3B1 2019

1(a) Radiation resistance, $R_r$ : power radiated $P_r = \frac{1}{2} I^2 R_r$

Gain, G : $\frac{\text{max. power density radiated}}{\text{power density from isotropic antenna}} = G$

Radiation efficiency, $e = \frac{P_r}{P_{in}}$ where $P_{in} = \frac{1}{2} I^2 (R_r + R_{ohmic})$

Effective Aperture, $A_e$ : power delivered into a matched load by antenna $= A_e \times$ power density

$G = 4\pi A_e / \lambda^2$

(b) Power density @ Earth $= \frac{\overset{P_r}{75} \cdot \overset{\text{Gain}}{1.5}}{4\pi \underbrace{(145 \times 10^9)^2}_{\text{Range}}}$

$= 4.26 \times 10^{-22}$ W/m²

(c)(i) $P_{rec} = \frac{\pi 35^2}{4} \cdot 4.26 \times 10^{-22} = 4.10 \times 10^{-19}$ W

$= V^2/R$ with $R = 150\Omega$

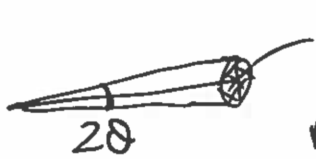
$\therefore V = (150 \times 4.10 \times 10^{-19})^{1/2} = 7.84$ nV rms

22.2 nV pk-pk

(ii) $G = 4\pi A_e / \lambda^2$ where $A_e = 962$ m²

$\lambda = \frac{c}{f} = \frac{3 \times 10^8}{11 \times 10^9} = 0.0273$m

$\therefore G = 16.2 \times 10^6$




$A_b = \pi(R\theta)^2$

$A_t = 4\pi R^2$

Estimate beam angle:

$G = \frac{4\cancel{\pi}\cancel{R^2}}{\cancel{\pi}\cancel{R^2}\theta^2} = \frac{4}{\theta^2} = 16.2 \times 10^6$ $\therefore \theta = 4.97 \times 10^{-4}$ rad.

$= 0.0285°$

or full beam angle $0.057°$

1(d) Lander antenna length $= \frac{\lambda}{4} = 6.83$ mm

skin depth $= \sqrt{\frac{2\rho}{\omega\mu}} = \left(\frac{2 \cdot 6.9\times10^{-7}}{2\pi \cdot 11\times10^{9} \cdot 4\pi\times10^{-7}}\right)^{1/2}$

$\delta$

$A = 2\pi r\delta$

$= 3.96\ \mu m$

Antenna resistance $R_{ohmic} = \frac{\rho l}{A} = \frac{6.9\times10^{-7} \cdot 6.83\times10^{-3}}{2\pi \cdot 0.25\times10^{-3} \cdot 3.96\times10^{-6}}$

$= 0.76\ \Omega$

$R_r = 30\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$ for cosine dist. $(74\Omega)$

OR $20\pi^2\left(\frac{\Delta z}{\lambda}\right)^2$ for linear dist. $(50\Omega)$

$\therefore$ Taking cosine distribution for current $e \simeq 99\%$

(e) For lander antenna $A_e = \frac{G\lambda^2}{4\pi} = \frac{1.5 \cdot 0.0273^2}{4\pi}$

