

1.

a)

$$\text{Gain: } \frac{\text{Max Power Radiated per unit area}}{\text{Power per unit area for isotropic antenna}} = \frac{P_{\text{max}}/\text{unit area}}{P_{\text{in}}/4\pi r^2}$$

Effective Aperture: Power into matched load = Effective Aperture × Power density of Radio Wave

$$\text{Related to gain by: } G = \frac{4\pi A_e}{\lambda^2}$$

Radiation Resistance: Equivalent resistance to the losses due to radiation

$$\text{Defined by - Power radiated } P_r = \frac{1}{2} I R_r^2, \text{ where } I \text{ is the antenna current}$$

Polarisation: Orientation of the  $E$  field component of an antenna. Can be linear with a direction (normally horizontal or vertical) or circular. If circular ( $E_x$  and  $E_y$  90° out of phase) polarisation is effectively rotating.

b) i)

See smith chart

$$50\Omega \times (1.4 + 1.2j) = 70 + 60j$$

$$\Gamma = 0.22$$

b) ii)

(2 methods of solution, one uses only the relations above the other uses the Friis Equation

$$\text{which must be known } P_{rx} = P_{tx} G_{rx} G_{tx} \left( \frac{\lambda}{4\pi R} \right)^2$$

$$\lambda = \frac{c}{f} = 0.2\text{m}$$

$$G = \frac{4\pi A_e}{\lambda^2}$$

Convert antenna gains to linear units:

$$G = 10^{\frac{G(\text{dB})}{10}}$$

Now get the effective area of the rx antenna

$$A_{e_{rx}} = 0.00636 \text{ m}^2$$

Power into the unmatched transmitter antenna:

$$P = (1 - \Gamma) P_{in}$$

$$P = 156 \text{ W}$$

Accounting for the Tx gain, our antenna looks like an isotropic antenna with  $P_{\text{equiv}} = 1560\text{W}$

At Rx, require  $-100\text{dBm} = 1 \times 10^{-13} \text{ W}$ , so required power density at receive antenna given by:

$$S_{rx} = 1 \times 10^{-13} / A_{e_{rx}} = 1.57 \times 10^{-11} \text{ W/m}^2$$

$$\frac{P_{equiv}}{S_{rx}} = 4\pi r^2$$

$$r = 2811 \text{ km}$$

ii)

$$P = 200\text{W}$$

$$r = 3183 \text{ km}$$

iii)

From smith chart:

$$0.1875 \lambda \text{ to } 0.164 \lambda$$

$$0.5 \lambda - (0.1875 \lambda - 0.164 \lambda)$$

$$= 0.4761 \lambda$$

$$1.5\text{Ghz}, \epsilon_r = 2 \rightarrow \lambda = 0.141\text{m}$$

$$\rightarrow 67.3\text{mm}$$

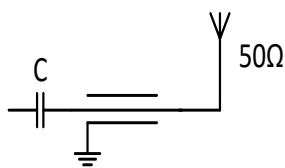
Capacitor:

Again from smith, reading off intersection to TL circle and unity circle

$$1.1j * 50\Omega = 55j \Omega$$

$$\frac{1}{j\omega C} = 55$$

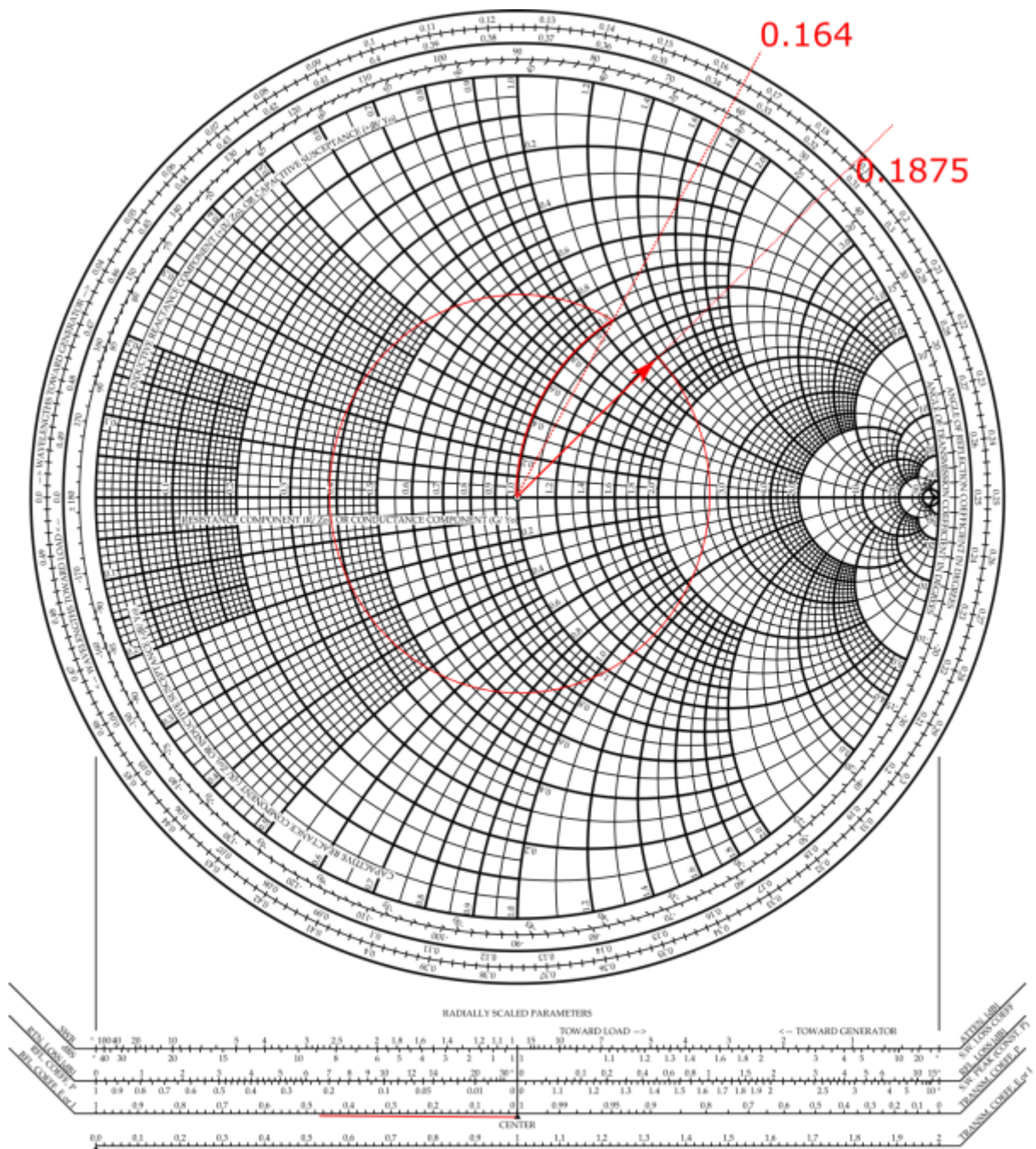
$$C = 1.9\text{fF}$$



iv) Large power will be reflected, results in a standing wave on the transmission line between the antenna and amplifier. Power must be dissipated in the amplifier. Risk to irreversible damage to the amplifier due to power dissipation or excessive voltage.

*Examiner's comment:*

*Antenna terms generally answered well, although some responses lacked precision. Polarisation caused the most trouble here. Smith chart was generally done correctly. The range calculation had a range of responses with different approaches to the problem yielding the correct answer, surprisingly frequent problems occurred converting gain in dB to a linear quantity even when in part (i) it was recognised that antenna gain is a power ratio. Recognition that the previously found mis-match power reflection factor had to be used separated the best candidates. Impedance match also generally well done (although a diagram of the circuit was often missing).*



2.

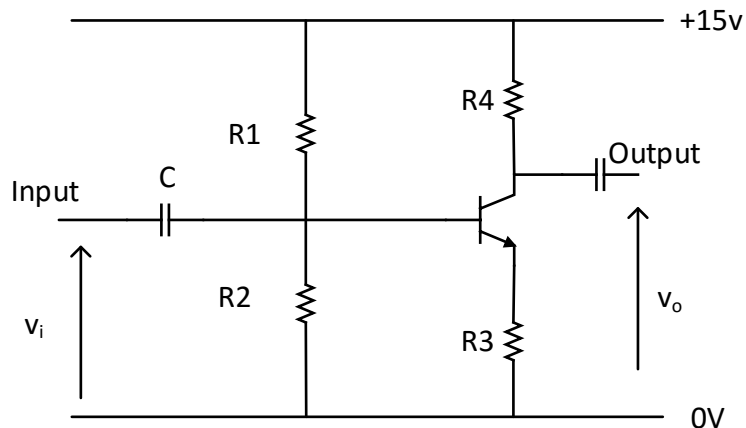
a) i)

