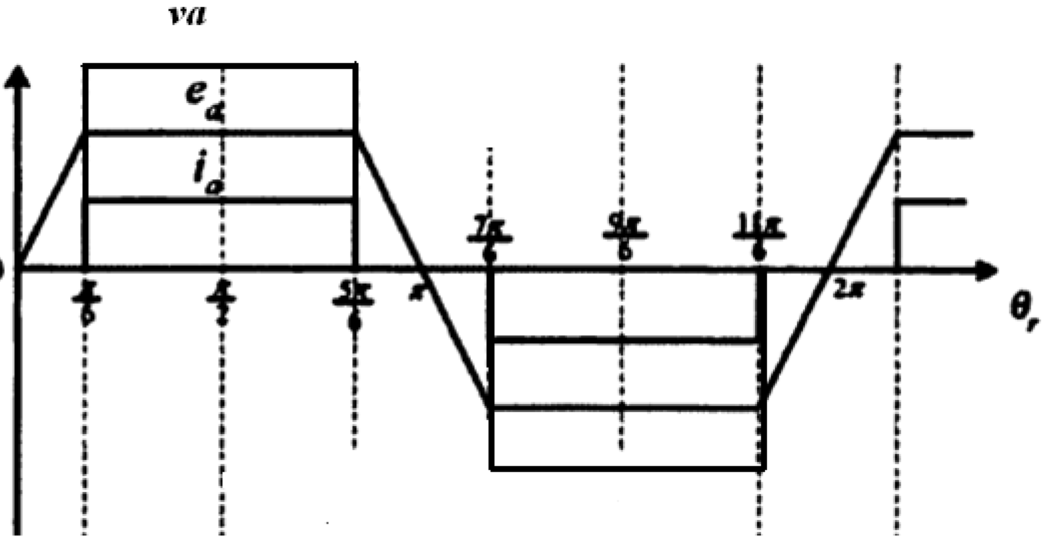
Module 3B4 2015 crib

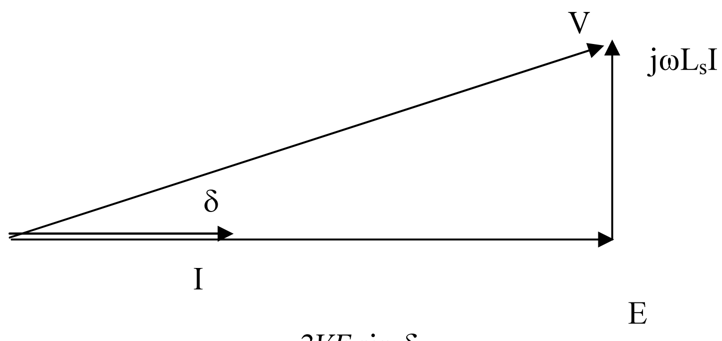
1. (a) The rotors of both machines are similar, with a set of radially-magnetised permanent magnets (PM) fixed to the surface of the back-iron, so that the poles alternate N-S-N-S etc. However, the stators are different. The stator of the trapezoidal BLDCM has a number of poles, which is different from the number of rotor poles in order to avoid cogging. The stator coils are then wrapped around each stator pole, giving rise to a concentrated stator winding. For the sinusoidal BLDCM, the stator is wound as for a conventional synchronous machine ie a distributed winding of the same pole number of the rotor, designed so that when the rotor rotates the back-emf induced is close to a perfect sinusoid. The sinusoidal BLDCM has a lower torque ripple than the trapezoidal BLDCM. However, it is more complex to control in an electrical drive system. [25%]

(b) The figure below shows the required waveforms. Note that the phase current and back-emf are in phase, meaning that the torque angle is 90 degrees. [15%, 5% for each waveform]



Hall-effect sensors produce a square-wave output which is aligned with the rotor position. There is one Hall-effect sensor per phase, and when the sensor output eg for Hall-effect sensor A goes high, the inverter transistors switch so that line A is high and line B is low with line C open-circuited. When Hall-effect sensor B goes high, line B is driven high, line C is low with line A open-circuited etc. This ensures that the rectangular pulses of current in the phases are in phase with their back-emfs, which in turn keeps the torque angle at 90 degrees and maximizes the torque per amp. [10%]

(c) The phasor diagram for operation at 90 degrees torque angle is shown below.



