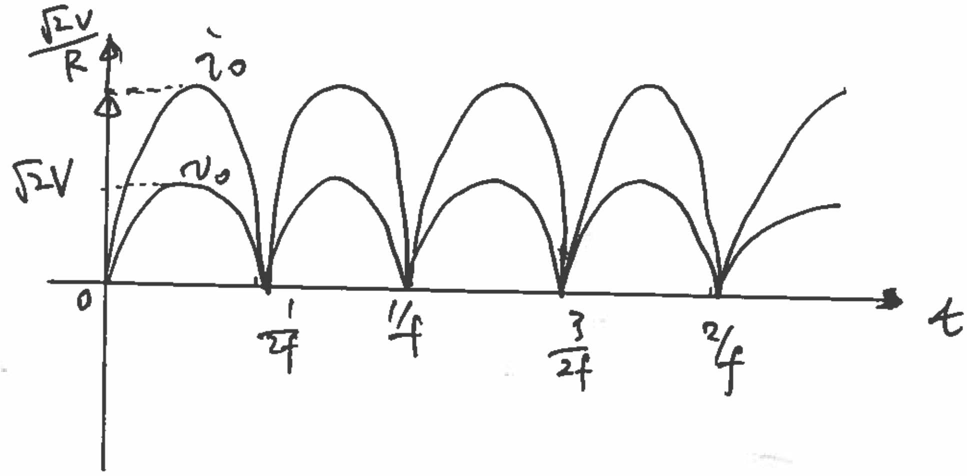
1.

(a)



(15%)

(b)

$$V_o = \frac{1}{\pi}\int_0^{\pi}\sqrt{2}V\sin 2\pi f t = \frac{2\sqrt{2}}{\pi}V$$

$$I_o = \frac{V_o}{R} = \frac{2\sqrt{2}V}{\pi R}$$

$$v_o(\omega t) = \frac{2\sqrt{2}}{\pi}V - \frac{4\sqrt{2}V}{\pi}\sum_{n=2,4,6\cdots}\left[\frac{1}{n^2-1}\cos n\omega t\right]$$

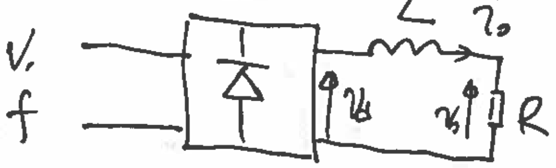
The largest harmonics is at $n=2$,

$$V_{o2}(\omega t) = \frac{2\sqrt{2}}{\pi}V - \frac{4\sqrt{2}V}{3\pi}\cos 2\omega t$$

$$V_{o2pk} = \frac{2\sqrt{2}}{\pi}V + \frac{4\sqrt{2}}{3\pi}V = \frac{10\sqrt{2}}{3\pi}V$$

(15%)

(C)



$$V_d(\omega t) = \frac{2\sqrt{2}}{\pi}V - \frac{4\sqrt{2}V}{\pi}\sum_{n=2,4,\dots}\left(\frac{1}{n^2-1}\cos n\omega t\right)$$

$$i_o(\omega t) = \frac{2\sqrt{2}V}{\pi R} - \frac{4\sqrt{2}V}{\pi}\sum_{n=2,4,\dots}\left(\frac{\cos(n\omega t-\theta_n)}{(n^2-1)\sqrt{R^2+(n\omega L)^2}}\right)$$