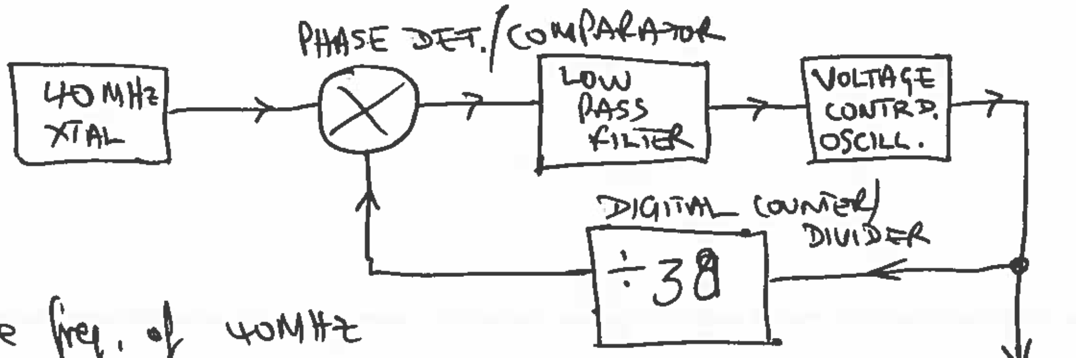
$A_e = 8.90\times10^{-5}\ m^2$

Power density @ lander $= \frac{P_r \cdot G}{4\pi R^2} = \frac{1000 \cdot 16.2\times10^{6}}{4\pi(145\times10^{9})^2} = 6.13\times10^{-14}$ W/m²

$\therefore P_{rec} = 8.9\times10^{-5} \cdot 6.13\times10^{-14} = \frac{V_s^2}{75}$

$\therefore V_s = 20.2\ nV_{rms}$ $(57.2\ nV\ pk\text{-}pk)$

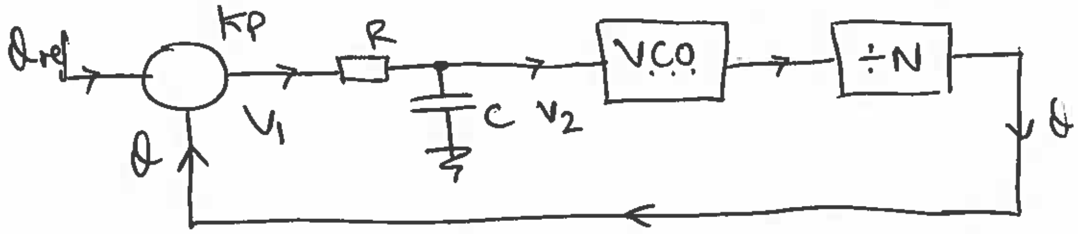
2(a)(i)




Reference freq. of 40MHz is compared to $\frac{1.56 \times 10^3}{38}$ MHz $= 40$MHz

VCO scaled output in phase detector.
The low-pass filter smooths the PD output to provide input to VCO. Before phase lock, the beat freq. puts side-bands on the VCO spectrum – one is at same freq. as reference and mixes in PD to provide DC offset to push the VCO to the correct frequency. Once locked the divided VCO freq. matches the reference.

(ii) (Note: if using XOR gate, best to ÷19 then ÷2 to get 50:50 square wave for phase comparison)




① — $V_1 = K_p(\theta_{ref} - \theta)$ $\qquad$ $K_p = 3.3$V for $180^\circ = \pi$ rad.
$\therefore K_p = 1.05$ V/rad.

with $\theta \equiv e^{j(\omega t + \theta)}$, $\frac{d\theta}{dt} = \dot{\theta} = j\omega\theta$, $\frac{d^2\theta}{dt^2} = \ddot{\theta} = -\omega^2\theta$

$V_2 = \frac{V_1}{1 + j\omega CR}$ $\quad \therefore V_1 = V_2 + j\omega CR V_2$ — ②

① & ②:

$\therefore \cancel{K_p}(\theta_{ref} - \theta) = \frac{V_2}{K_p} + j\omega CR \frac{V_2}{K_p}$ $\quad$ with $\frac{K_o V_2}{N} = \frac{d\theta}{dt} = j\omega\theta$

$\therefore \theta_{ref} = \frac{j\omega N}{K_p K_o}\theta + \frac{j\omega CR}{K_p}\frac{j\omega N}{K_o}\theta + \theta$

2(a)(ii) contd. $\theta_q = \dot{\theta}\frac{N}{K_p K_o} + \theta + \ddot{\theta}\frac{CRN}{K_p K_o}$

See page 6 Mechuwes Data Book :

