381 2010

1(a) $G = \frac{4\pi A_e}{\lambda^2}$

G, gain is the increase in radiated power density in a selected direction relative to that achieved by an isotropic antenna radiating the same total power - expressed as a linear multiple or in dB

Ae, affective aperture determines the power delivered into a matched load by a receiving antenna when an incident power density P_i falls on the antenna, such that:

$$P_{rec} = P_i \times A_e = V^2/R \Leftarrow \text{matched load.}$$

[15%] For a small antenna (dipole) the gain $\simeq 1.5$ (1.76 dB)

(b)(i) Power density $= \frac{4W}{4\pi R^2} \times 1.5$ where range, $R = 375 \times 10^3$ m

$$\therefore \underline{P = 3.40 \text{ pW/m}^2}$$

$P = \frac{\frac{1}{2} E^2}{\eta}$ where $\eta = 120\pi$, impedance of free space

[15%] $\therefore E = \sqrt{2P\eta} = \underline{50.6 \ \mu V/m}$

(ii) $\lambda = \frac{c}{f} = 3 \times 10^8 / 2 \times 10^9 = 0.15$ m

$$-100 \text{ dBm} \equiv 10^{-13} \text{ W} = A_e P = A_e . 3.4 \times 10^{-12}$$

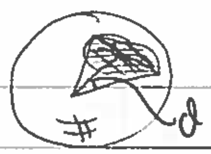
$$\therefore A_e = 0.0294 \text{ m}^2$$

$G = \frac{4\pi A_e}{\lambda^2}$ $\therefore$ $\underline{G = 16.4}$ (or 12.2 dB)

[20%] (actually not so large, possible with a dish a few feet across)

[10%] (iii) $10^{-13} = \frac{V^2}{75}$ $\therefore$ $\underline{V = 2.74 \ \mu V_{rms}}$

1(c) Gain = 16·4 ∴ power is concentrated into $\frac{1}{16 \cdot 4}$ × surface area of sphere



$$A_{tot} = 4\pi R^2 \quad \therefore \quad A_G = \frac{4\pi R^2}{16 \cdot 4} = \frac{\pi R^2}{4 \cdot 1}$$

Approximate circle area to sphere patch

$$A_G = \pi (R \sin\theta)^2 \quad \therefore \quad \sin^2\theta = 0{\cdot}244$$

$$\therefore \theta = 28^\circ \qquad \text{beam } \tfrac{1}{2}\text{-angle}$$

Hence, total/viewing angle across sky for fixed dish ≈ 56°, so call

time ~ $\frac{56}{360} \times 93$ = 14 minutes per orbit (if passing directly overhead)

[20%]

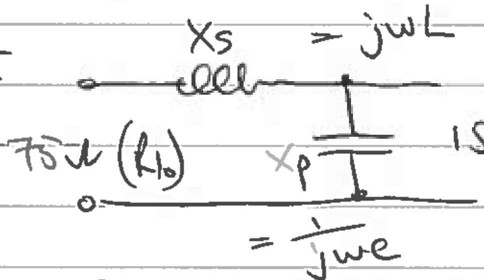
(d) with impedance mismatch, ρ, (voltage refl. coeff) = $\left|\frac{75-150}{75+150}\right|$

∴ voltage reflection of 0·33

(power reflection 11%)

So, the signal will be reduced in amplitude and waveforms distorted by reflections, possible standing waves too if the cable impedance is mismatched.

For a matching circuit



@ 2GHz, $\omega = 4\pi \times 10^9$ rad/s

$$Q = \sqrt{\frac{150}{75} - 1} = 1 \qquad \text{(low Q, so broad matching)}$$

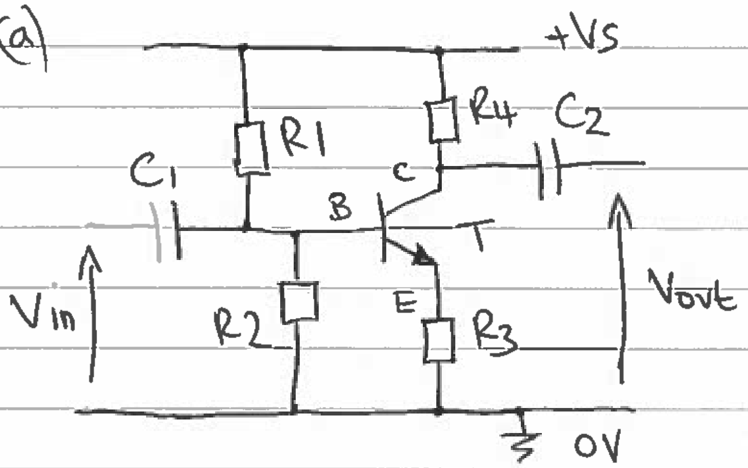
$$Q = \frac{R_{hi}}{X_p} = \frac{X_s}{R_{lo}} \quad \therefore \quad X_p = \frac{1}{4\pi \times 10^9 C} = 150 \quad \therefore C = 0{\cdot}53\text{pF}$$