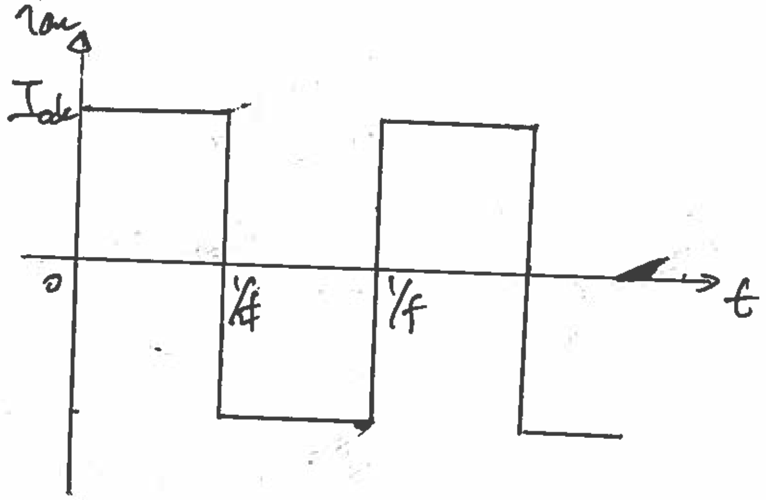
$\omega L >> R$,
$$i_o(\omega t) \simeq \frac{2\sqrt{2}V}{\pi R} - \frac{4\sqrt{2}V}{\pi}\sum_{n=2,4,\dots}\left(\frac{\cos(n\omega t-\theta_n)}{(n^2-1)n\omega L}\right)$$

$$\cancel{I_{orms} = \sqrt{I_o^2 + I_{2rms}^2 + I_{4rms}^2 + \cdots} = I}$$

$$I_{orms} = I_{odc} + \sqrt{\sum_{n=2,4,\dots} I_{nrms}^2}$$

$$= \frac{2\sqrt{2}V}{\pi R} + \sqrt{\sum_{n=2,4,\dots} \frac{16V^2}{\pi^2(n^2-1)^2 n^2 \omega^2 L^2}}$$ (20%)

(d)



Fourier series of i_{ac}:

$$i_{ac} = \sum_{n=1,3,5\dots}^{\infty} \frac{4}{n\pi} I_{odc} \sin(n\omega t)$$

$$I_{acrms} = \sqrt{\sum_{n=1,3,5\dots}^{\infty} \left(\frac{8}{n^2\pi^2} I_{odc}^2\right)} = I_{odc} = \frac{2\sqrt{2}V}{\pi R}$$

$$I_{ac1rms} = \frac{2\sqrt{2}}{\pi} I_{odc} = \frac{2\sqrt{2}}{\pi} \cdot \frac{2\sqrt{2}V}{\pi R} = \frac{8V}{\pi^2 R}$$

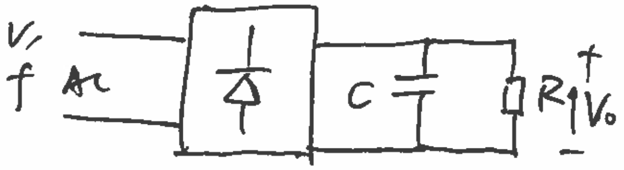
$$THD_i = \sqrt{\frac{I_{acrms}^2 - I_{ac1rms}^2}{I_{ac1rms}^2}} = 0.48, \quad THD_i = 48\%$$

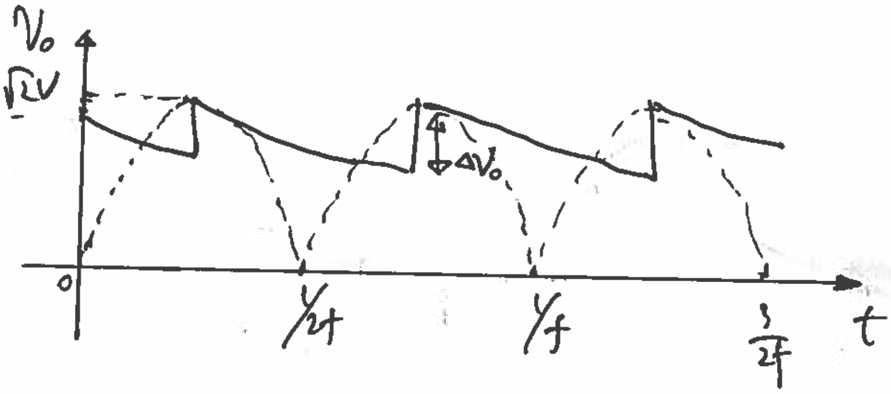
The source is ideal, and no displacement between current and voltage of 50Hz,

$$P.f = \frac{1}{\sqrt{1+THD_i^2}} = \frac{1}{\sqrt{1+(0.48)^2}} = 0.9.$$

(20%)