$\frac{CRN}{K_o K_p} = \frac{1}{\omega_n^2}$ , $\frac{N}{K_o K_p} = \frac{2c}{\omega_n}$ with $c=1$ for critical damping

$K_p = 1.05$ V/rad , $N = 38$ , $K_o = 2\pi \times 10^9$ rad/s/V

$\therefore \omega_n = \frac{2 K_o K_p}{N} = 347 \times 10^6$ rad/s

$\therefore CR = 1.44 \times 10^{-9}$ s $\quad \therefore \underline{R = 100\Omega, \; C = 14.4\text{ pF}}$

2(b)(i)

$C$/unit length $\simeq \frac{2(w+2t)\varepsilon_o\varepsilon_r}{t}$ (fringing approx.)

for $C \simeq \frac{A\varepsilon}{t}$

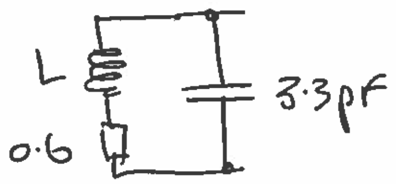
velocity of propagation, $v = \frac{1}{\sqrt{\mu_o\varepsilon_o\varepsilon_r}} = \frac{1}{\sqrt{L}\sqrt{C}}$

characteristic imp., $Z = \frac{\sqrt{L}}{\sqrt{C}} = \frac{1}{vC} = \frac{\sqrt{\mu_o\varepsilon_o\varepsilon_r}\, t}{2(w+2t)\varepsilon_o\varepsilon_r}$

$\therefore 50 = \frac{\sqrt{4\pi \times 10^{-7}} \cdot 10^{-3} \cdot 0.6}{2(w + 1.2 \times 10^{-3})\sqrt{8.854 \times 10^{-12} \cdot 2.5}}$

$\therefore \frac{w}{10^{-3}} + 1.2 = \frac{71.48}{50} \Rightarrow \underline{w = 0.23\text{ mm}}$

(ii) Impedance mismatch will cause voltage reflection of magnitude $\frac{25}{75+50} = 0.2$ (= 4% power reflection) but more of a problem is the reflected wave, which can cause 'ringing' = edge spikes on transitions, distorting the signals.

3(a) 

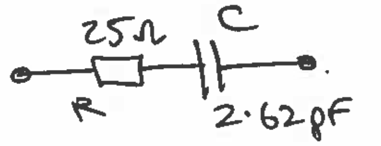

$f = 868 \times 10^6 = \dfrac{1}{2\pi\sqrt{LC}}$

$\therefore L = 10.2\ nH$

$Q = \dfrac{\omega L}{r} = \dfrac{55.6}{0.6} = 92.7 = \dfrac{868}{B.W.}$ $\therefore B.W. = 9.37\ MHz$

(b)(i) From Smith chart, normalised imp. = $0.5 - j1.4$ = 'A'

$\therefore$ de-normalise from $50\Omega$ $\Rightarrow Z = 25 - j70\ \Omega$




$\dfrac{1}{2\pi f C} = 70$ with $f = 868 \times 10^6$ Hz

$\therefore C = 2.62\ pF$

(ii) From impedance point 'A' $0.5 - j1.4$, move 180°
to admittance point 'B' $0.22 + j0.64$
Move 'B' → 'D' with parallel inductor: inductive
suceptance of $-j0.22$ such that 180° around
from 'D' is the impedance point 'C' $1 - j1.9$.
Then cancel $-j1.9$ with series inductor of $+j1.9$
to get to match point 'O' @ centre.

