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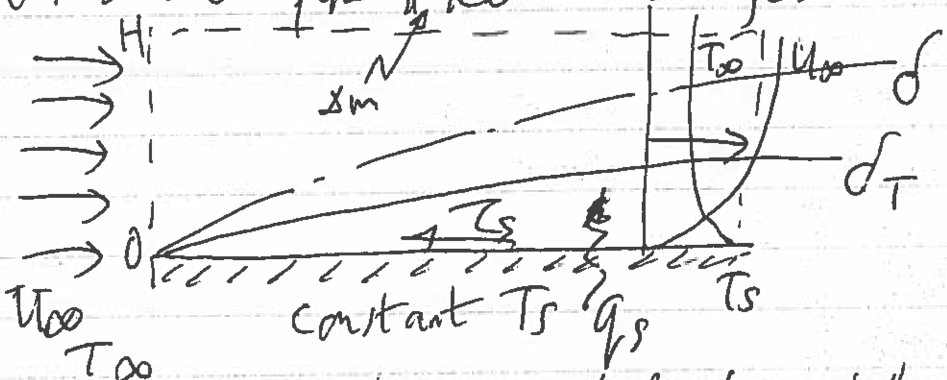
1. (a) Sketch flow & thermal b'layers:



The relative thickness of the flow & thermal b'layers, δ & δ_T, depends on the Prandtl number, $Pr = \mu c_p/\lambda$. The Prandtl number measures the viscous w.r.t. the thermal diffusion; for $Pr > 1$, $\delta > \delta_T$ & vice versa.

The Stanton number, $St = h/\rho c_p U_\infty$, Nusselt number, $Nu = hx/\lambda$, and Reynolds number, $Re = \rho U_\infty x/\mu$ are related by:

$$St = Nu/Pr\,Re \quad , \text{where } h = q_s/(T_\infty - T_s)$$

St is "the heat transfer per unit convection"
Re is "the convection vs. the viscous diffusion"
Nu is "the non-dimensional heat transfer".

(b) There are various ways to derive the integral equations; here we use "A" from the L/Notes:

$$\int_0^H (\underbrace{\rho U_\infty c_p T_\infty}_{h_\infty} - \underbrace{\rho u c_p T}_{h})\,dY - \underbrace{\Delta m}_{\int(\rho U_\infty - \rho u)\,dY} c_p T_\infty = \int_0^X q_s\,dX$$

$$\therefore \rho c_p U_\infty \underbrace{\int_0^H (T_\infty - T)\frac{u}{U_\infty}\,dY}_{} = \int_0^X q_s\,dX$$

$$\delta_\theta = \int \left(\frac{T_\infty - T}{T_\infty - T_s}\right)\frac{u}{U_\infty}\,dY$$

$\partial/\partial x$: $\quad q_s = \rho c_p u_\infty (T_\infty - T_s)\dfrac{d}{dx}(\delta_\theta)$

$St = \dfrac{q_s/(T_\infty - T_s)}{\rho c_p u_\infty}$ $\therefore$ $\underline{\dfrac{d}{dx}(\delta_\theta) = St}$.

(c) Given: $\dfrac{u}{u_\infty} = \dfrac{3}{2}\left(\dfrac{y}{\delta}\right) - \dfrac{1}{2}\left(\dfrac{y}{\delta}\right)^3$ & $\theta = \dfrac{T - T_s}{T_\infty - T_s} = \dfrac{3}{2}\left(\dfrac{y}{\delta_T}\right) - \dfrac{1}{2}\left(\dfrac{y}{\delta_T}\right)^3$

and that $Pr = 1$ $\quad$ $T = T_s @ y = 0$, $T = T_\infty @ y = \delta_T$

$\rightarrow$ assume $\delta = \delta_T$

(otherwise have to split y integration - just algebra...) ε_{xs}

So, $\delta_\theta = \displaystyle\int_0^1 \underbrace{\left(\frac{3}{2}\hat{y} - \frac{1}{2}\hat{y}^3\right)}_{u/u_\infty}\underbrace{\left(1 - \frac{3}{2}\hat{y} + \frac{1}{2}\hat{y}^3\right)}_{1-\theta} d\hat{y}$

$\dfrac{T_\infty - T}{T_\infty - T_s} = \dfrac{T_\infty - T_s - (T - T_s)}{T_\infty - T_s}$