(e)





$$\Delta V_o = \sqrt{2}V - \sqrt{2}V\left(1 - \frac{1}{2fRC}\right) = \frac{\sqrt{2}V}{2fRC}$$

Assume

1° Linear discharge of capacitor C,
$RC >> \frac{\pi}{2\pi f} = \frac{1}{f}$

2° The discharge occurs over the whole half cycle, i.e. charging is effectively instanteous.

(2%)

(f)

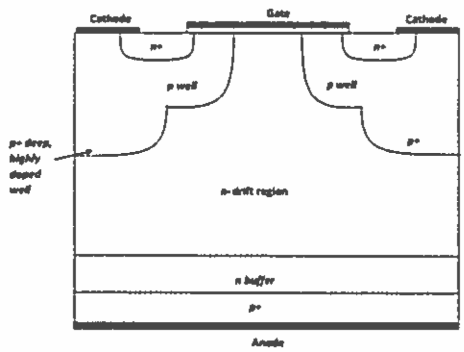
① The inductor smooth gives much increased power factor at the input because of less current distortion. (5%)

② The ripple voltage of the load is linearly proportional to the load R when it is smoothed by an inductor. The larger the load (i.e. smaller resistance R), the smaller of the ripple voltage.

However, when using capacitor smooth, the ripple voltage at the load is reverse propotional to the load. The larger the load, (i.e. smaller resistance R) the larger the ripple voltage.

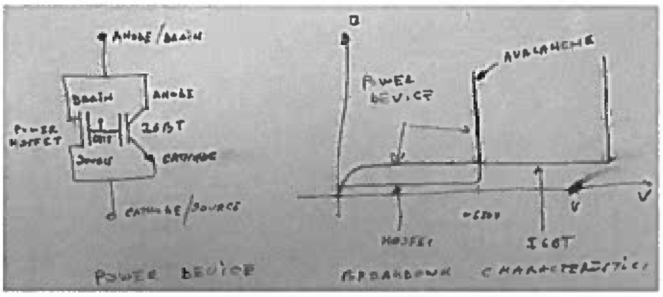
The ripple voltage causes losses. The higher the load current (i.e. larger the load), the capacitor smooth method will result in more losses. Not as good as inductor smooth. (5%)

2. (a)



The conductivity modulation refers to the enhancement of the conductivity of the drift region due to the presence of additional mobile carrier charge called plasma and containing holes and electrons in equilibrium. The additional charge is the result of (i) hole injection from the p+ anode, which acts as an emitter for an internal pnp transistor and as the anode for the internal PIN diode, and (ii) electron injection from the accumulation layer into the drift region, which acts as the cathode of the internal PIN diode. The conductivity modulation could result in an increase in the conductivity of the drift region by one to two order of magnitude. [20%]

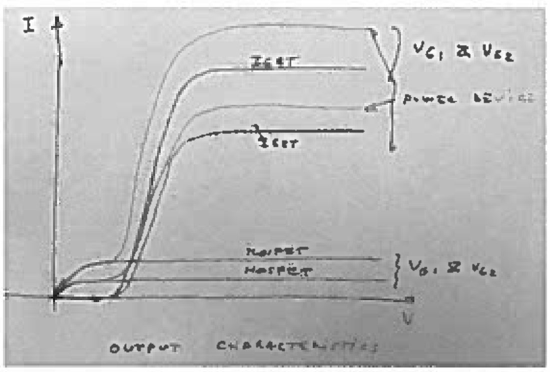
(b)



