

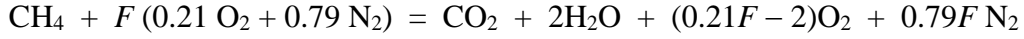
**ENGINEERING TRIPOS PART IIA 2015**

**MODULE 3A5 – THERMODYNAMICS AND POWER GENERATION**

**SOLUTIONS TO TRIPOS QUESTIONS**

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1 (a) The chemical reaction is,



The SFEE for an adiabatic combustor is,

$$\bar{q} - \bar{w}_x = 0 = [\bar{h}_{\text{CO}_2}(T_2) + 2\bar{h}_{\text{H}_2\text{O}}(T_2) + (0.21F - 2)\bar{h}_{\text{O}_2}(T_2) + 0.79F\bar{h}_{\text{N}_2}(T_2)] \\ - [\bar{h}_{\text{CH}_4}(T_1) + 0.21F\bar{h}_{\text{O}_2}(T_1) + 0.79F\bar{h}_{\text{N}_2}(T_1)]$$

By definition,  $\Delta\bar{H}_{T_1}^0 = \bar{h}_{\text{CO}_2}(T_1) + 2\bar{h}_{\text{H}_2\text{O}}(T_1) - \bar{h}_{\text{CH}_4}(T_1) - 2\bar{h}_{\text{O}_2}(T_1)$

Eliminating  $\bar{h}_{\text{CH}_4}(T_1)$  gives,

$$-\Delta\bar{H}_{T_1}^0 = [\bar{h}_{\text{CO}_2}(T_2) - \bar{h}_{\text{CO}_2}(T_1)] + 2[\bar{h}_{\text{H}_2\text{O}}(T_2) - \bar{h}_{\text{H}_2\text{O}}(T_1)] \\ + (0.21F - 2)[\bar{h}_{\text{O}_2}(T_2) - \bar{h}_{\text{O}_2}(T_1)] + 0.79F[\bar{h}_{\text{N}_2}(T_2) - \bar{h}_{\text{N}_2}(T_1)]$$

As all species behave as perfect gases with the same isobaric molar heat capacity,

$$-\Delta\bar{H}_{T_1}^0 = [1 + 2 + (0.21F - 2) + 0.79F] \bar{c}_p (T_2 - T_1)$$

Hence,  $T_2 - T_1 = \frac{-\Delta\bar{H}_{T_1}^0}{(1 + F)\bar{c}_p}$  [25 %]

(b) All chemical species enter or leave the chamber separately at standard pressure  $p_0$ . The SFSE for the combustor is thus,

$$\Delta\bar{s}_{irrev} = [\bar{s}_{\text{CO}_2}(T_2, p_0) + 2\bar{s}_{\text{H}_2\text{O}}(T_2, p_0) + (0.21F - 2)\bar{s}_{\text{O}_2}(T_2, p_0) + 0.79F\bar{s}_{\text{N}_2}(T_2, p_0)] \\ - [\bar{s}_{\text{CH}_4}(T_1, p_0) + 0.21F\bar{s}_{\text{O}_2}(T_1, p_0) + 0.79F\bar{s}_{\text{N}_2}(T_1, p_0)]$$

By definition,  $\Delta\bar{S}_{T_1}^0 = \bar{s}_{\text{CO}_2}(T_1, p_0) + 2\bar{s}_{\text{H}_2\text{O}}(T_1, p_0) - \bar{s}_{\text{CH}_4}(T_1, p_0) - 2\bar{s}_{\text{O}_2}(T_1, p_0)$

Eliminating  $\bar{s}_{\text{CH}_4}(T_1, p_0)$  gives,

$$\Delta\bar{s}_{irrev} = \Delta\bar{S}_{T_1}^0 + [\bar{s}_{\text{CO}_2}(T_2, p_0) - \bar{s}_{\text{CO}_2}(T_1, p_0)] + 2[\bar{s}_{\text{H}_2\text{O}}(T_2, p_0) - \bar{s}_{\text{H}_2\text{O}}(T_1, p_0)] \\ + (0.21F - 2)[\bar{s}_{\text{O}_2}(T_2, p_0) - \bar{s}_{\text{O}_2}(T_1, p_0)] + 0.79F[\bar{s}_{\text{N}_2}(T_2, p_0) - \bar{s}_{\text{N}_2}(T_1, p_0)]$$