$$X_s = 4\pi \times 10^9 L = 75 \quad \therefore L = 5{\cdot}97\text{nH}$$

[or 1·06 pF series and 11·9 nH parallel i.e. with L and C swapped.]

[20%]

2 (a)



$h_{fe} \simeq 250$
$V_S = 15\,V$

R_1 and R_2 set bias voltage on base (dc) for $V_C \simeq \frac{V_S}{2}$
C_1 decouples input from dc bias on base
R_3 emitter feedback to stabilise bias point and control gain
R_4 output resistor: set o/p impedance $\simeq R_4$ and gain $\simeq -\frac{R_4}{R_3}$

[20%]

(b) 20 dB of power gain ⇒ 1 transistor should be ok
0.1 W into 50Ω ⇒ $V_{sig} = 2.2\,V_{rms}$ (6.3 V_{pp} ✓ ok.)
$P = V_{sig}^2/R$

$R_4 = 50\Omega$ $f = 2\,GHz$ ∴ C_1 and C_2 choose 10nF for small impedance re. 50Ω

$= 10\log_{10}\left(\frac{V_o^2/50}{V_i^2/75}\right)$

For 20 dB gain (compared to no amplifier) = ×10 amplitude, but we add an extra inter-stage coupling with the amplifier, so we design for ×20 linear voltage gain. (×16.4 with some spare margin) ×8.215

∴ $R_3 \simeq \frac{50}{20} = 2.5\Omega$ initial trial. now with $V_C = 7.5\,V$ d.c. ($\frac{1}{2}$ supply)

$I_C = 0.15\,A$ and $r_e = \frac{0.025}{0.15} = 0.17\Omega$

Actual gain $\simeq \frac{R_4}{(R_3 + r_e)}$ ∴ set $R_3 = 2.2\Omega$ (std value)

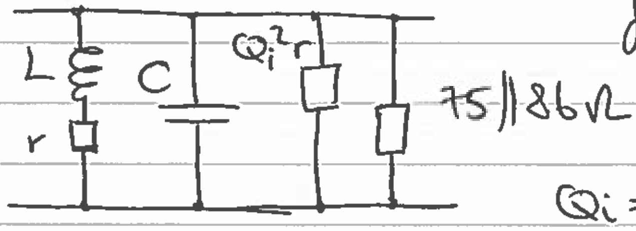
(112.5)

Choose $R2 = 1.5 \times 75\Omega \simeq 110\Omega$ and $V_{BE} = 0.65V$, so with
$V_E = 0.15 \times 2.2 = 0.33V$, set base at $\simeq 1V$ dc

2(b) contd $\therefore 1 = \frac{R_2}{R_1+R_2} \times 15$ (110) $\therefore$ $\underline{R_1 = 1500\,\Omega}$

[30%] check: $R_{in} = R_1 \| R_2 \| h_{fe}R_3 = 86\,\Omega$ (✓ ok for 75 Ω)

(c)



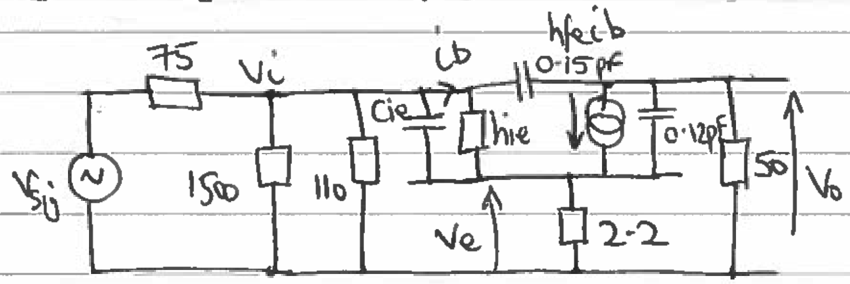
$f_{res} = 2\times10^9 = \frac{1}{2\pi\sqrt{LC}}$, $L = 0.47n$