(i)

The devices are connected in parallel. Since the MOSFET has a lower voltage rating, the breakdown (which is ~ the rated voltage + a 10% margin) will be dictated by the MOSFET. The MOSFET device will be first to enter the avalanche regime. In other words the power device, combining the two devices will be limited by the breakdown of the MOSFET (see picture). In terms of the leakage, given that the two devices have the same surface area, the IGBT leakage will be significantly larger than the MOSFET leakage. This is because the leakage is amplified by the bipolar action of the pnp transistor that exists in the IGBT. Therefore the device will have mostly the leakage of the IGBT and the avalanche breakdown of the MOSFET [20%]

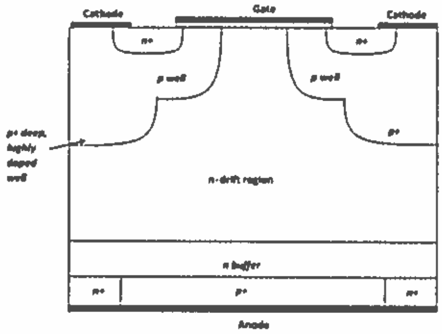
(ii)



The MOSFET output characteristics start from 0V while the IGBT output characteristics start from ~0.7V when the anode junction is forward-biased. The current densities in the IGBT are much higher due to conductivity modulation. The power device will have an on-state current equivalent to the sum of the MOSFET and IGBT currents. Initially, up to 0.7 V only the MOSFET will conduct, while above the 0.7 V, most of the current will flow through the IGBT [20%]

(iii) During the reverse conducting mode, only the MOSFET will conduct through its intrinsic PIN diode (also known as the anti-parallel diode). The internal PIN diode in the MOSFET is formed between the p well (as the anode of the PIN diode), the n- drift region (as the intrinsic region of the PIN diode) and the n+ drain (as the cathode of the PIN diode). No reverse conduction can occur in through the IGBT, as the open-base PNP transistor will block the current during the reverse mode operation. Note that there is no anti-parallel diode present in the IGBT [20%]

(iv)

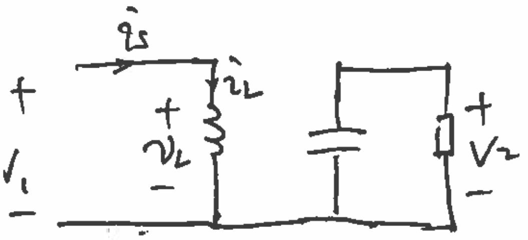


The structure of a combined IGBT/MOSFET integrated in the same chip will look as if n+ anode shorts are introduced (see picture).

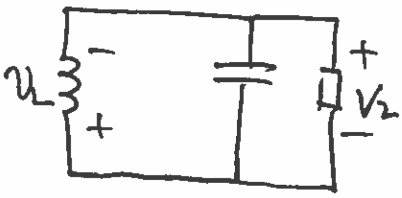
The structure is called a Reverse Conducting IGBT .From an area (and cost) point of view this structure offers a better solution compared to a device formed of discrete components. However, one has to pay attention to the snap-back issue present in the on-state output characteristics. In the on-state, the electron current will flow initially through the MOS channel, the n- drift region and eto the n+ shorts and only when the p+/n well anode junction becomes forward biased, the current will eventually flow through the IGBT structure (and conductivity modulation could be established as a result). This leads to a snap-back voltage which could result in instabilities during switching.
[20%]

3. a

i) Q ON:



Q OFF:



$$V_1 = L\frac{\Delta i_L}{kT} \quad \text{— ①}$$

$$-V_2 = L\frac{\Delta i_L}{(1-k)T} \quad \text{— ②}$$

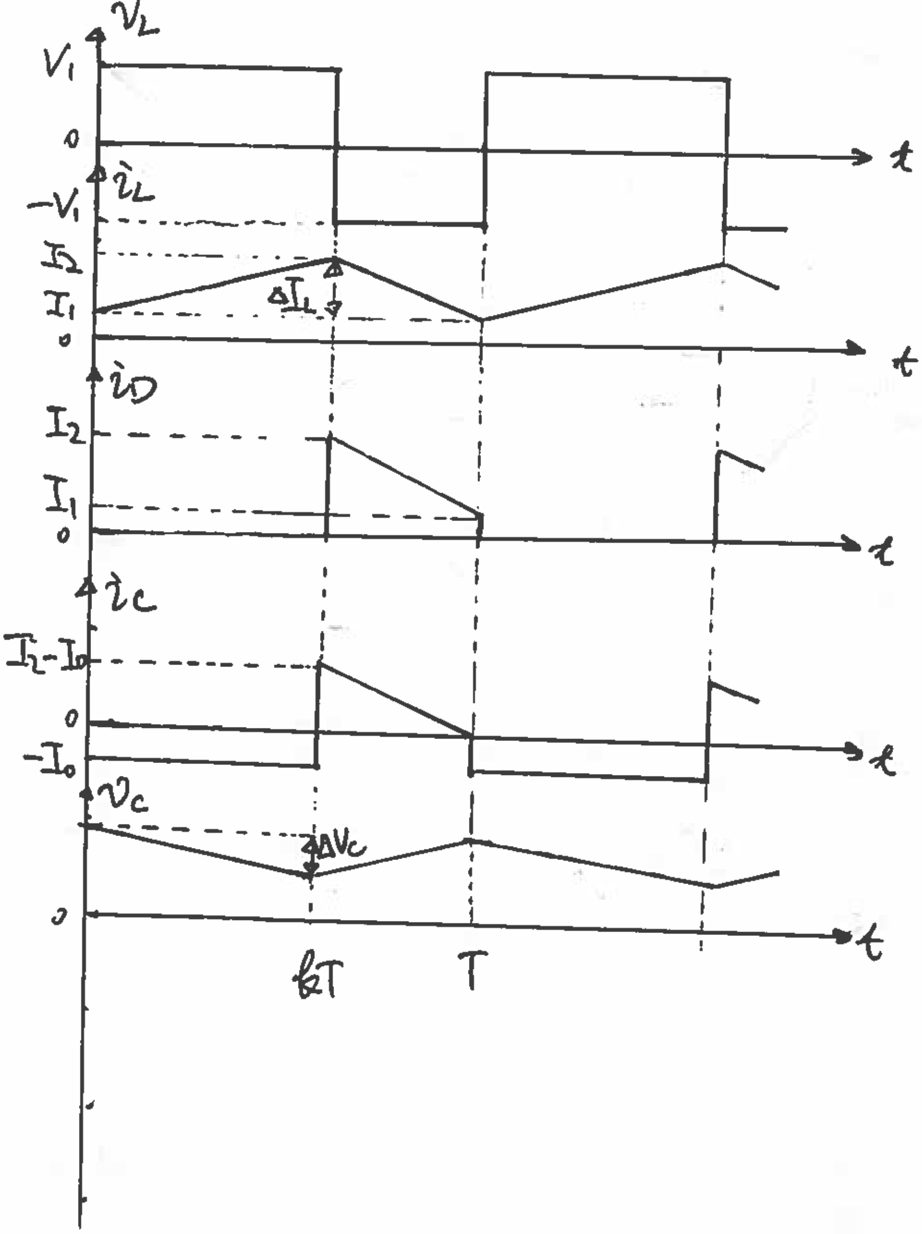
combining ① and ②.

$$V_2/V_1 = -\frac{k}{(1-k)}.$$

using the polarity of the circuit shown on the question,

$$\frac{V_2}{V_1} = \frac{k}{1-k}.$$

ii)



iii) The average current of L. I_L is the same to output current I_o. i.e. $I_L = I_o$.