$= \left[\dfrac{3}{2}\dfrac{y^2}{2} - \dfrac{9}{4}\dfrac{y^3}{3} + \dfrac{3}{4}\dfrac{y^5}{5} - \dfrac{1}{2}\dfrac{y^4}{4} + \dfrac{3}{4}\dfrac{y^5}{5} - \dfrac{1}{4}\dfrac{y^7}{7}\right]_0^1$

~~δ_θ~~ $\therefore$ $\underline{\delta_\theta = 0.14\,\delta}$ [Lecture 3, slide 21]

(d) Given: Blasius, $\delta/x = 5.0/\sqrt{Re_x}$

$\rightarrow$ then, $Nu = \overbrace{St}^{St \text{ from (b)}}\,Pr\,Re_x$

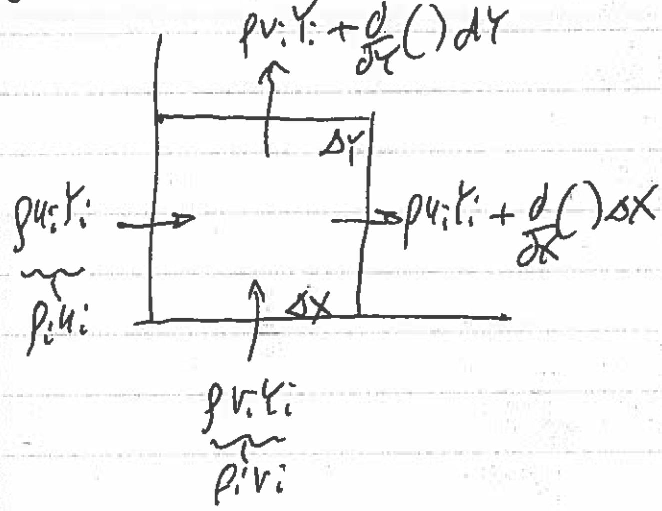
$= Pr\,\dfrac{\rho u_\infty x}{\mu} \cdot 0.14\,\dfrac{d}{dx}\left(\underbrace{5.0\sqrt{\dfrac{\mu}{\rho u_\infty}}\,x^{1/2}}_{(c)}\right)$

$= Pr \cdot \dfrac{\rho u_\infty x}{\mu} \cdot 0.14 \times 5.0 \times \sqrt{\dfrac{\mu}{\rho u_\infty}} \cdot \dfrac{1}{2}\,\dfrac{1}{x^{1/2}}$

$\therefore$ $\underline{Nu = 0.35\,Pr\,Re_x^{1/2}}$

cf. correlation ($Pr > 0.6$), $Nu = 0.45\,Pr^{1/3}\,Re^{1/2}$.

2. (a) Consider $\delta x, \delta y$ control volume :



<u>contents</u> : $\delta x \delta y \dfrac{\partial}{\partial t}(\rho Y_i)$

<u>net flux</u> : $\dfrac{\partial}{\partial x}(\rho u_i Y_i)\delta x \delta y + \dfrac{\partial}{\partial y}(\rho v_i Y_i)\delta y \delta x$

<u>Source</u> : $\dot{w}_i \, \delta x \delta y$

$$\therefore \quad \frac{\partial}{\partial t}(\rho Y_i) + \nabla \cdot \rho \underline{u}_i Y_i = \dot{w}_i$$

$$\underline{u}_i = \underline{u} + \underline{u}_{d_i}$$

<u>bulk</u>   <u>diffusive</u> fluxes

$$\underline{G}_{d_i} = \rho_i(\underline{u}_i - \underline{u})$$

$$\therefore \quad \frac{\partial}{\partial t}(\rho Y_i) + \nabla \cdot \rho \underline{u} Y_i = -\nabla \cdot \underline{G}_{d_i} + \dot{w}_i$$

(b) Sum over all species:

$$\frac{\partial}{\partial t}\rho \underbrace{\sum_{i=1}^{N} Y_i}_{=1} + \nabla \cdot \rho \underline{u} \underbrace{\sum Y_i}_{=1} = -\nabla \cdot \underbrace{\sum \underline{G}_{d_i}}_{=0} + \underbrace{\sum \dot{w}_i}_{=0} \; !$$

$$\therefore \quad \frac{\partial \rho}{\partial t} + \nabla \cdot \rho \underline{u} = 0$$

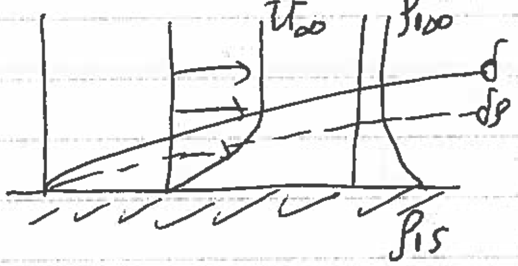
, standard bulk mass conservation eq^{n}.