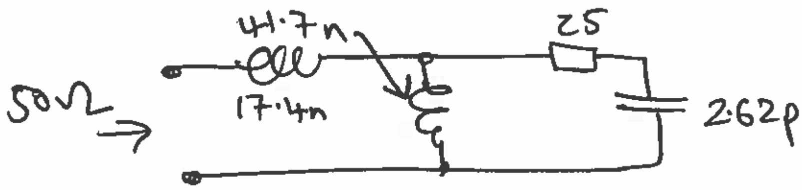
Inductive suscept. $-j0.22/50 = -j4.4 \times 10^{-3}$

$\Rightarrow$ impedance $= \dfrac{1}{-j4.4 \times 10^{-3}} = j227\ \Omega = j2\pi f L$

$\therefore$ @ $868 \times 10^6$ Hz, $L = 41.7\ nH$ in parallel

Then series impedance $= j1.9 \times 50 = j95 = j2\pi f L$

$\therefore L = 17.4\ nH$ in series



Version PAR/3

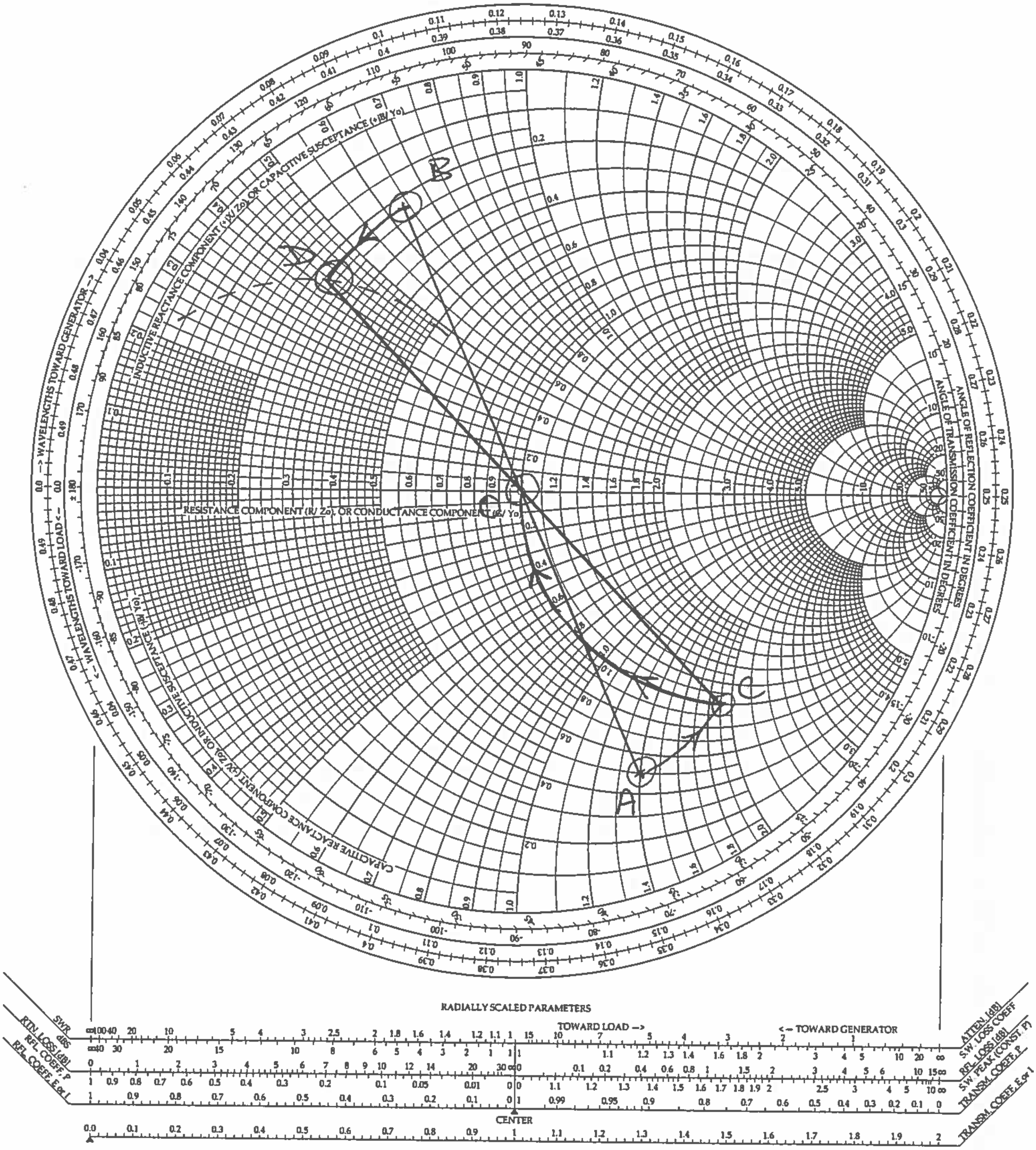
EGT2

ENGINEERING TRIPOS PART IIA