Power gain = 10 dB (10×)

5V pk-pk into 75 ohms -&gt; 1.76Vrms, 0.0416W

Input = 41.6mW -&gt; 0.0056V rms

$$\text{Voltage Gain} = \frac{1}{0.5^2} \times \frac{1.76}{0.0056} = 12.66$$



Note: decoupling capacitors required on input and output

$$R_{out} = R_4 = 75\Omega$$

$$\text{Gain} = \frac{-R_4}{R_3 + r_e}$$

Assume  $r_e \ll R_3$

$$\rightarrow R_3 = 5.9\Omega$$

For maximum potential output swing:

$$V_C = \frac{V_S}{2} = 7.5V$$

$$V_E \sim \frac{V_S}{20} = 0.75V$$

$$V_B = V_E + 0.6$$

$$V_B = 1.25V$$

$R_1, R_2$  form potential divider to set  $V_B$

$$V_B = V_S \frac{R_2}{R_1 + R_2} \text{ (+ margin for loading)}$$

$$R_{in} = R_1 \parallel R_2 \parallel h_{fe}R_3 \text{ (} h_{fe}R_3 \text{ is big and can be ignored)}$$

$$75\Omega = R_1 \parallel R_2$$

$R_1, R_2 \sim 150\Omega$

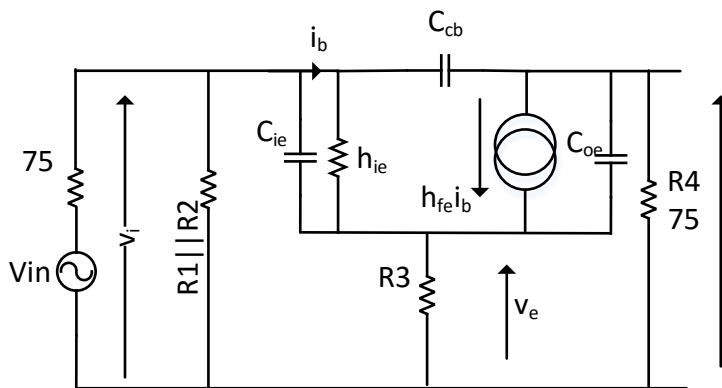
Try  $R_2 = 150\Omega$  gives  $R_1 = 1650\Omega$ , not great for  $R_{in}$  but probably ok

C's need to be 'big' e.g. 10nF

ii)

Assume that input side dominates response

Small signal model:



$I_c = \frac{V_E}{R_4} = 100\text{mA}$  use approximation  $r_e = \frac{25}{I_c} = 0.25\Omega$  (note this depends on assumptions in part (i))

Also  $h_{ie} = h_{fe} r_e$

$$f_t = \frac{1}{2\pi c_{ie} r_e}$$

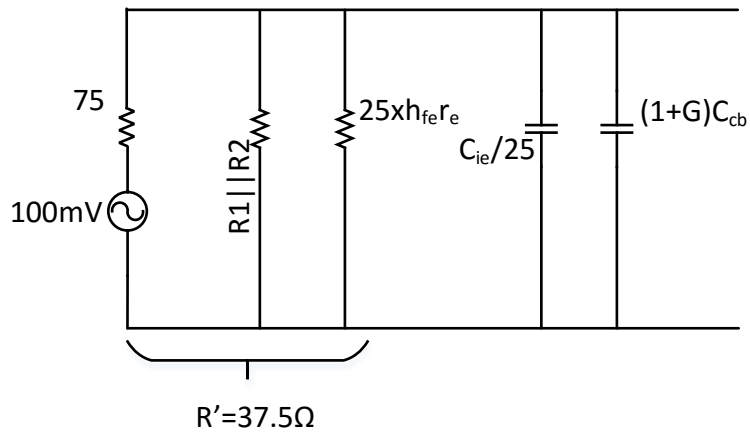
Gives:

$C_{ie} = 39.8 \text{ pF}$  for high performance part

$C_{ie} = 159 \text{ pF}$  for low performance part

To get 3dB freq need to simplify SSM by referring  $h_{ie}, c_{ie}$  to gnd

$\frac{v_e}{v_c} = \frac{R_3}{R_3 + r_e} = 0.96$  so to refer  $c_{ie}, h_{ie}$  to gnd need factor of  $1/(1-0.96) = 25$



$$f_{3dB} = \frac{1}{2\pi R' C'}$$

Low performance part:

$$C' = 8.5\text{pF} \rightarrow f_{3dB} = 500\text{ MHz}$$

High performance part:

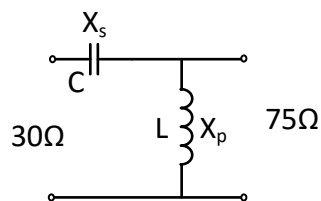
$$C' = 4.7\text{pF} \rightarrow f_{3dB} = 903\text{MHz}$$

Low performance part will not cover the full band of interest so the high performance part will be required

(Note, the results are quite dependant on the assumed values in part (i))

b)

i)



$$Q = \frac{R_{hi}}{X_p} = \frac{X_s}{R_{lo}} = \sqrt{\frac{R_{hi}}{R_{lo}}} - 1$$

$$Q = 1.22$$

2 potential solutions

$$X_p = 61.47$$

$$X_s = 36.6$$

For series C, parallel L

$$C=6.7\text{pF}, L=15\text{nH}$$

For parallel C, series L

C=4pF, L=9nH

ii)

$$B = \frac{2f_0}{Q} = 1065 \text{ MHz}$$

Or 3dB points at 117MHz, 1182MHz

Smaller bandwidth by increasing the impedance ratio by changing to much higher or lower impedance than back again. (simply more stages might increase bandwidth)

*Examiner's Comments:*

*Amplifier circuit was generally well done, although the gain was often mis-calculated (failing to account for coupling, or mistaking power gain for voltage gain). The transition frequency part required referring to ground, which caused some problems. The matching circuit was generally well answered, but again some answers arrived at correct component values but failed to explicitly show clearly the circuit. Bandwidth estimation – only a few remembered that there is a factor of 2, answers to the final part were often vague (e.g. filter, or use multiple impedance match stages).*

3.

a)

Chebyshev – fastest roll off out of band in the frequency domain and some unwanted pass band ripple, but at the expense of overshoot in the time domain step response.

Bessel – slower roll off out of band in the frequency domain, but best response in the time domain (no overshoot, maximally flat phase response)

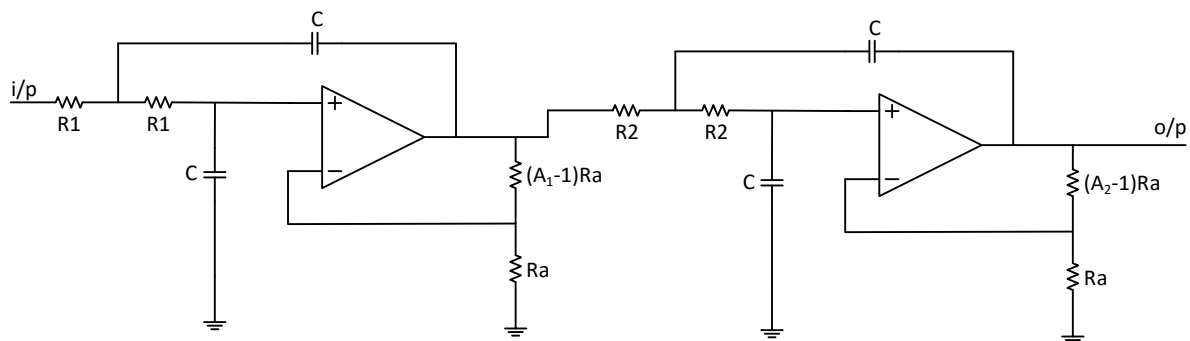
Butterworth – compromise in terms of time domain and frequency domain responses.

b)

Best frequency roll off required so select Chebyshev

Bandpass by cascade of 2pole lowpass and high pass sections

Lowpass (200kHz cut off)