(c) <u>Fick's Law</u> : $\underline{G}_{d_i} = -\rho D_{ij} \nabla Y_i$

D_{ij} "Diffusion coefficient"

$Y_i = \rho_i/\rho$ so $\underline{G}_{d_i} = -D_{ij} \nabla \rho_i$

Hence, for mass transfer from a flat plate:



mass transfer coefficient:

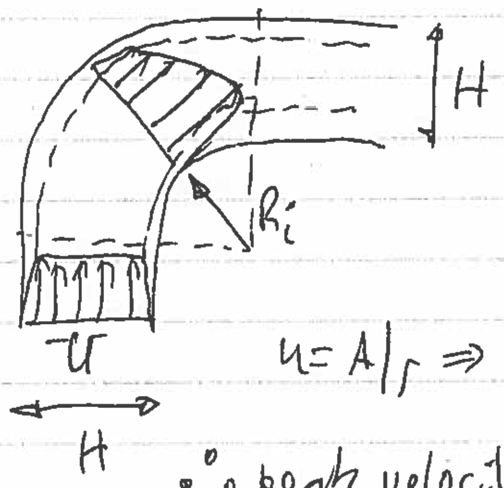
$$h_{m_1} = \frac{-D(\partial\rho_1/\partial y)_{y=0}}{(\rho_{1s}-\rho_{1\infty})}$$

cf. heat transfer coefficient, $h = \frac{-\lambda(\partial T/\partial y)_{y=0}}{\rho c_p U_\infty (T_s - T_\infty)}$

& momentum transfer coeff. $C_f = \frac{\mu(\partial u/\partial y)_{y=0}}{\frac{1}{2}\rho U_\infty^2}$.

(d) Consider developing flow in a rectangular channel with thin, turbulent b'layers and

$$Sh_x \sim Sc^{1/3} Re_x^{4/5}$$



assuming free-vortex flow outside the thin b'layers

$$u = A/r \Rightarrow UH = \int_{R_i}^{R_i+H} \frac{A}{r}\,dr = A\log_e\left(1+\frac{H}{R_i}\right)$$

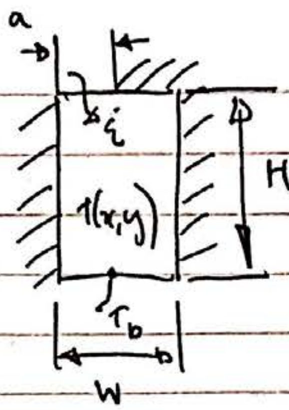
∴ peak velocity at inside of bend $u_{pk} = \frac{UH}{R_i}\cdot\frac{1}{\log(1+H/R_i)}$.

∴ ratio of mass transfer coefficient for case $R_i = H$ & $R_i = 2H$ is $\frac{h_{mH}}{h_{m2H}} = 2\frac{\log(1.5)}{\log(2)}$

$= 1.13$.

So, 13% more mass transfer

a)



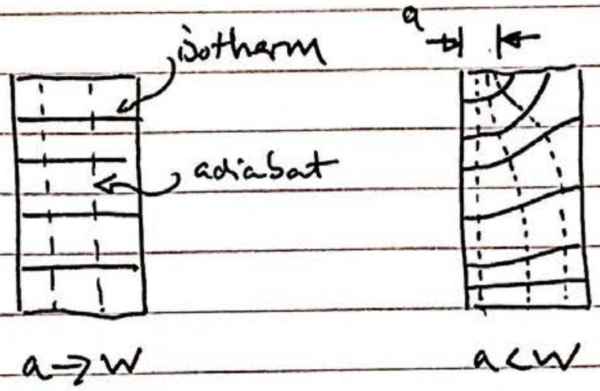
i) As $a \to W$ the problem can be approximated as 1-D.