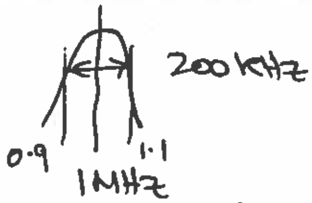
Monday 29th April 2019, Module 3B1, Question 3

Candidate No. CRIB

Smith Chart to be detached and handed in with script if required



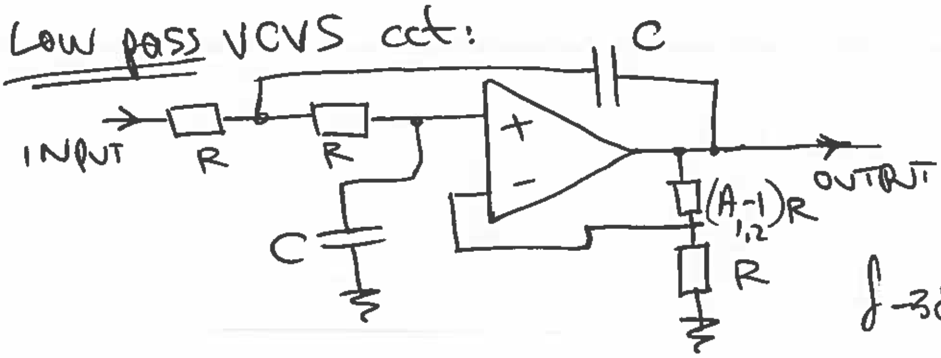
3(c) I.F. filter




∴ low pass to 1.1 MHz
high pass from 0.9 MHz

Use Chebyshev. for steep band edges

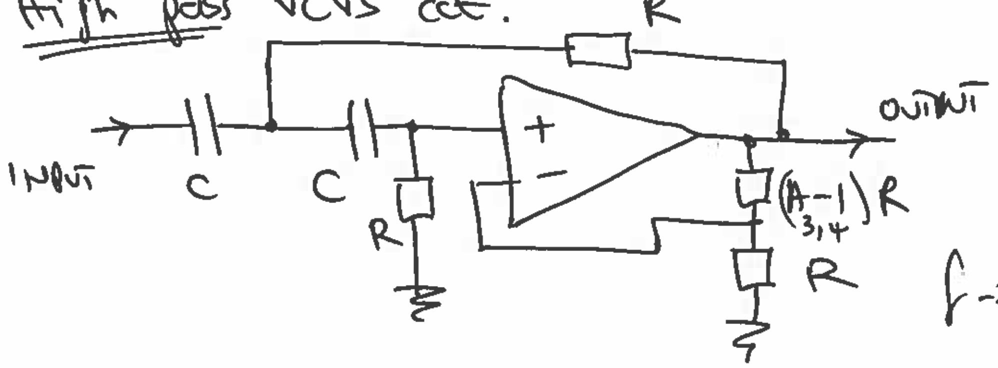
Low pass VCVS cct:




1,2 stages
x2 cascaded

$$f_{-3dB} = \frac{1}{2\pi f_n RC}$$

High pass VCVS cct.