As all species behave as perfect gases with the same isobaric molar heat capacity, and as all species are at the same pressure  $p_0$ , we have,

$$\Delta\bar{s}_{irrev} = \Delta\bar{S}_{T_1}^0 + [1 + 2 + (0.21F - 2) + 0.79F] \bar{c}_p \ln\left(\frac{T_2}{T_1}\right) = \Delta\bar{S}_{T_1}^0 + (1 + F)\bar{c}_p \ln\left(\frac{T_2}{T_1}\right)$$
 [30 %]

(c) As all gas species are ideal  $K_p = K_p(T)$ . Also,  $\Delta\bar{H}_T^0$  is independent of temperature. Integrating van 't Hoff's equation :

$$\frac{d \ln(K_p)}{dT} = \frac{\Delta\bar{H}_T^0}{RT^2} \rightarrow \ln(K_p) = -\frac{\Delta\bar{H}_T^0}{RT} + \text{const.}$$

From the definition of  $K_p$  we have  $\Delta\bar{G}_T^0 = -RT \ln(K_p)$  (see data book). Also, from the definitions of  $\Delta\bar{G}_T^0$ ,  $\Delta\bar{H}_T^0$  and  $\Delta\bar{S}_T^0$  we have  $\Delta\bar{G}_T^0 = \Delta\bar{H}_T^0 - T \Delta\bar{S}_T^0$ . Thus,

$$-\frac{\Delta\bar{G}_T^0}{RT} = -\frac{\Delta\bar{H}_T^0}{RT} + \text{const.} \rightarrow \frac{\Delta\bar{S}_T^0}{R} = \text{const.}$$

Hence,  $\Delta\bar{S}_T^0$  is independent of temperature. [20 %]

(d) As in part (c), we assume  $\Delta\bar{H}_{T_1}^0 = \Delta\bar{H}_{T_0}^0 = 802.3 \text{ MJ kmol}^{-1}$ . Then, from part (a),

$$1 + F = \frac{-\Delta\bar{H}_{T_1}^0}{\bar{c}_p (T_2 - T_1)} = \frac{802.3 \times 1000}{30.0 \times (1750 - 800)} = 28.15$$

We have  $\Delta\bar{G}_{T_0}^0 = \Delta\bar{H}_{T_0}^0 - T_0 \Delta\bar{S}_{T_0}^0$  and hence,

$$\Delta\bar{S}_{T_0}^0 = \frac{[-\Delta\bar{G}_{T_0}^0] - [-\Delta\bar{H}_{T_0}^0]}{T_0} = \frac{800.8 - 802.3}{298.15} = -0.005031 \text{ MJ kmol}^{-1} \text{ K}^{-1}$$

From the result of part (c),  $\Delta\bar{S}_{T_1}^0 = \Delta\bar{S}_{T_0}^0 = -5.031 \text{ kJ kmol}^{-1} \text{ K}^{-1}$ .

From the result of part (b),

$$\begin{aligned} \Delta\bar{s}_{irrev} &= \Delta\bar{S}_{T_1}^0 + (1 + F) \bar{c}_p \ln\left(\frac{T_2}{T_1}\right) \\ &= -5.031 + 28.15 \times 30.0 \times \ln\left(\frac{1750}{800}\right) = 656.01 \text{ kJ kmol}^{-1} \text{ K}^{-1} \end{aligned}$$

Hence, the maximum rational efficiency of the power plant is,

$$\eta_{rat,max} = 1 - \frac{T_0 \Delta\bar{s}_{irrev}}{[-\Delta\bar{G}_{T_0}^0]} = 1 - \frac{298.15 \times 656.01}{800.8 \times 1000} = 1 - 0.244 = 0.756 \quad [25\%]$$