$\therefore \underline{C = 13.5\,pF}$

$Q_i = \frac{\omega L}{r} = 59.1 \therefore Q_i^2 r = 349\,\Omega$

[20%] $\therefore Q_{tot.} = \frac{349\|75\|86}{\omega L} = \frac{35.9}{5.9} = 6.1 = \frac{2\times10^9}{\Delta f} \therefore \underline{\Delta f = 328\ MHz}$

(d) Small signal circuit model:



300 0.17

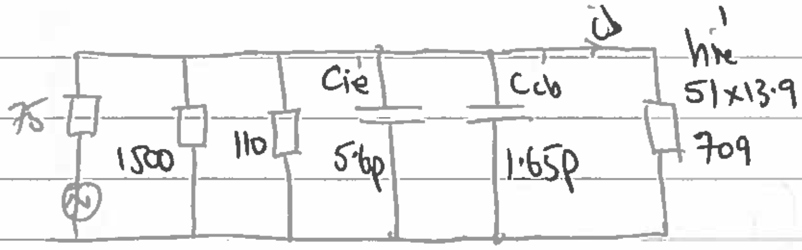
$h_{ie} = h_{fe} \times r_e$
$= 51\,\Omega$

$V_e = \frac{R_3}{R_3 + r_e} . V_i = \frac{2.2}{(2.2+0.17)} . V_i = 0.928\, V_i$

$\therefore \frac{1}{1-0.928} = 13.9$, $f_t = 12\times10^9 = \frac{1}{2\pi r_e C_{ie}}$

$\therefore C_{ie} = 78\,pF$ $\div 13.9$ when referred to gnd.

Loaded stage gain $= \times 10$ $\therefore$ input SSM becomes:



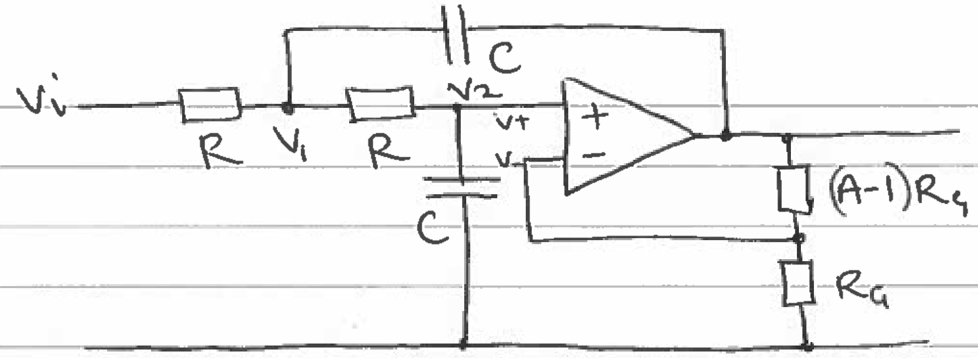
where $C_{cb}' = 11 \times 0.15$ (GAIN+1)
$= 1.65\,pF$
from Miller effect

Hence f_{-3dB} is given by: $\frac{1}{2\pi R'C'}$ where $R' = 75\|1500\|110\|709\,\Omega$
$= 40.8\,\Omega$

[30%] $\therefore f_{-3dB} = 538\ MHz$ $C' = 5.6 + 1.65\,pF$

$\therefore$ it is not fast enough

3(a)



$$V_o = AV_2 \quad \text{and} \quad V_2 = \frac{V_1 \frac{1}{j\omega C}}{R + 1/j\omega C} = \frac{V_1}{1 + j\omega CR} \qquad *$$

Sum currents at V_1 node:

$$\frac{V_i - V_1}{R} = \frac{V_1 - V_o}{1/j\omega C} + \frac{V_1 - V_2}{R} \Rightarrow V_i = V_1(2 + j\omega CR) - V_o j\omega CR - V_2$$

Subst. for V_1 and V_2 from $*$ in above:

$$V_i = \left(\frac{1 + j\omega CR}{A}\right) V_o (2 + j\omega CR) - V_o j\omega CR - V_o/A$$

$$\therefore V_i = \frac{V_o}{A}\left[1 - (\omega CR)^2 + j\omega CR(3 - A)\right]$$