3,4 stages
x2 casc.

$$f_{-3dB} = \frac{f_n}{2\pi RC}$$

Low pass stages 1 & 2

$$1.1 \times 10^6 = \frac{1}{2\pi . 0.597 . 10^3 . C_1} = \frac{1}{2\pi . 1.031 . 10^3 . C_2}$$

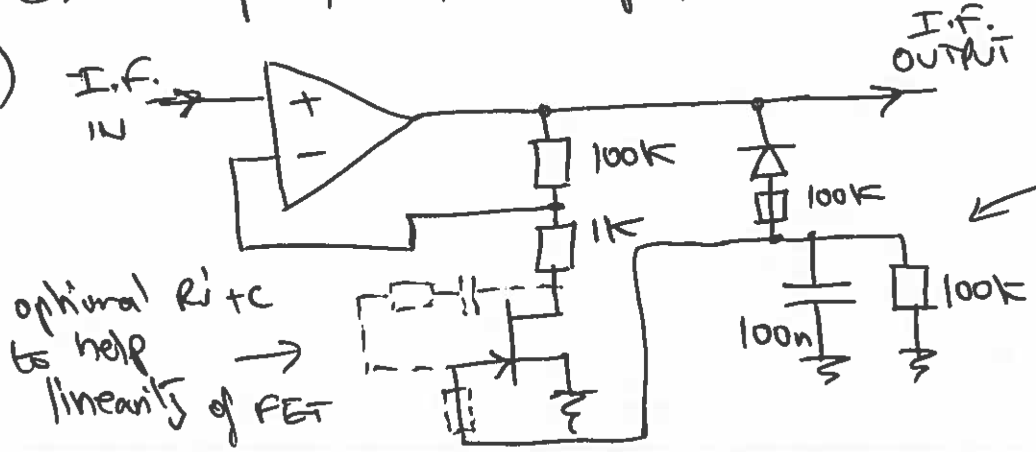
$C_1 = 242\,pF$, $C_2 = 140\,pF$, $R_1 = 582\,\Omega$, $R_2 = 1.66\,k\Omega$

High pass stages 3 & 4

$$0.9 \times 10^6 = \frac{0.597}{2\pi . 10^3 . C_3} = \frac{1.031}{2\pi . 10^3 . C_4}$$

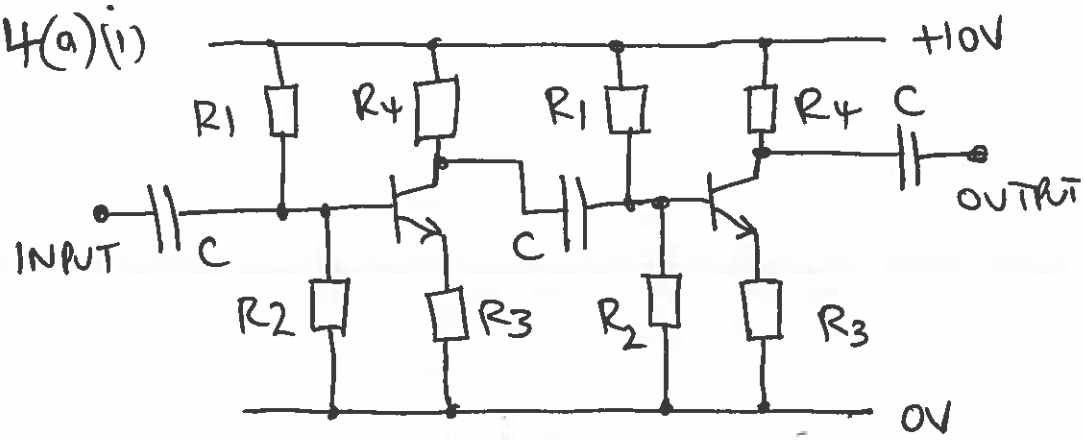
$C_3 = 106\,pF$, $C_4 = 182\,pF$, $R_3 = 582\,\Omega$, $R_4 = 1.66\,k\Omega$

(d)




time constant ~ 10ms

4(a)(i)




R1, R2 : base bias resistors, set base-emitter voltage at $\geq 0.6V$ and $V_c$ at $\simeq 1/2$ Supply

R3 : emitter feedback resistor, sets gain and -ve feedback for bias to set $V_c$ at $\simeq 1/2$ supply

R4 : output resistor    C : coupling (dc blocking) caps.