### Q1. Examiner's Comment:

This popular question was done very badly indeed and had to be re-marked to improve the average (*i.e.*, most candidates got an extra two or three free marks for nothing). It is not clear why there were difficulties because parts (a), (b) and (d) were simplified versions of example sheet problems. In part (a) candidates were asked to apply the SFEE to a combustion chamber in order to find the molar air/fuel ratio. Despite writing out the chemical reaction correctly most then insisted in working in mass rather than molar units and arrived at the stated result by loose physical reasoning rather than precise application of the SFEE. In part (b), it was clear that few had any idea how to apply the steady-flow entropy equation or even the meaning of the quantities  $\bar{s}$ ,  $\Delta\bar{s}_{irrev}$  and  $\Delta\bar{S}_T^0$  some, or all, of which appeared, correctly or incorrectly, in their attempts. Part (c) was moderately well done as most of the information required was given in the question but few staggered on to part (d) and those who did were generally unaware that the exergy loss in the chamber was simply  $T_0 \Delta\bar{s}_{irrev}$ .

2. (a) The conditions to be satisfied at the critical point are,

$$\left(\frac{\partial p}{\partial v}\right)_T = \left(\frac{\partial^2 p}{\partial v^2}\right)_T = 0 \quad [10 \%$$

*i.e.*, the critical isotherm has a ‘horizontal’ tangent and a point of inflection in the  $p$ - $v$  diagram.

(b) The Dieterici equation of state is,

$$p = \frac{RT}{(v-b)} \exp\left(-\frac{a}{RTv}\right)$$

Applying the first condition,

$$\left(\frac{\partial p}{\partial v}\right)_T = \exp\left(-\frac{a}{RTv}\right) \left[ \frac{RT}{(v-b)} \frac{a}{RTv^2} - \frac{RT}{(v-b)^2} \right] = p \left[ \frac{a}{RTv^2} - \frac{1}{(v-b)} \right]$$

Thus, at the critical point,

$$\frac{a}{RT_C v_C^2} = \frac{1}{(v_C - b)} \quad [10 \%$$

Applying the second condition,

$$\left(\frac{\partial^2 p}{\partial v^2}\right)_T = \left(\frac{\partial p}{\partial v}\right)_T \left[ \frac{a}{RTv^2} - \frac{1}{(v-b)} \right] + p \left[ \frac{-2a}{RTv^3} + \frac{1}{(v-b)^2} \right]$$

Thus, at the critical point,

$$\frac{-2a}{RT_C v_C^3} = \frac{-1}{(v_C - b)^2} \quad [10 \%$$

Combining the two conditions to solve for  $a$  and  $b$  gives,

$$b = \frac{v_C}{2} \quad \text{and} \quad a = 2RT_C v_C \quad [5 \%$$

Substituting for  $a$  and  $b$  in the equation of state gives,

$$p = \frac{2RT}{(2v - \bar{v}_C)} \exp\left(-\frac{2T_C v_C}{T \bar{v}}\right)$$

Setting  $p = p_C$ ,  $T = T_C$  and  $v = v_C$  then gives,

$$\frac{p_C v_C}{RT_C} = 2e^{-2} = 0.271 \quad [10 \%$$

(c) Introducing the ‘reduced’ variables  $p_R = p/p_C$ ,  $T_R = T/T_C$  and  $v_R = v/v_C$ , we obtain the universal form,

$$p_R = \frac{RT_C}{p_C v_C} \frac{2T_R}{(2v_R - 1)} \exp\left(-\frac{2}{T_R v_R}\right) = \frac{e^2 T_R}{(2v_R - 1)} \exp\left(-\frac{2}{T_R v_R}\right) \quad [15 \%]$$