The ripple current of L is twice of average current when boundary continuous. i.e. $\Delta I_L = 2I_L = 2I_o$

$$2I_o = \frac{2kV_1}{(1-k)R} \quad \text{—— ①}$$

During ON, $V_1 = L_{cri}\dfrac{2I_L}{kT}$ —— ②

combining ① and ②. by using $I_L = I_o$.

$$L_{cri} = \frac{(1-k)R\cdot T}{2}.$$

iv) When the switch is ON, the capacitor C supplies the load current for kT.

The average discharge current $I_c = I_o$

The peak-to-peak capacitor voltage is

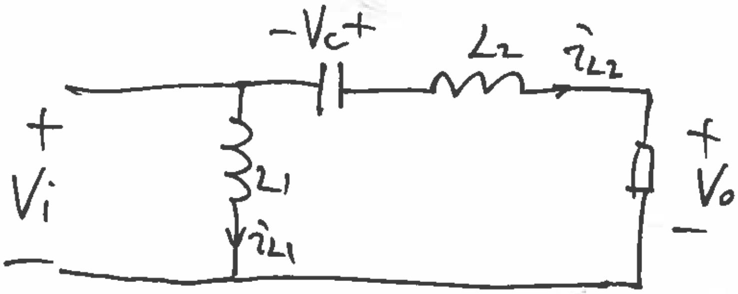
$$\Delta V_c = \frac{1}{C}\int_0^{kT} I_c\,dt = \frac{1}{C}\int_0^{kT} I_o\,dt = \frac{I_o kT}{C_{cri}} \quad \text{—— ①}$$

When boundary continuous, the ripple voltage of C is twice of the average output voltage.

i.e. $\Delta V_c = 2I_oR$. —— ②

Combining ① and ②, $C_{cri} = \dfrac{kT}{2R}$.

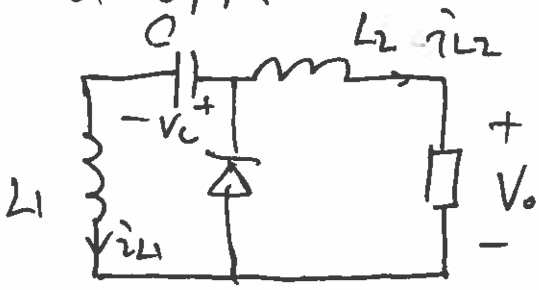
b) i) Q ON:



$$V_i = -V_c + L_2 \frac{\Delta i_{L2}}{DT} + V_o \quad \text{—— ①}$$

$$V_i = L_1 \frac{\Delta i_{L1}}{DT} \quad \text{—— ②}$$

Q OFF:



$$L_1 \frac{\Delta i_{L1}}{(1-D)T} = V_c \quad \text{—— ③}$$

$$L_2 \frac{\Delta i_{L2}}{(1-D)T} = V_o \quad \text{—— ④}$$

combining ①, ②, ③, ④,

$$\frac{V_2}{V_1} = \frac{D}{1-D}$$

ii)

1) The circuit can be grounded. The output has the same polarity of the voltage with the input.

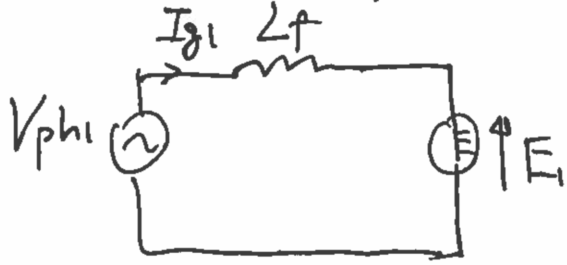
2). The output capacitor can be saved because the load current is provided by the L_2 when the Q is OFF.