Start from the data book expression $P_{out} = \dfrac{3VE\sin\delta}{\omega L_s}$

By trigonometry: $V\sin\delta = \omega L_s I$

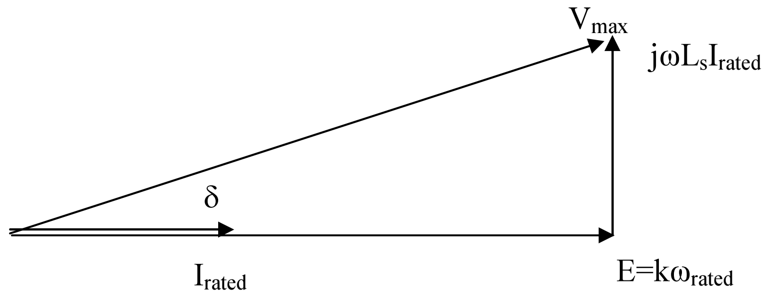
Also: $E = k\omega_s = k\dfrac{\omega}{p}$

Substituting: $P_{out} = T\omega_s = \dfrac{3k\omega_s \omega L_s I}{\omega L_s} = 3k\omega_s I$ giving $T = 3kI$ [20%]

(d) (i) Rated torque corresponds to the motor operating at rated current:

$T_{rated} = 3kI_{rated} = 3\times 1.2\times 40 = 144$ Nm

Rated speed is the maximum speed at which the motor can still deliver its rated torque. This speed is limited in turn by the maximum inverter voltage, or possibly, frequency. Assume that it is limited by the inverter voltage and then check that the resulting speed is not greater than the maximum possible speed of the drive. Letting ω_{rated} be the rated angular speed of the drive, the phasor diagram becomes as shown in the figure below.



Noting that $V_{max}=415/\sqrt{3}$, $I_{rated}=40$ A, Ls=8 mH and $\omega = p\omega_{rated}$ where p is the number of pole-pairs which is 2:

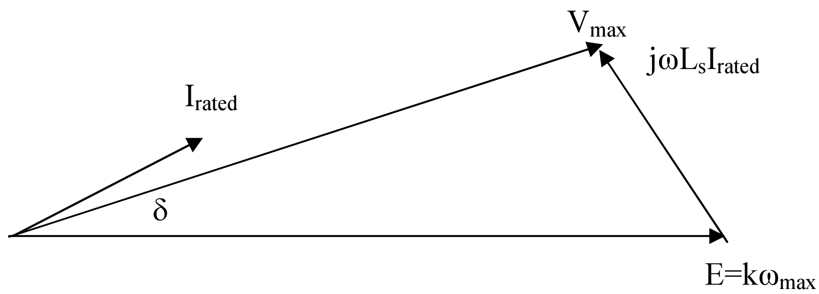
$$V_{max}^2 = (k\omega_{rated})^2 + (p\omega_{rated} L_s I_{rated})^2$$

Putting in the numbers: $\left(\dfrac{415}{\sqrt{3}}\right)^2 = (1.2\omega_{rated})^2 + (2\omega_{rated}\times 8\times 10^{-3}\times 40)^2$
giving $\omega_{rated} = 176$ rads^{-1} = 1682 rpm

Check: The maximum speed of the drive is $60f_{max}/p = 60\times 80/2 = 2400$ rpm so the maximum speed is indeed limited by the maximum inverter voltage.

$P_{rated} = T_{rated}\omega_{rated} = 25.3$ kW [10%]

(ii) The maximum speed of the drive is dictated by the maximum inverter frequency of 80 Hz and is given above as 2400 rpm. Above rated speed the drive operates at the maximum load angle possible, and so the torque angle is no longer 90 degrees giving the phasor diagram shown below.



First, derive the expression for torque in terms of load angle:

$$P_{out} = \frac{3VE\sin\delta}{\omega L_s} = \frac{3V_{max}k\omega_{max}\sin\delta}{\omega L_s} = \frac{3V_{max}k\sin\delta}{pL_s} \text{ and so } T = \frac{3V_{max}k\sin\delta}{\omega_{max}pL_s}$$

Everything is known in this expression except for the load angle. This can be found from the cosine rule, noting that it is maximized by operating the drive at its rated current:

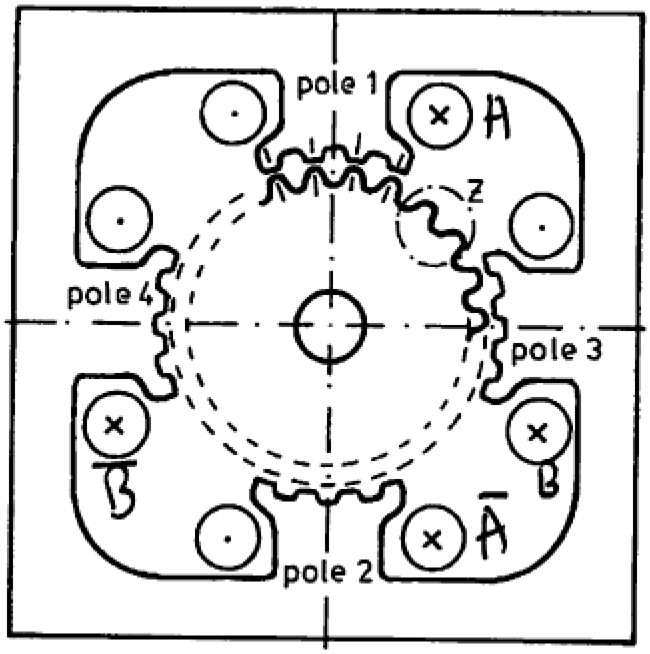
$$(\omega L_s I_{rated})^2 = V_{max}^2 + (k\omega_{max})^2 - 2V_{max}k\omega_{max}\cos\delta$$

Putting in the numbers and solving for δ gives δ=32.30 and substituting this into the torque equation gives T = 115 Nm. This is substantially less than the rated torque of 144 Nm and is the result of field-weakening. [20%]

Examiner's comments on Q1

This question concerned sinusoidal and trapezoidal brushless DC motor drive systems. Parts (a) and (b) were largely bookwork, and were generally well-attempted. Most students were able to derive the expression for torque, part (c). Part (d) caused the most problems. Common errors were to determine the maximum rather than the rated speed by ignoring the voltage drop across the synchronous reactance, and in part (ii) the condition that the drive operates at maximum load angle was often ignored, and the speed found at zero load torque with maximum field-weakening, which is a much easier calculation.

2(a)



The rotor wheels are offset by half a rotor tooth pitch so that in the position shown the north pole rotor wheel will be aligned with the teeth of pole 1 (south poles) and the south pole rotor wheel will be aligned with the teeth of pole 2. This means that when phase B is excited so that pole 3 becomes a south pole, the rotor will move by one quarter of a rotor tooth pitch. When phase A is then excited but in the opposite sense, pole 2 will be a south pole, pole 1 a north pole and so again the rotor will move round by a quarter of a rotor tooth pitch. Phase B is then excited, but in the oppostite sense to the previous time, completing the cycle and the rotor moves a further quarter of a rotor tooth pitch, meaning that 4 steps of excitation results in the rotor moving by a complete rotor tooth pitch.

One rotor tooth pitch = 360/50 = 7.2^0 and so the full step size is 7.2/4 = 1.8^0.

(b) The reluctance torque on the rotor is given by:

$T = -\hat{T}\sin(N_t\theta)$ in which N_t is the number of rotor teeth per wheel and $\hat{T}$ is the peak restoring torque. Assuming that the motor drive a purely inertial load so that the combined moment of inertia of the rotor and the load is J, and that the system damping is negligible:

$$T = -\hat{T}\sin(N_t\theta) = J\frac{d^2\theta}{dt^2}$$

For small displacements about the equilibrium angular position of zero, $\sin(N_t\theta)$ can be approximated as $N_t\theta$ giving:

$$\frac{d^2\theta}{dt^2} = -\frac{N_t\hat{T}}{J}\theta$$

This is the differential equation for simple harmonic motion and the solution is a pure sinusoid with natural frequency given by:

$$\omega_0^2 = \frac{N_t \hat{T}}{J} \quad \text{giving}$$

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{N_t \hat{T}}{J}}$$

(c) (i) The maximum angular position error occurs when the load torque is equal to the restoring torque, and is equal to one quarter of a rotor tooth pitch, which is the same thing as the step angle of the stepper motor. Thus, for a maximum error of 0.6^0 the gearbox must be a reduction gearing of 3:1 since the stepper motor stepping angle is 1.8^0.