Pole 1:

$$f_n = 0.597 A = 1.582$$

$$f_c = 200\text{kHz} = \frac{1}{2\pi RCf_n}$$

$$R_1 = 1.33\text{k}\Omega$$

Ra can be anything (but R1 value reduces number of required different values – used throughout this solution), 1k also a popular choice)

With Ra = 1.33kΩ

$$(A_1-1)Ra = 775\Omega$$

Pole 2:

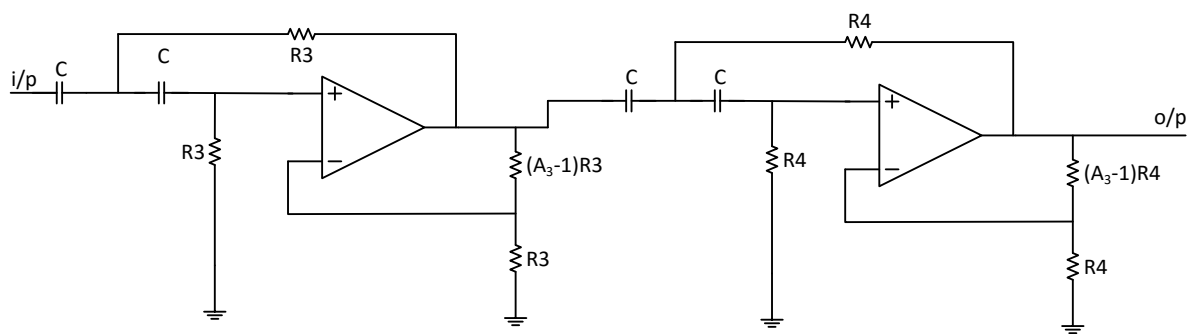
$$f_n = 1.031 A = 2.66$$

$$R_1 = 772\Omega$$

$$(A_2-1)Ra = 1.1281\text{k}\Omega$$

High pass (10kHz cut off):





Pole 3:

$$1/f_n = 1.675 \mu\text{s} = 1.582$$

$$f_c = 10\text{kHz} = \frac{f_n}{2\pi RC}$$

$$R_3 = 9.5\text{k}\Omega$$

$$(A_3-1)R_a = 5.52\text{k}\Omega$$

Pole 4:

$$R_4 = 16.4\text{k}\Omega$$

$$(A_4-1)R_a = 27.3\text{k}\Omega$$

c)i)

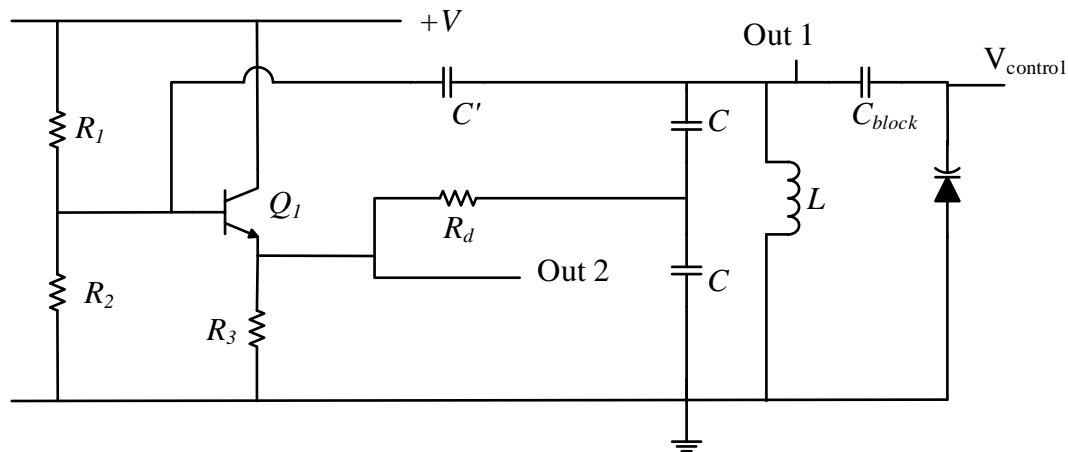
$R_1$ ,  $R_2$ ,  $R_3$  and transistor form an emitter follower circuit with a gain of 1 and positive feedback to the tank circuit.

