} = 1200 - \frac{(1200 - 1080)}{.9} = 1066\,K$$

$$\frac{p_{03}}{p_{01}} = \left(\frac{T_{03}^{is}}{T_{01}}\right)^{\frac{\gamma}{\gamma-1}} = \left(\frac{1066}{1200}\right)^{\frac{1.333}{.333}} = .6232$$

Hence $p_{03} = 8 \times 10^5 \times .6232 = \underline{\underline{4.986}}$ bar

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(b) $\phi = \dfrac{V_x}{\bar{U}} \Rightarrow V_x = .7 \times 314.2 = \underline{219.9}$ m/s

Euler Work Eq^n: $\dfrac{\Delta h_0}{\bar{U}^2} = \dfrac{\bar{U}(V_{\theta 2} - \overset{0}{V_{\theta 3}})}{\bar{U}^2} = \dfrac{V_{\theta_2}}{\bar{U}}$

$\Rightarrow V_{\theta_2} = 314.2 \times 1.4 = \underline{439.8}$ m/s

$$\tan \alpha_2 = \frac{V_{\theta_2}}{V_{x_2}} \Rightarrow \alpha_2 = \tan^{-1}\left(\frac{439.8}{219.9}\right) = \underline{\underline{63.4^\circ}}$$

$$T_2 = T_{01} - \frac{V_{x_2}^2 + V_{\theta_2}^2}{2c_p} = 1200 - \frac{219.9^2 + 439.8^2}{2 \times 1149}$$

$$= 1095 \text{ K}$$

$$\Rightarrow M_2 = \sqrt{\frac{V_x^2 + V_\theta^2}{\gamma R T_2}} = \left[\frac{219.9^2 + 439.8^2}{1.333 \times 287 \times 1095}\right]^{1/2}$$

$$\underline{\underline{M_2 = .760}}$$

(c) $$Y_p = \frac{p_{01} - p_{02}}{p_{02} - p_2} \quad (\text{stator})$$

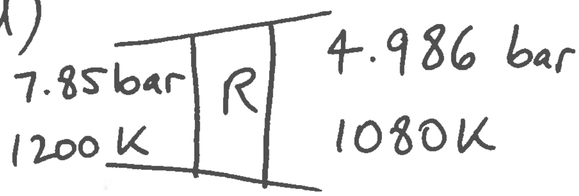
$$p_2 = p_{02}\left(\frac{T_2}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}} = 7.85 \times 10^5 \left(\frac{1095}{1200}\right)^{\frac{1.333}{.333}} = \underline{5.437 \text{ bar}}$$

$$Y_p = \frac{8 - 7.85}{7.85 - 5.437} = \underline{\underline{.062}}$$

[$Y_p = \dfrac{p_{01} - p_{02}}{p_{01} - p_2}$ also sometimes used & acceptable here

$$= \frac{8 - 7.85}{8 - 5.437} = 0.059$$]

(d)



(Rotor only)

$$T_{03}^{ISEN} = T_{03}\left(\frac{P_{03}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}} = 1200\left(\frac{4.986}{7.85}\right)^{\frac{.333}{1.333}}$$

$$= 1071 \text{ K}$$

$$\eta_{tt}^{rotor} = \frac{T_{02} - T_{03}}{T_{02} - T_{03}^{ISEN}} = \frac{1200 - 1080}{1200 - 1071}$$

$$= .93$$

(e) (i) zero exit swirl into combustion chamber means a smaller wasted exit KE ⇒ good choice

(ii) Another turbine downstream means exit K.E. is useful, so allowing $V_{\theta 3} < 0$ would permit larger stage 1 work output without compromising cycle efficiency

⇒ zero exit swirl not necessary.

Examiner's Note Attempts 30 Mean 12.2

Quite well done by those who did it seriously. The commonest error was to use formulae for loss coefficient & η applicable to compressors.