4. (a)

The phase current I_g is:

$$I_{g1} = \frac{P}{\sqrt{3} \cdot V_{LL}} = \frac{22 \times 10^3}{\sqrt{3} \times 400} = 31.8\,A.$$

At 50 Hz, the AC-DC rectifier is:



$$|E_1| = |V_{ph1} + j\omega L_f I_{g1}|$$

$$= \left|\frac{400}{\sqrt{3}} + j\,100\pi \cdot 10 \times 10^{-3} \times 31.8\right|$$

$$= 251\,V.$$

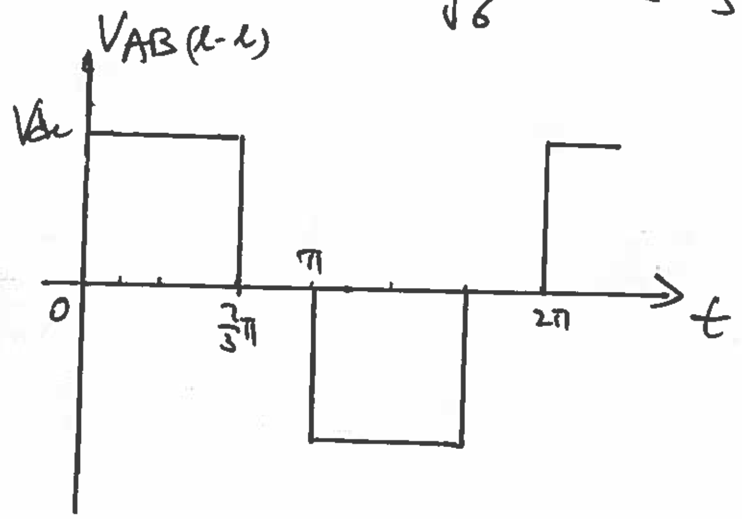
SPWM in use:

$$M = \frac{\hat{V}_{LL}}{V_{dc}} = \frac{251 \times \sqrt{3} \times \sqrt{2}}{740} = 0.831.$$