Hence, $\left|\frac{V_o}{V_i}\right| = A\left[(1 - (\omega CR)^2)^2 + (\omega CR)^2 (3 - A)^2\right]^{-1/2}$ for magnitude

$$= A\left[1 + (\omega CR)^2\left[(3 - A)^2 - 2\right] + (\omega CR)^4\right]^{-1/2}$$

with $\omega = 2\pi f$ and $f_c = \frac{1}{2\pi CR}$, then we can obtain a Butterworth response when $(3 - A)^2 - 2 = 0 \quad \therefore A = 1.586$
(ie: no $(f/f_c)^2$ terms)

[40%] For a high pass response, the input part R's and C's should be swapped over.

(b) Choose a Butterworth response with $f_c = 12$ MHz, so that it passes the fundamental freq. at $16/2 = 8$ MHz but is in cut-off for higher harmonics. Butterworth gives ~10% overshoot on transients, ok.

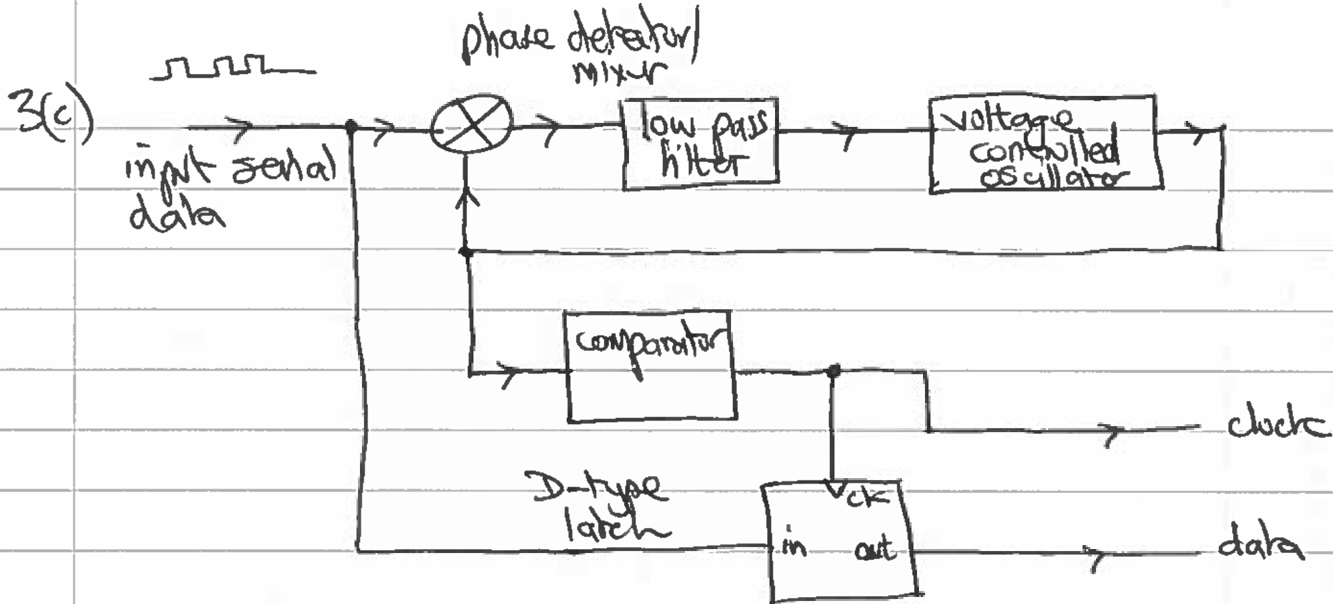
Select $R = 100\,\Omega$ (all) and $R_G = 1\,k\Omega$ then $(A-1)R_G = 152\,\Omega$ — ①
and $1.24\,k\Omega$ — ②

with 2 cascaded stages of the circuit in (a) above

[20%] C (all) $= \dfrac{1}{2\pi f_n R f_c} = \dfrac{1}{2\pi \times 100 \times 12 \times 10^6} = 133\,pF$

as $f_n = 1$

3(c)



A comparator is included if the VCO has an analogue output - to synchronize the clock and data streams of bits a D-type latch is included.

The phase detector compares the incoming bitstream with a regular waveform from the VCO. The resolved component from the bitstream fundamental mixes with the VCO output to give a dc component - keeping the frequencies aligned and the 'free-wheeling' inertial effect of the low pass filter allows 'missing' clock bits from the datastream to be tolerated whilst still keeping the loop in lock.