(ii) $R_4 = 50\Omega$ for matched impedance

then $I_c = 0.1A$ for $V_c$ @ +5V d.c. (1/2 supply)

$\therefore r_e = \frac{0.025}{0.1} = 0.25\Omega$

For 30 dB extra gain when loaded $30 = 20\log_{10}\left(\frac{V_o}{V_i}\right)$

$\therefore \frac{V_o}{V_i} = 31.6$ with 2 extra couplings to compensate

$\therefore$ open cct. gain total $= 31.6 \times 2 \times 2 = 127$

$\therefore$ each stage gain (linear) $= \sqrt{127} = 12$ say (a bit of spare)

$\therefore 12 = 50/R_3 + r_e \quad \therefore R_3 = 3.9\Omega$

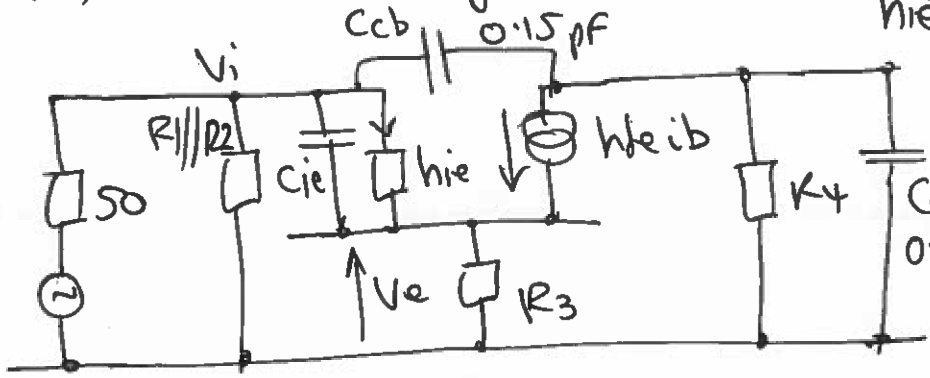
Choose $R_2$ at $68\Omega$ say, then $R_1 = 560\Omega$ to give $V_B \simeq 1.1V$ and $R_{in} = 68 \| 560 \| 250 \times 4.2 \simeq 57\Omega$ ok.

Choose C for low impedance $\therefore$ say 10nF $(0.02\Omega)$

(iii) $A = 12 \times 10^9 = \frac{1}{2\pi C_{ie} r_e}$ ← 0.25 $\therefore C_{ie} = 53pF$

$\frac{V_e}{V_i} = \frac{R_3}{R_3 + r_e} = 0.94 \quad \therefore$ base-emitter impedances ⇒ gnd are $\times \frac{1}{(1-0.94)}$
$= \times 16.6$

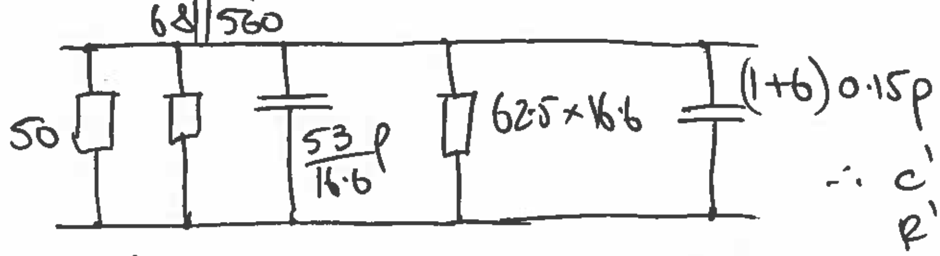
4(a)(iii) contd. small sig. model.