$$\dot{Q} = -\lambda \,\text{Area} \frac{dT}{dy} \Rightarrow -\dot{q}a = -\lambda W \frac{d\theta}{dy}$$

so $$\theta = \frac{\dot{q}}{\lambda}\frac{a}{W} y$$

this is a lower bound on the full 2-D temperature field, and will be the 0^{th} term in the series.

ii)



iii) $$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} = 0 \Rightarrow \frac{\partial^2 \theta}{\partial x^2} + \frac{\partial^2 \theta}{\partial y^2} = 0$$ where $\theta(x,y) = T(x,y) - T_b$

Let $\theta = XY$ so $$Y\frac{\partial^2 X}{\partial x^2} + X\frac{\partial^2 Y}{\partial y^2} = 0 \Rightarrow \frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -\frac{1}{Y}\frac{\partial^2 Y}{\partial y^2} = -m^2$$

(we need a negative separation constant for homogeneous x boundaries).

$$\Rightarrow \theta = \left(C_1 \sin(mx) + C_2 \cos(mx)\right)\left(C_3 \sinh(my) + C_4 \cosh(my)\right)$$

for $\theta(0,y) = \theta(W,y) = 0$

$$\theta(x,y) = \sum_{n=0}^{\infty} \cos(m_n x)\left(A_n \sinh(m_n y) + B_n \cosh(m_n y)\right)$$

?

iv) Consider: $\theta(x,0) = 0$

$$\theta(x,0) = \sum_{n=0}^{\infty} \cos(m_n x)\left(A_n \sinh(0) + B_n \cosh(0)\right) = 0$$

$$\Rightarrow B_n = 0$$

Now look at 0^{th} term carefully:

$$\theta(x,y) = \underbrace{\lim_{n\to 0}\left\{\cancel{\cos(m_n x)}^{\,0} A_n \cancel{\sinh(m_n y)}^{\,m_n y}\right\}}_{A_0} + \sum_{n=1}^{\infty} \cos(m_n x) A_n \sinh(m_n y)$$

$A_0 = \lim_{n\to 0}\left\{A_n m_n\right\} y$ so we need to find $\lim_{n\to 0}\left\{A_n m_n\right\}$

Consider: $\lambda \dfrac{\partial\theta(x,H)}{\partial y} = \begin{cases} \dot{q} & x < a \\ 0 & \text{otherwise} \end{cases}$

$$\frac{\partial\theta(x,H)}{\partial y} = \lim_{n\to 0}\left\{A_n m_n\right\} + \sum_{n=1}^{\infty} \cos(m_n x) A_n m_n \cosh(m_n H) = \begin{cases} \dot{q}/\lambda \\ 0 \end{cases}$$

to find $\lim_{n\to 0}\left\{A_n m_n\right\}$ and A_n we need to multiply both sides by $\cos(m_k x)$ and integrate from $x = 0 \Rightarrow W$.

$$\int_0^W \lim_{n\to 0}\{A_n m_n\}\cos(m_0 x)\,dx + \int_0^W \sum_{n=1}^{\infty}\cos(m_n x)\cos(m_k x)\,A_n m_n \cosh(m_n H)\,dx \ldots$$
$$= \int_0^a \frac{\dot{q}}{\lambda}\cos(m_k x)\,dx + \int_a^W 0\cos(m_k x)\,dx$$

For $k=0$ we have:

$$\int_0^W \lim_{n\to 0}\{A_n m_n\}\,dx = \int_0^a \frac{\dot{q}}{\lambda}\cos(m_0 x)\,dx$$

$$\lim_{n\to 0}\{A_n m_n\}\,W = \frac{\dot{q}a}{\lambda} \Rightarrow \lim_{n\to 0}\{A_n m_n\} = \frac{\dot{q}a}{\lambda W} \quad //$$