[25%]

(d) 16 Mbit/s $\Rightarrow$ 8MHz fundamental, but filtered to 12MHz so we shall work with this frequency

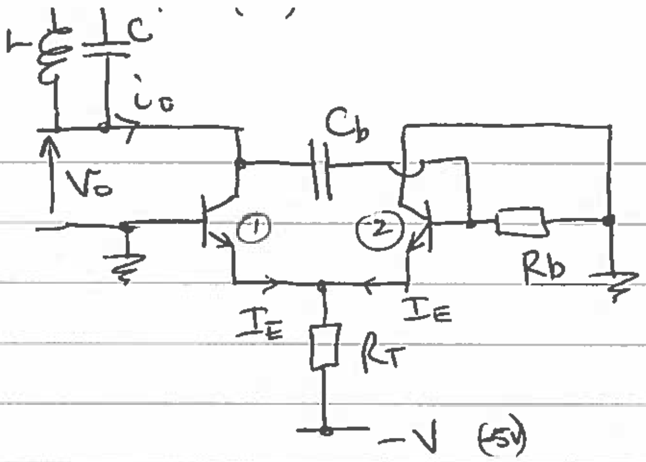
$$\text{skin depth, } \delta = \sqrt{\frac{2}{\omega\mu\sigma}} = \sqrt{\frac{2\rho}{\omega\mu}} = \sqrt{\frac{\rho}{\pi f\mu}}$$

For Cu; $\rho = 1.2\times10^{-8}\ \Omega m$, $\mu = 4\pi\times10^{-7}$ H/m, $\omega = 2\pi f$

$\therefore$ @ 12MHz, $\underline{\delta \doteq 16\ \mu m}$ $\therefore$ choose say $3\times\delta$ for minimum losses $\underline{t \doteq 50\ \mu m}$

[15%]

4(a)



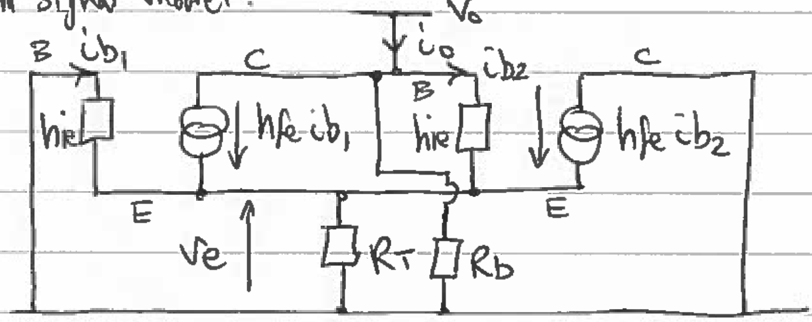
$$f_0 = \frac{1}{2\pi\sqrt{LC}}$$

$$1.5\times10^9 = \frac{1}{2\pi\sqrt{10^{-9}\cdot C\cdot 3}}$$

$\therefore$ $C = 3.75$ pF

$$I_E \approx \frac{(5-0.65)}{2R_T}$$

Small signal model:



$$i_{b_1} = \frac{-v_e}{h_{ie}} \qquad i_{b_2} = \frac{V_o - V_e}{h_{ie}} \qquad v_e \simeq R_T h_{fe}(i_{b_1}+i_{b_2})$$

$$* \quad i_o = h_{fe} i_{b_1} + i_{b_2} + V_o/R_b$$

subs. for i_{b_1} and i_{b_2} in above:

$$\therefore V_e = -2R_T h_{fe}\frac{V_e}{h_{ie}} + R_T h_{fe}\frac{V_o}{h_{ie}}$$

$$\therefore V_e\left(\overset{\text{small}}{\cancel{1}} + \frac{2R_T h_{fe}}{h_{ie}}\right) = \frac{R_T h_{fe}}{h_{ie}} V_o \qquad \therefore V_e \simeq \frac{V_o}{2}$$

$$\Rightarrow *: \quad i_o = -h_{fe}\frac{V_o}{2h_{ie}} + \overset{\text{small}}{\cancel{\frac{V_o}{2h_{ie}}}} + \frac{V_o}{R_b}$$

$$\therefore i_o = V_o\left(\frac{1}{R_b} - \frac{h_{fe}}{2h_{ie}}\right) \qquad \therefore Z_o = \frac{V_o}{i_o} = \left(\frac{1}{R_b} + \frac{1}{-2r_e}\right)^{-1}$$