(ii) The gearbox reduces the moment of inertia of the load as seen by the stepper motor by the factor $1/(\text{Gear ratio})^2$ i.e. 1/9. Thus, the total moment of inertia seen by the stepper motor, J_{tot}, is:

$J_{tot}=(0.1+2/9)\times10^{-5}=0.322\times10^{-5}$ kgm^2

Thus, the natural frequency of oscillation is:

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{N_t \hat{T}}{J}} = \frac{1}{2\pi}\sqrt{\frac{50\times0.1}{0.322\times10^{-5}}} = 198\,\text{Hz}$$

(iii) This frequency corresponds to 198/200 revolutions per second 0.992 rps=59.5 rpm. Operating the motor at this speed would excite its natural resonance, and if this was continued then the oscillations would build up to the point where the motor started to miss steps. It would then become uncontrollable.

(d) For a speed of 100 rpm at the paper feed mechanism the stepper motor would be rotating at 300 rpm = 5 rps. Thus, the frequency of excitation would be given by:

f/200=5 so f = 1 kHz.

Clearly this means that the stepper motor will have to pass through the speed which corresponds to its natural resonance. Two strategies for dealing with this problem: use microstepping or even just half-stepping when operating close to this speed, which will have the effect of changing the natural frequency; operate the motor at maximum torque so that it accelerates through the problem speed as fast as possible, thereby not allowing oscillations time to build up; a third possibility is the use of special damper couplings which are able to absorb and dissipate the energy contained in the oscillations i.e. increasing the mechanical damping.

Examiner's comments on Q2

This question was on stepper motors, and most candidates scored heavily on parts (a) and (b), which are both mainly bookwork. Part (c) caused more problems. Most candidates got the gearbox ratio correct, but many failed to determine the correct combined moment of inertia correctly in (c) part (ii). A common error was to confuse the step angle with the tooth pitch in part (d), giving a factor of 4 error in the excitation frequency. However, most candidates knew the consequences of operating the stepper motor at excitation frequencies that excite resonance, and the techniques for mitigating against this.

Q3

(a) Specific electric loading is the total current per unit circumferential length averaged around the air-gap. The value used in the output formula should be the effective total current but the gross value is usually used as winding factors are normally close to unity. The value of the electric loading is limited by the temperature rise caused by losses and depends on cooling.

The specific magnetic loading is the air-gap flux density averaged over one pole. Its value is limited by the need to avoid excessive saturation in the iron circuit to around 0.5 T in normal electrical steels. [10%]

(b) (i) Using the output formula with the given values of specific electric and magnetic loadings and a VA rating of 22,000/0.8 = 27.5 kVA gives

$$27500=\frac{\pi}{\sqrt{2}}.\pi\left(\frac{0.175}{2}\right)^2.\frac{2\pi.50}{2}0.55.30000$$

giving a stack length of 198.6 mm. [5%]

(ii) There are 48 slots, three phases and four pole pairs so each phase belt will occupy four slots. The winding factor can then be evaluated using m = 4 and beta = 7.5 degrees as

$$K_w=\frac{\sin\left(\frac{mp\beta}{2}\right)}{m\sin\left(p\beta/2\right)}$$

The number of turns can be found by rearranging the emf relationship and noting that

$$B_{rms}=\frac{2\sqrt{2}}{\pi}\bar{B}=0.605\text{T}$$

$$N_{ph}=E\frac{p}{\ell\omega}.\frac{1}{d}.\frac{1}{K_w}.\frac{1}{B_{rms}}=400\frac{2}{2\pi 50}.\frac{1}{0.1968}.\frac{1}{0.175}.\frac{1}{0.9977}.\frac{1}{0.605}$$

Dividing by 8 (4 coils per phase belt, two phase belts) gives 15.8. Obviously fractional turns are not acceptable so 16 turns per coil would be used. [25%]

(iii) The power of the machine is given by 3 V I cos(Øi). The current is then

I = 22,000/(3.400.0.8) = 22.9 A

The required conductor section, assuming all turns in series, is 22.9/5 = 4.58 mm^2, equivalent to a wire diameter of 2.4 mm.

This diameter is acceptable from a manufacturing point of view in terms of flexibility and bend radius but putting the two phase belts in parallel would double the number of turns a phase belt and reduce the wire diameter to 1.6 mm which is more flexible. [10%]

(iv) A double layer winding would allow short pitching, giving a further reduction in the harmonic content of the air-gap field beyond that given by having a distributed winding. It would also allow the manipulation of the series/parallel connection of coils. [10%]

(c) (i) The governing equation is

$$\theta = \frac{P}{k}\left(1 - e^{-t/\tau}\right)$$

where theta is the temperature rise and tau the thermal time constant, c/k

At steady state theta = P/k giving k = 3000/120 = 25 W/K

Using the second piece of information

$$90 = \frac{3000}{25}\left(1 - e - \frac{30.60}{\tau}\right) = 1299\,s$$ corresponding to a c of 32468 J/K

[15%]

(ii) The key assumption is that losses are proportional to the square of the machines current. A 50% overload will increase the current by 50% and the losses by 125%, giving a total dissipation of 6750 W. Putting this into the equation gives

120 = 6750/25 (1 – e-(t/1299) so t = 765 s or 12.75 minutes.