For $k=1,2\ldots$ we have: for $\underline{\underline{n=k \text{ only}}}$

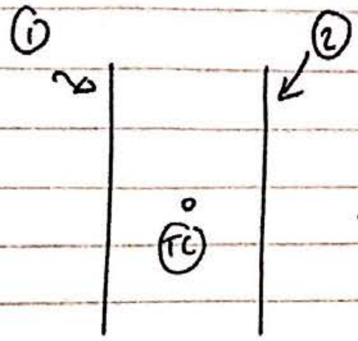
$$\int_0^W \cos^2(m_n x)\,A_n m_n \cosh(m_n H)\,dx = \int_0^a \frac{\dot{q}}{\lambda}\cos(m_n x)\,dx$$

$$\int_0^W \cos^2(m_n x)\,dx = \frac{W}{2}, \quad \int_0^a \cos(m_n x)\,dx = \frac{\sin(m_n a)}{m_n}$$

$$\Rightarrow A_n = 2\frac{\dot{q}}{\lambda W}\,\frac{\sin(m_n a)}{m_n^2}\cdot\frac{1}{\cosh(m_n H)} \quad //$$

$$\theta(x,y) = \frac{\dot{q}}{\lambda W}\left\{ay + 2\sum_{n=1}^{\infty}\cos(m_n x)\,\frac{\sinh(m_n y)}{\cosh(m_n H)}\,\frac{\sin(m_n a)}{m_n^2}\right\} \quad //$$

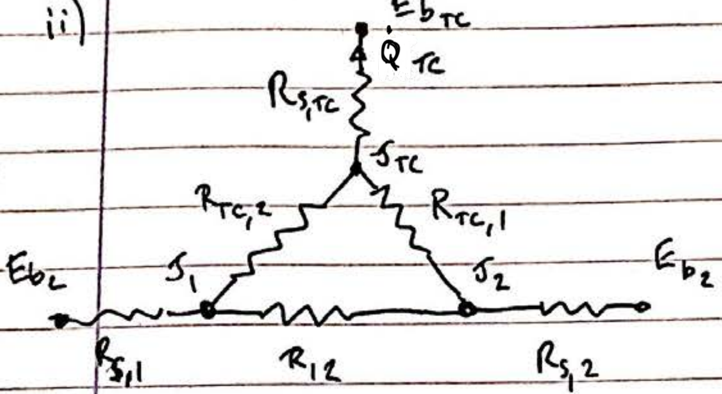
i)



$$F_{TC,TC} + F_{TC,1} + F_{TC,2} = 1$$

By symmetry $F_{TC,1} = F_{TC,2} = 0.5$ //

ii)



$$R_{s,TC} = \frac{1-\varepsilon_{TC}}{\varepsilon_{TC} A_{TC}}$$

$$R_{TC,1} = \frac{1}{F_{TC,1} A_{TC}} = \frac{2}{A_{TC}}$$

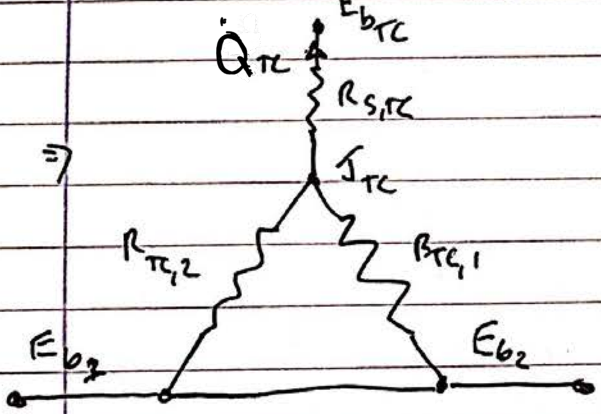
$$R_{TC,2} = \frac{1}{F_{TC,2} A_{TC}} = \frac{2}{A_{TC}}$$

$$R_{12} = \frac{1}{F_{12} A_1} \rightarrow 0$$

$$R_{s,1} = \frac{1-\varepsilon_1}{\varepsilon_1 A_1} \rightarrow 0$$

$$R_{s,2} = \frac{1-\varepsilon_2}{\varepsilon_2 A_2} \rightarrow 0$$

$\Rightarrow$



iii) Energy balance at J_{TC}

$$\frac{E_{b_2} - J_{TC}}{R_{TC,2}} + \frac{E_{b_1} - J_{TC}}{R_{TC,2}} = \frac{J_{TC} - E_{b_{TC}}}{R_{s,TC}}$$

$$\Rightarrow \quad J_{TC} = \frac{\dfrac{E_{b2}}{R_{TC,2}} + \dfrac{E_{b1}}{R_{TC,1}} + \dfrac{E_{bTC}}{R_{s,TC}}}{\left(\dfrac{1}{R_{s,TC}} + \dfrac{1}{R_{TC,2}} + \dfrac{1}{R_{TC,1}}\right)}$$