$h_{ie} = 250 \times 0.25\ (r_e)$
$= 62.5\ \Omega$

loaded gain
$\simeq 12/2$
$= 6$

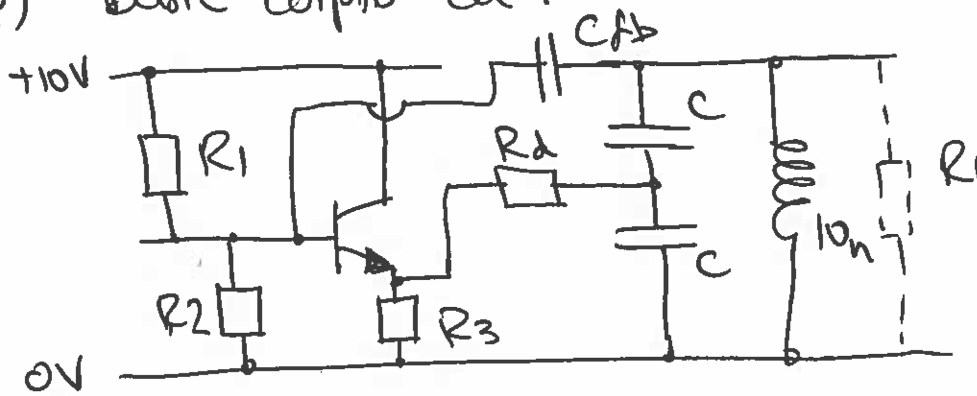
Looking @ input only with values referred to gnd :-




$\therefore\ c' = 4.25\,pF$
$R' = 26.7\,\Omega$

$\Rightarrow f_{-3dB} = \dfrac{1}{2\pi R'C'} = \underline{1.4\ GHz}$ ✓ o.k.

4(b) Basic Colpitts cct.




$R_{loss} = \omega LQ$
(1.9kΩ)

$R_1$ and $R_2$ set base @ ~5.6V and are ~ few kΩ each (to match loss resistor). $\therefore\ \underline{R_1 = 2k7},\ \underline{R_2 = 3k3}$

$f = \dfrac{1}{2\pi\sqrt{L\,C/2}} = 868\ MHz \ \therefore\ \underline{C = 6.72\,pF}$ (−0.1pF for Cie compensation *)

Resistance referred to C-C midpoint $\triangleq \dfrac{2k7\,||\,3k3\,||\,1k9}{4}$

$\approx 200\ \Omega$

$\therefore$ Choose $R_d < 200$ $\therefore$ say $\underline{150\ \Omega}$ with $\underline{R_3 = 220\ \Omega}$

$\therefore r_e \approx 11\ \Omega$, $C_{ie} = 12\,pF$

check $h_{fe} \times R_3 \gg$ few kΩ $= 250 \times 220 = 55k\Omega$ ✓

Cfb for small impedance $\Rightarrow \underline{1\,nF}$ and Cie referred $= \dfrac{12\,p}{221}$

* $\approx 0.05\,pF$ ✓ ok.

Examiner's comments

**Q1 Antennas**

A popular and straightforward question, well-answered by most candidates. Most candidates knew the antenna term definitions and the antenna equation, and applied it correctly to calculate signal magnitudes. The beam angle was also estimated correctly in many cases.

**Q2 Phase Locked Loop & waveguide**

A less popular question; only attempted by about a third of the cohort. The basic PLL architecture and description was handled quite well although the detail of including a 'divide by 38' function to achieve the correct frequency was often omitted in the analysis. The stripline impedance calculation was generally well done.

**Q3 Impedance matching & filters**

This question was attempted by all candidates and quite well answered. There were several correct solutions available in the Smith Chart matching section, and the attempts seen covered all of them – although in some cases they started with the analytical matching equation and then illustrated the solution on the chart. The VCVS filter section was quite straightforward and well attempted in most cases.

**Q4 RF amplifier & oscillator**

A fairly popular question with good attempts on the whole. The 2-stage amplifier design was well answered, although the gain was often incorrect by a factor of 2 either way. The frequency response was also quite well attempted in many cases, although the unloaded gain was often considered rather than the loaded value. The oscillator design was also quite straightforward.