(b) When it is square-wave operation, at 50 Hz,

$$E_{llRms1} = \frac{\sqrt{6}}{\pi} V_{dc}, \quad E_{ph1} = 251\,V.$$

$$V_{dc} = \frac{E_{ph1} \cdot \sqrt{3} \cdot \pi}{\sqrt{6}} = 558\,V$$

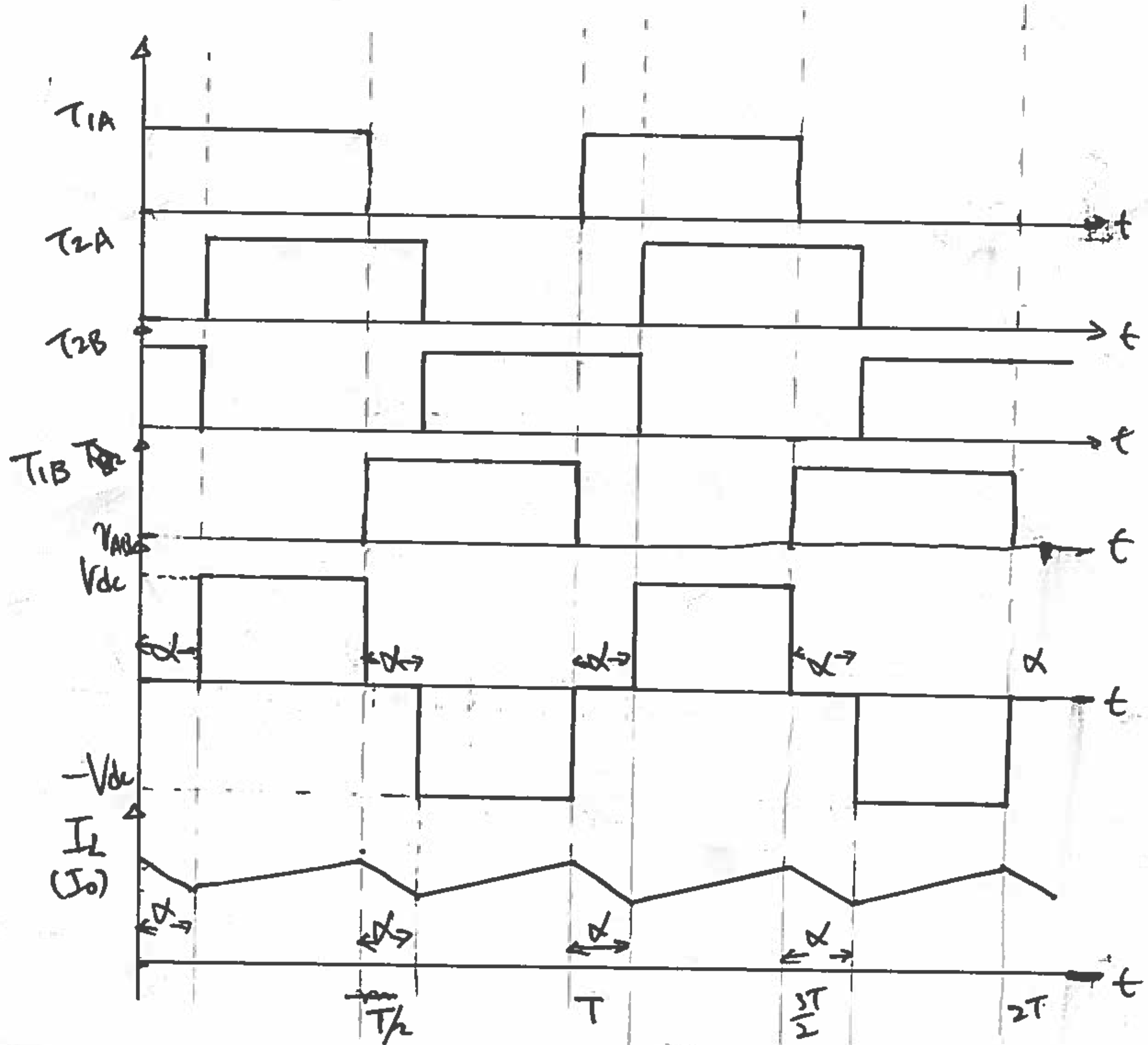


At 250Hz, (5^{th} harmonics, the largest harmonics)

$$E_{ph5} = \frac{\sqrt{6}}{\pi} V_d \cdot \frac{1}{\sqrt{3}} \cdot \frac{1}{5} = \frac{\sqrt{2}}{5\pi} \cdot 558 = 50\,V.$$

$$I_{g5} = \frac{E_{ph5}}{\omega_5 L_f} = \frac{50}{5 \times 50 \times 2\pi \times 10 \times 10^{-3}} = 3.18\,A.$$

(c)



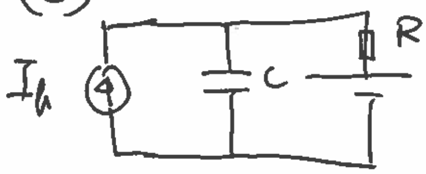
(d) The H-Bridge and Diode rectifier form a Buck converter.

$V_o = (1 - \frac{\alpha}{\pi}) \cdot V_{dc}$, $(1 - \frac{\alpha}{\pi}) = D$, $0 \le D \le 1$

for a Buck converter, when $D = \frac{1}{2}$, the ripple current of the L is at the largest. $\Delta I_L = \frac{V_{dc}(1-D) \cdot D}{Lf}$, when $D = \frac{1}{2}$, $V_o = \frac{1}{2} \cdot 740 = 370\,V.$

$\Delta I_L = \frac{V_{dc}}{4Lf} = 1\,A$, $f = \frac{V_{dc}}{4L\Delta I_L} = 18.5\,kHz$

(e)



I_h is the ripple current.

if the battery internal impedance R is very small.

The battery branch always shows a lower impedance to the filter capacitor. Therefore, the ripple current will not flow into the capacitor but into the battery. The capacitor becomes less necessary.