(d) The conditions for phase equilibrium are,

$$T_g = T_f = T, \quad p_g = p_f = p_{sat}(T), \quad g_g = g_f \quad [10 \%]$$

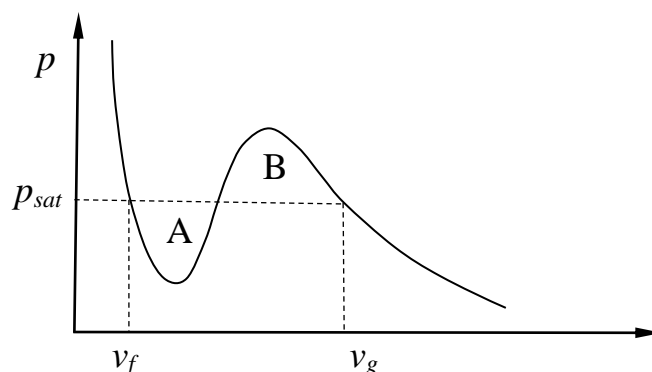
From  $dg = v dp - s dT$  applied along an isotherm, the third condition can be written,

$$g_g - g_f = \int_f^g dg = \int_f^g v dp = 0 \quad [5 \%]$$

To introduce  $p dv$  rather than  $v dp$  we note that  $d(pv) = p dv + v dp$ . Hence,

$$\int_f^g d(pv) = p_{sat}(T)(v_g - v_f) = \int_f^g p dv + \int_f^g v dp \rightarrow \int_f^g p dv - p_{sat}(T)(v_g - v_f) = 0 \quad [15 \%]$$

(e) The diagram shows an isotherm with  $T < T_C$  sketched on the  $p$ - $v$  diagram.



The final result of part (d) implies that area A is equal to area B. This is the Maxwell construction for defining the saturated vapour pressure. It ensures that  $g_g = g_f$ .

[10 %]

## Q2. Examiner's Comment:

Most candidates recognised that, although similar to an examples paper question, this question was a new departure for a 3A5 Tripos paper and decided to give it the cold shoulder. Most of the few who did not failed miserably. With hindsight, the question should have informed the candidates that the critical point on a  $p$ - $v$  diagram is defined by  $(\partial p / \partial v)_T = (\partial^2 p / \partial v^2)_T = 0$  (though this was stated in the lecture notes and used in an examples paper question related to the van der Waals equation). But even candidates who knew this were incapable of differentiating the given equation of state correctly and no-one arrived at the correct result. The second half of the question was about phase equilibrium and was genuinely tricky so it was no surprise by then to find that candidates had given up.

3. (a) In feedheating, steam is extracted ('bled') from the turbine and used to heat the feedwater delivered by the feed pump. Feedheaters can either mix the bled steam and feedwater directly (direct contact feedheater) or use a heat exchanger to transfer the heat (indirect feedheater). The reduction in external heat input needed to raise the feedwater to the saturated vapour temperature more than offsets the reduced work output from the turbine so that the cycle efficiency is increased. [10%]

(b) *Properties of steam not supplied in Table 1 are obtained from the tables in the Thermofluids Data Book.*

(i) Without feedheating: assuming the feed pump and turbine are adiabatic, the cycle efficiency is given by,

$$\eta_c = \frac{w_x}{q_{in}} = \frac{(h_3 - h_4) + (h_5 - h_6)}{(h_3 - h_1) + (h_5 - h_4)} = \frac{(3450.4 - 3060) + (3560.3 - 2180)}{(3450.4 - 121.4) + (3560.3 - 3060)} = \frac{390.4 + 1380.3}{3329 + 500.3} = 0.462$$

(ii) With feedheating: assuming that feedheating raises the temperature of the feedwater all the way to the saturation curve at state 2, that all components are reversible between states 5 and 2 (including the feedheaters), and that the environment temperature is the same as the condenser temperature, then the maximum work that can be extracted between states 5 and 2 is given by the change in the steady flow availability function,

$$w_{x52\max} = (h_5 - T_1 s_5) - (h_2 - T_1 s_2) = (h_5 - h_2) - T_1 (s_5 - s_2)$$

The cycle efficiency is then,

$$\begin{aligned} \eta_c &= \frac{(h_3 - h_4) + (h_5 - h_2) - T_1 (s_5 - s_2)}{(h_3 - h_2) + (h_5 - h_4)} \\ &= \frac{(3450.4 - 3060) + (3560.3 - 1610.3) - 302.11 \times (7.235 - 3.685)}{(3450.4 - 1610.3) + (3560.3 - 3060)} \\ &= \frac{390.4 + 1950 - 1072.5}{1840.1 + 500.3} = 0.542 \end{aligned}$$