$R_d$  acts as a 'gain control component'

Blocking C prevents  $R_1$ ,  $R_2$  applying a DC bias to the tank circuit

C, L resonant tank – resonates at desired frequency and provides an effective gain in ideal case of 2.

Since the tank has gain of 2, and emitter follower gain of 1, total gain around circuit can be  $> 1$  even when losses are present  $\rightarrow$  oscillation.



Note 'extra' blocking cap required for operation with varactor.

c)ii)

1.5MHz - 2MHz tank circuit

$f_{mid} = 1.75\text{MHz}$

$$L = \frac{1}{(2\pi f_{mid})^2 C_{mid}}$$

$$C_{ratio} = \left(\frac{f_{max}}{f_{min}}\right)^2 = 1.78$$

Using full tuning range of supplied varactor:

$$C_{max} = 300\text{pF} + \frac{C}{2}, C_{min} = 10\text{pF} + \frac{C}{2}$$

So  $C = 723\text{pF}$  (720pF std val)

$$C_{mid} = 516.2\text{pF}$$

$$L = 16\ \mu\text{H}$$

$C_{block}, C'$  need to be 'big' e.g. 10 n

Now bias transistor:

$$V_E = V_s/2 \text{ to get max swing (= 5V)}$$

$$\rightarrow V_B = 5.6\text{V}$$

$$\frac{V_B}{10} = \frac{R_2}{R_1 + R_2}$$

Require a high Z in to avoid loading the tank circuit, so pick  $R_1$  fairly big (e.g. 1k)

$$R_2 = 12.7\text{k}$$

Note that the varactor will reduce the effective step up of the tank circuit so our gain will be a bit less than 2.

Assume 2:1 for now

$$\text{Referred load} = \frac{15\text{k}}{2^2} = 3.75\text{k}$$

Overall loop gain > 1 ->  $R_d < 3.75\text{k}$  e.g. 2k to allow for reduced setup up.

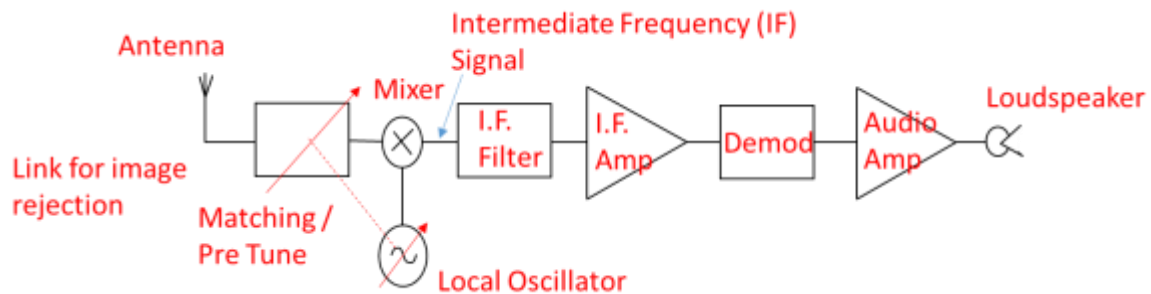
R3, not overly critical. 1.5-2 x load at resonance e.g. 3.75k

*Examiner's Comment:*

*Most correctly designed the VCVS part (b). In the voltage controlled collpitts (c), a blocking capacitor was frequently missing or misplaced. Explanations of the operation were varied in the level of detail with many missing the need for a loop gain > 1. Range of solutions for part (ii).*

4

a)



(optional RF amplifier stage)

Antenna – converts EM wave to electric current

LC tank – initial tuning can provide some frequency selection and image rejection, also voltage gain from Q factor

RF amplifier – boost signal level before mixer

LO – must be tunable and track LC tank if tuned.  $RF_{\text{tuned}} = LO - IF$ 

Mixer – generates sum and difference frequencies of the signals applied to its ports

IF-amp – boost signal after mixing to improve SNR.

IF filter – selects the difference frequency which is constant due to tracking of LO. Fixed frequency allows good quality filter to be used.

Demodulator – recover original signal.

Advantages – IF filter is fixed frequency – allows a very high Q filter to be used giving good discrimination between stations compared to crystal set style radio where the filter is tuned. Also keeps filter Q constant with tuning.