[20%]

(iii) A device to sense the machine's temperature such as a thermistor could be employed or if the motor were used in a drive the inverter controller could have a temperature estimation function which would keep the motor in a safe state. [5%]

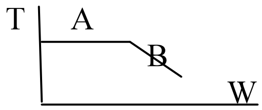
Examiner's comments on Q3

Every candidate attempted this question on machine design and performance, and generally produced good answers. Marks were mostly lost through minor errors but the part on the overloaded motor caused some difficulty – few candidates realized that a 50% power overload implies a 50% current overload and hence a 125% increase in resistive losses; most took the increase in loss to be 50%.

Q4

(a) The drive can be operated with the motor fully fluxed, achieved by keeping the volts per Hertz ratio constant, up to a point where the maximum output voltage from the inverter is reached. This is the constant flux region. Above this point, the machine's flux falls in inverse proportion to the frequency – this is the field weakening regime. As the frequency increases further the stator reactance can no longer be taken to be negligible and the fall in flux accelerates. Also, at low frequencies the voltage drop in the stator resistance cannot be neglected. [15%]

(b) [10%]



A constant torque (machine fully fluxed)
B field weakening

(c) The first assumption is that the stator impedance (R_1 and X_1) can be neglected so that the terminal voltage appears across the magnetizing branch. The second assumption is that the referred rotor leakage reactance (X_2') can be neglected in comparison with R_2'/s – valid for small slip and up to a frequency likely to be beyond the normal operating range. [15%]

(d) (i) Neglect the stator impedance ($R_1 + X_1$). The stator current I_1 is the quadrature sum of I_2' ', assuming the rotor branch to be essentially resistive, and the magnetizing current. The magnetizing current is $\frac{400}{\sqrt{3}}.\frac{1}{j120} = 1.92\,A$

$$I_2' = \sqrt{\left(4^2 - 1.54^2\right)} = 3.51\,A$$

Also $I_2' = V_1 \Big/ \left(R_2'/s\right)$ so $s = \frac{R_2' I_2'}{V_1} = 2.7.\frac{3.69}{400/\sqrt{3}} = 0.041$

The torque formula then gives

$$T = \frac{3VI_1^2 s}{\omega_s R_2'} = \frac{3\left(\frac{400}{\sqrt{3}}\right)^2 0.041}{50\pi.2.7} = 15.4\,Nm$$ [20%]

(ii) Speed is $60/2\pi$ 60/(2 π) (1 – s) ω_s = 1500 (1 – 0.041) = 1439 rpm as this is when it is assumed that the limit of the inverter output at 400 V is reached. The output frequency is 50 Hz. [10%]

(iii) Torque is proportional to $s\omega_s$ so this will be the same at zero speed as at maximum speed.

Hence SWs = 0.041 and the slip frequency is 2.05 Hz. Stator current is as before, magnitude = 4 A, giving a drop across the stator resistance of 4x3.4 = 13.6 V (the stator reactance is small at this frequency). The voltage across the magnetizing branch on a constant V/Hz basis is $\frac{2.05}{50}.\frac{400}{\sqrt{3}}$ = 9.4 V. Considering the phase angle of the voltage across the magnetizing branch relative to that across the stator resistance (21 °) a reasonable approximation for the required terminal voltage is 13.6 + 9.4 = 23.1 V, considerably higher than that calculated from the constant volts per Hertz consideration alone. [20%]

(iv) This is simply found from operation at -50 Hz (reverse phase sequence) and so is 1439 rpm.

(e) Encoderless drives are attractive as the encoder is a relatively costly part, especially on a small drive, and is vulnerable to damage. Mounting the encoder can be inconvenient.

Whilst open loop control is acceptable for some drives, vector control and direct torque control are proven approaches. [15%]

Examiner's comments on Q4

Most candidates attempted this question on induction motor drives and there were both some good answers and some very weak attempts. There were no obviously unduly challenging parts to the question although the last part on encoderless subsystems did not lead to many thoughtful answers.

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14/5/2015