So, output impedance $\frac{V_o}{i_o} = R_b \,||\, -2r_e$

The inductor parasitic resistance (parallel loss) $= \overset{2\pi f}{\omega} L \overset{30}{Q} \simeq 848\,\Omega$

Choose $R_b = 1\text{k}\Omega$ $\quad\therefore$ required $r_e < 229\,\Omega$ $\quad\therefore$ say $100\,\Omega \simeq \frac{0.025}{I_E}$

So $I_E \simeq 0.25$ mA $\quad\therefore R_T = 8700\,\Omega$ (say $8.2\text{k}\Omega$ std)

[35%]

4(b)(i) from Smith chart $\rho = 0.35\angle 40° = 1.50 + j0.80$ normalised

[20%] $\times 50\,\Omega$ for actual input impedance $= \underline{75 + j40\,\Omega}$

(ii) From Smith chart, length of transmission line $\equiv \frac{330°}{360°} \times \lambda/2$

$\lambda = \frac{c}{\sqrt{\varepsilon_r}} \cdot \frac{1}{f} = \frac{3\times10^8}{\sqrt{2}\,.\,2\times10^9} = 0.106\,m$

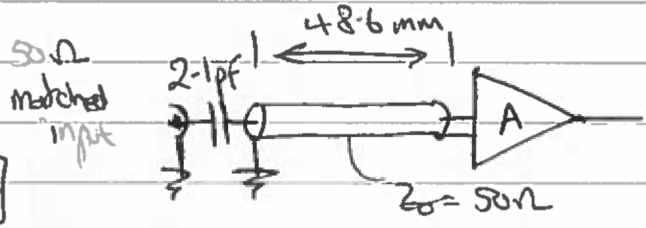
$\Rightarrow 0.458\lambda = \underline{48.6\,mm}$

Impedance 'P' to unity circle 'A' gives

$1 + j0.75$ $\therefore$ requires $-j0.75$ to cancel Im part too

$-j0.75 \times 50 = -j37.5\,\Omega = \frac{-j}{2\pi f C}$ with $f = 2\times10^9$ Hz

$\therefore \underline{C = 2.1\,pF}$



[20%]

(iii) $75 + j40\,\Omega$ at 2GHz $\equiv$ 75 Ω resistor in series with an inductor of 3.18 nH

$\underbrace{75}_{R} + j\underbrace{40}_{\omega L}$

at 1GHz $Z_{in} = 75 + j20\,\Omega \Rightarrow 1.5 + j0.40$ normalised

$\Rightarrow$ Point 'B' on Smith chart $\Rightarrow$ gives 0.25 reflection coeff @ 1GHz

[15%]

(iv) At 1GHz λ is doubled vs. 2GHz $\therefore$ 48.6 mm $\equiv 0.229\lambda$ which is 165° on the Smith chart.

$\therefore 30° - 165° = -135°$ to point 'C' $= 0.66 - j0.25$

at 1GHz, the series capacitor of 2.1 pF gives $-j75\,\Omega = -j1.5$ norm which moves the normalised impedance to $0.66 - j1.75$: point D