[35%]

(c) Assume all feedpump work input can be neglected.  
SFEE for the 20 bar feedheater:

$$(1 - x)h_{2b} + xh_{6a} = h_{2a} \rightarrow x = \frac{h_{2a} - h_{2b}}{h_{6a} - h_{2b}} = \frac{908.5 - 640.1}{3330 - 640.1} = \frac{265.4}{2689.9} = 0.099$$

SFEE for the 5 bar feedheater:

$$(1 - x - y)h_1 + yh_{6b} = (1 - x)h_{2b} \rightarrow y = \frac{(1 - x)(h_{2b} - h_1)}{h_{6b} - h_1} = \frac{(1 - 0.099) \times (640.1 - 121.4)}{(2950 - 121.4)} = 0.165$$

The cycle efficiency is,

$$\begin{aligned} \eta_c &= \frac{(h_3 - h_4) + (h_5 - h_{6a}) + (1 - x)(h_{6a} - h_{6b}) + (1 - x - y)(h_{6b} - h_6)}{(h_3 - h_{2a}) + (h_5 - h_4)} \\ &= \frac{(3450.4 - 3060) + (3560.3 - 3330) + (1 - x)(3330 - 2950) + (1 - x - y)(2950 - 2180)}{(3450.4 - 908.5) + (3560.3 - 3060)} \\ &= \frac{390.4 + 230.3 + (0.901 \cdot 380) + (0.736 \cdot 770)}{3042.2} \\ &= 0.503 \end{aligned}$$

[35%]

(d) Feedheating reduces boiler efficiency because the temperature of the flue gases leaving the boiler cannot drop below the temperature of the feedwater (i.e. after the final feedheater). In order to bring the flue gas temperature closer to the environment temperature (while still keeping above the dew point temperature to avoid corrosion of the stack) a preheater is fitted. The preheater is a heat exchanger that transfer heat from the flue gases to the air entering the combustor.

[20%]

### Q3. Examiner's Comment:

Most candidates understood, broadly, the rationale behind feedheating. Almost all were able to evaluate the cycle efficiency of a steam cycle without feedheating. However, to obtain the maximum possible cycle efficiency with feedheating (*i.e.*, with an infinite number of reversible feedheaters), required an exergy analysis and very few candidates spotted this. A  $T-s$  diagram for a cycle with two direct contact feedheaters was given in the question and this helped many candidates to derive an expression for the cycle efficiency and approximately half were able to obtain the correct numerical value. Unfortunately, in the final part of the question, almost all candidates confused the preheating process used in steam cycles with feedheaters, with the HRSG preheating loop used in combined cycles.

4. (a) If the polytropic efficiency is used, a multi-stage turbine or compressor where each stage has a polytropic efficiency of  $\eta_p$  will have an overall polytropic efficiency for the whole turbine or compressor of  $\eta_p$ . If isentropic efficiency is used, and each stage has an isentropic efficiency of  $\eta_s$ , then the isentropic efficiency for the whole turbine will be greater than  $\eta_s$  and for the whole compressor will be less than  $\eta_s$ . [15%]

(b) Starting from the following expression for the second law for a small change,

$$Tds = dh - dp/\rho$$

if the expansion is isentropic we obtain  $dh_s = dp/\rho$ . For a small expansion in a turbine,  $dh_s = dh/\eta$  and so,

$$\frac{dh}{\eta} = \frac{dp}{\rho}$$

The gas is ideal,  $p = rRT$ , and semi-perfect,  $dh = c_p(T)dT$ , so we may write,

$$\frac{c_p dT}{T} = \eta R \frac{dp}{p}$$

Integrating between states 1 and 2 yields,

$$\int_{T_1}^{T_2} \frac{c_p}{T} dT = \eta R \ln\left(\frac{p_2}{p_1}\right)$$

[20%]

(c) (i) Using the model for  $c_p(T)$  we have,

$$\frac{c_{p0}}{T_0^n} \int_{T_1}^{T_2} T^{n-1} dT = \eta R \ln\left(\frac{p_2}{p_1}\right)$$