$$= \frac{\cancel{A_{TC}}\frac{1}{2}\sigma\left(T_2^4 + T_1^4\right) + \dfrac{\varepsilon_{TC}\cancel{A_{TC}}\sigma T_{TC}^4}{1-\varepsilon_{TC}}}{\dfrac{\varepsilon_{TC}\cancel{A_{TC}}}{1-\varepsilon_{TC}} + \dfrac{\cancel{A_{TC}}}{2} + \dfrac{\cancel{A_{TC}}}{2}}$$

$$= \frac{\frac{1}{2}\sigma\left(T_2^4 + T_1^4\right) + \dfrac{\varepsilon_{TC}\sigma T_{TC}^4}{1-\varepsilon_{TC}}}{\varepsilon_{TC}/_{1-\varepsilon_{TC}} + 1}$$

$$J_{TC} = \frac{\left(1-\varepsilon_{TC}\right)\frac{1}{2}\sigma\left(T_2^4 + T_1^4\right) + \varepsilon_{TC}\sigma T_{TC}^4}{\cancel{\varepsilon_{TC}} + 1 - \cancel{\varepsilon_{TC}}}$$

$$\dot{Q}_{TC} = \frac{J_{TC} - E_{bTC}}{R_{s,TC}} = \frac{\left(1-\varepsilon_{TC}\right)\frac{1}{2}\sigma\left(T_2^4 + T_1^4\right) + \left(\varepsilon_{TC}-1\right)\sigma T_{TC}^4}{\left(1-\varepsilon_{TC}\right)/\varepsilon_{TC}A_{TC}}$$

$$= \frac{\cancel{\left(1-\varepsilon_{TC}\right)}}{\cancel{\left(1-\varepsilon_{TC}\right)}}\left[\frac{1}{2}\sigma\left(T_2^4 + T_1^4\right) - \sigma T_{TC}^4\right]\varepsilon_{TC}A_{TC}$$

$$\dot{Q}_{TC} = A_{TC}\varepsilon\sigma\left[\frac{1}{2}\left(T_2^4 + T_1^4\right) - T_{TC}^4\right]$$

$\frac{3}{4}$

iv) In equilibrium : $\dot{q}_{TC} = 0$

$$\Rightarrow \quad \frac{1}{2}\left(T_2^4 + T_1^4\right) = T_{TC}^4$$

$$\Rightarrow T_{TC} = \left(\frac{1}{2}\left(T_2^4 + T_1\right)^4\right)^{1/4}$$

b)
i) Convective cooling reduces temperature at the bead until the radiation is in balance, so result from (a)(iii) still valid:

$$\dot{Q}_{radiation} = \dot{Q}_{convection}.$$

$$\cancel{A_{TC}}\, h \left(T_{air} - T_{TC}\right) = \cancel{A_{TC}}\, \frac{\varepsilon_{TC}}{4} \sigma \left[\frac{1}{2}\left(T_2^4 + T_1^4\right) - T_{TC}^4\right]$$

$$T_{TC} = T_{air} - \frac{\varepsilon_{TC}\sigma}{h}\left[\frac{1}{2}\left(T_2^4 + T_1^4\right) - T_{TC}^4\right]$$

or $$T_{air} - T_{TC} = \frac{\varepsilon_{TC}\sigma}{h}\left[\frac{1}{2}\left(T_2^4 + T_1^4\right) - T_{TC}^4\right]$$

ii) $h = 50\ \mathrm{W m^{-2} K^{-1}}$

$T_1 = 500\mathrm{K}$

$T_2 = 300\mathrm{K}$

T_{TC}	$T_{air} - \frac{\varepsilon_{TC}\sigma}{h}\left[\frac{1}{2}\left(T_2^4 + T_1^4\right) - T_{TC}^4\right]$
Guess 1: $T_{TC} = 400\mathrm{K}$	394.5 K
Guess 2: $T_{TC} = 394.5$	393.72 K
Guess 3: $T_{TC} = 393.72$	393.61 K
Guess 4: $T_{TC} = 393.61$	393.6 K.

so $\Delta T = 7.4\mathrm{K}$

iii) Reduce ε_{TC}, increase h, use a radiation shield at an intermediate temperature.