(Gain distributed through system also helps with SNR)

b)

$$RF = LO - IF$$

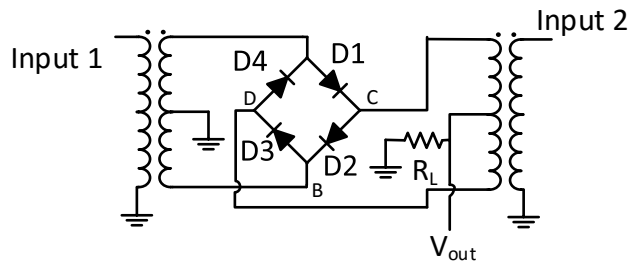
$$RF = 89.545\text{MHz}, \text{ Image at } 90.455\text{MHz}$$

IF can be suppressed if the front end filter won't significantly pass the image (requires  $Q \sim 200$ )

Design can be modified to increase the IF freq to relax front end filter Q.

c)

Double balanced Diode Ring



IN1: LO

IN2: RF

Input 1 positive half cycle, A becomes + with respect to B

D1, D2 conduct, D3, D4 block -> C effectively at GND

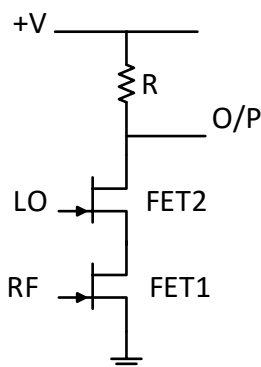
Vout becomes - IN2/2

Input 1 negative half cycle, A negative with respect to B

D1, D2 block, D3, D4 conduct -> D effectively at GND

Vout becomes + IN2/2

FET based mixer:



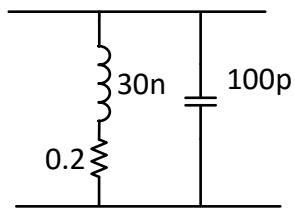
FET1 modulates drain current of both FET1 and FET2

FET2 biased as voltage variable resistance so R changes with LO

$V=IR$  so we have multiplication of RF and LO

Double balanced mixer, high cost due to the transformers, but very good isolation of the RF and LO from IF output. FET mixer – low cost and possibly integrated, but IF will contain a lot of the LO output. For a superhet LO leakage to the IF isn't overly important as it will be rejected by the IF filter.

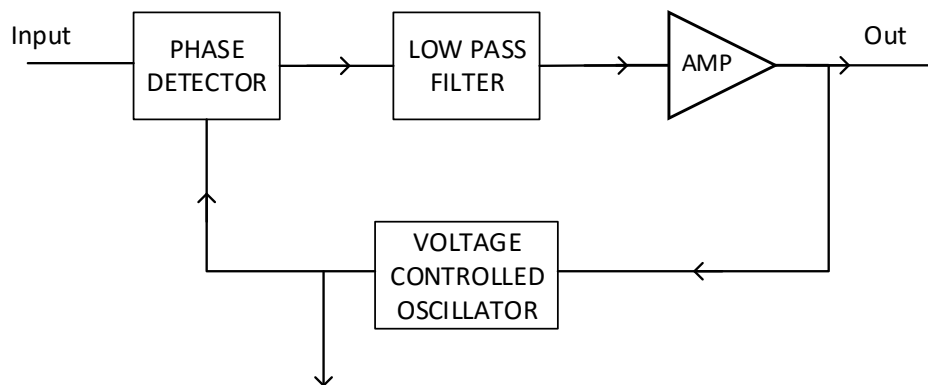
d)



$$\omega = \frac{1}{\sqrt{LC}} = 577 \times 10^6$$

$$Q = \frac{\omega L}{r} = 86.6$$

e)



PLL sets the VCO to be the long term average of the frequency of the input signal.

Hence the control voltage will be proportional to the phase (frequency) of input signal – frequency of VCO

Leads to demodulation of the output provided low pass filter is able to track changes in the demodulated signal.

*Examiners Comments:*

*Most drew a correct superhet and could explain basics, relatively few demonstrated why a fixed IF is important. In part (b) a common misunderstanding was between the image frequency and the sum and difference components yielded by mixing. Answers to the mixers (c) were varied with many failing to draw a balanced diode ring correctly. Parts (d) and (e) were well answered by those that attempted them*