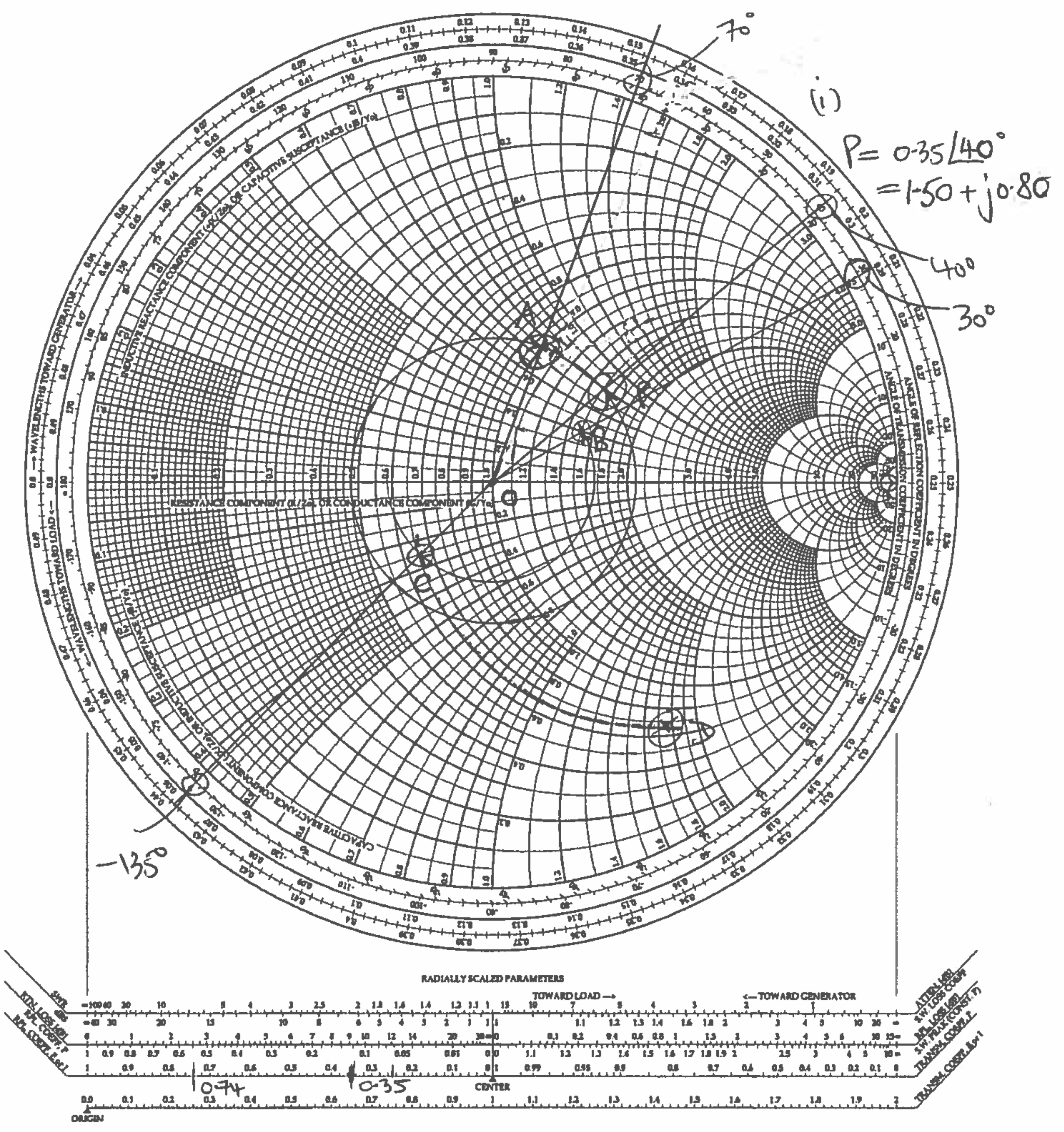
$\Rightarrow$ read off radius against scale $\Rightarrow \rho = 0.74$ reflection coeff. @ 1GHz

[10%]

Candidate No. CRIB

4/(b)

Smith Chart for Question 4 – to be detached and handed in with script.



Q1 Antennas & impedance matching

The first part on antenna terms was well answered and similarly so for the signal calculation, although some candidates confused the antenna areas and/or used the effective aperture incorrectly in deriving the beam angle. The final part on matching design was well attempted by most candidates.

Q2 Transistor RF amplifier

The basic amplifier circuit was remembered correctly by most candidates and many could select suitable values, although the gain required was often miscalculated; not taking into account the net gain including an extra coupling stage. Most attempts on the resonant circuit parts correctly found the capacitance value, but very few coped with the Q-factor calculation including the stage input and output impedances.

Q3 VCVS filter, phase locked loop (PLL)

Another well answered question in general. Most candidates could design the VCVS filter correctly although a number of candidates did not select the correct type (Butterworth). The PLL schematic was recalled correctly in most cases; with some realising that an XOR gate can be used as the phase detector, given the digital nature of the signals. The final part on skin depth was well attempted in general, but few realised that several skin depths are required to minimise conductor resistance.

Q4 Negative impedance circuit, Impedance matching, Smith chart

Most candidates correctly drew the negative impedance circuit, but not all managed the bookwork analysis - but some remembered the result. The Smith chart section was generally well attempted for the first part although the following sections presented more of a challenge and there were very few fully correct attempts.

P. A. Robertson (Principal Assessor)