Integration leads to,

$$T_2^n = \frac{nT_0^n}{c_{p0}} \eta R \ln\left(\frac{p_2}{p_1}\right) + T_1^n$$

so that,

$$T_2 = \left[ \frac{0.12 \times 298.15^{0.12}}{1010} \times 0.9 \times 287 \times \ln\left(\frac{1}{45}\right) + 1800^{0.12} \right]^{1/0.12} = 789.6 \text{ K}$$

Assuming an adiabatic expansion in the turbine,

$$w_x = \int_{T_1}^{T_2} c_p dT = \frac{c_{p0}}{T_0^n} \int_{T_1}^{T_2} T^n dT = \frac{c_{p0}}{(n+1)T_0^n} (T_2^{n+1} - T_1^{n+1})$$



so that,

$$-w_x = \frac{1010}{(0.12+1) \times 298.15^{0.12}} (789.6^{1.12} - 1800^{1.12}) = -1214 \text{ kJ kg}^{-1}$$

and the specific work *output* from the turbine is 1214 kJ kg<sup>-1</sup>.

[30%]

(ii) If the expansion were isentropic, we would have  $h = 1$  and the turbine exit temperature would be,

$$T_2 = \left[ \frac{0.12 \times 298.15^{0.12}}{1010} \times 287 \times \ln\left(\frac{1}{45}\right) + 1800^{0.12} \right]^{1/0.12} = 716.7 \text{ K}$$

The specific work output from this isentropic turbine would be,

$$-w_x = \frac{1010}{(0.12+1) \times 298.15^{0.12}} (716.7^{1.12} - 1800^{1.12}) = -1296 \text{ kJ kg}^{-1}$$

and so the *isentropic* efficiency of the turbine is  $h_s = 1214/1296 = 0.937$ .

[15%]

(d) Aero-derivative gas turbines (50 MW scale) can respond rapidly (minutes) to changes in electricity demand compared to the hours that can be taken by the large units (300 MW scale) of combined cycle power plants. This means that aero-derivative gas turbines can be used to 'fill the gap' between electricity demand and the fluctuating output of most renewable power generation, particularly wind.

The principal source of lost exergy (apart from combustion) is the GT exhaust. This can be reduced by:

- raising steam from the exhaust and injecting this into the combustion chamber – a STIG cycle;
- intercooling the compressor (spray water into the inlet – evaporative inter cooling, or use heat exchangers). For an ideal Joule cycle, intercooling will *reduce* cycle efficiency (equivalent to adding an additional Joule cycle but at a lower pressure ratio) but there are benefits in a real aero-derivative GT because the lower temperature at compressor exit means less cooling air is needed in the turbine;
- intercooling and recuperation. If the compressor exit temperature can be reduced to below the GT exhaust temperature, then a recuperator can be used to transfer heat from the GT exhaust to the compressor exit flow (so the cycle requires a reduced external heat input). Adding a reheat stage in the turbine is not practicable due to the scale of design changes required from the unmodified aero-engine.

[20%]

#### Q4. Examiner's Comment:

Having described why the polytropic efficiency definition is often preferred to isentropic efficiency in the analysis of (multistage) turbomachines, candidates were then asked to obtain a stated result for the temperature change associated with the expansion, at polytropic efficiency  $\eta$ , of a semi-perfect gas. Rather than take the intended derivation route via small expansions, many candidates forced the result starting from the definition of polytropic efficiency for the expansion of a perfect gas. Working with a supplied function for  $c_p(T)$ , most candidates were able to obtain the correct temperature at the end of the expansion but a significant minority then incorrectly assumed a constant  $c_p$  when evaluating the turbine work output. Similarly, many candidates used the temperature-based, perfect gas definition of isentropic efficiency. Listing modifications to improve the cycle efficiency of aero-derivative gas turbines proved straightforward, but few candidates stated that such machines have rapid startup times (10s of minutes) that allow them to provide flexible power generation capability (much-needed as intermittent renewables are added to the generation mix).