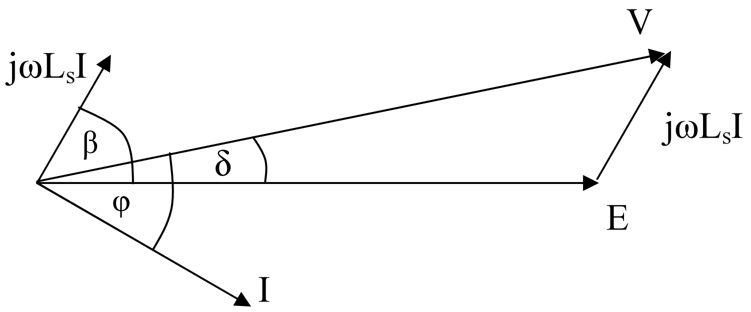
**Module 3B4 2016 crib** **Dr T J Flack, May 2016**

1. (a) Three advantages: higher power and torque density; smooth torque production (minimal reluctance torque); fast speed of response. Three disadvantages: greater sensor requirement (sensorless control is possible for the trapezoidal BLDCM whereas rotor position sensing is required for the sinusoidal BLDCM); stator current has to be resolved into orthogonal components and so greater hardware requirement and hence cost; control strategies are more complex.

General phasor diagram is shown below.




Start from the data book expression $P_{out} = \dfrac{3VE\sin\delta}{\omega L_s}$

By trigonometry: $V\sin\delta = \omega L_s I \sin\beta$

Also: $E = k\omega_s = k\dfrac{\omega}{p}$

Substituting: $P_{out} = T\omega_s = \dfrac{3k\omega_s \omega L_s I\sin\beta}{\omega L_s} = 3k\omega_s I\sin\beta$ giving $T = 3kI\sin\beta$

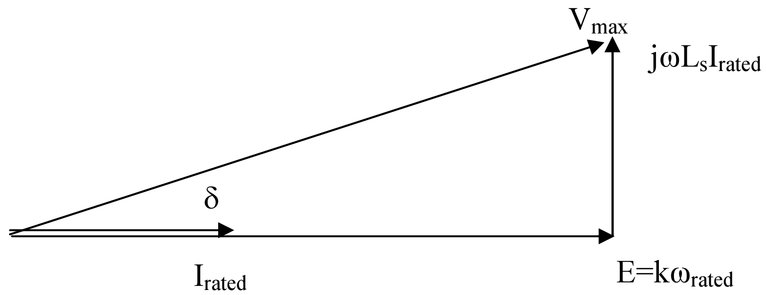
By operating with a torque angle of $90^0$ the maximum torque per stator current is obtained. This means that for a given torque and power output, power losses are minimised and so efficiency is maximised.

The drive system utilises a variable voltage, variable frequency inverter so that the amplitude, phase and frequency of the applied phase voltages can be controlled. By feeding back the instantaneous rotor position and stator currents, the phase, amplitude and frequency of the applied voltages can be calculated by the microprocessor which controls the switching of the inverter transistors using the phasor diagram above but with a torque angle of $90^0$. Note that for speeds above rated speed, the torque angle condition would be relaxed (field-weakening), but the controller software will account for this. [30%]

(b) (i) Rated torque corresponds to the motor operating at rated current:

$T_{rated} = 3kI_{rated} = 3\times 0.8\times 50 = 120$ Nm

Rated speed is the maximum speed at which the motor can still deliver its rated torque. This speed is limited in turn by the maximum inverter voltage, or possibly, frequency. Assume that it is limited by the inverter voltage and then check that the resulting speed is not greater than the maximum possible speed of the drive owing to the inverter frequency limit. Letting $\omega_{rated}$ be the rated angular speed of the drive, the phasor diagram becomes as shown in the figure below.




Noting that $V_{max}=415/\sqrt{3}$, $I_{rated}=50$ A, $L_s=4$ mH and $\omega=p\omega_{rated}$ where p is the number of pole-pairs which is 3:

$$V_{max}^2 = (k\omega_{rated})^2 + (p\omega_{rated} L_s I_{rated})^2$$

Putting in the numbers: $\left(\frac{415}{\sqrt{3}}\right)^2 = (0.8\omega_{rated})^2 + (3\omega_{rated} \times 4 \times 10^{-3} \times 50)^2$

giving $\omega_{rated}$ = 240 rads$^{-1}$ = 2291 rpm

Check: The maximum speed of the drive is $60f_{max}/p=60\times150/3=3000$ rpm so the maximum speed is indeed limited by the maximum inverter voltage.

[20%]

(ii) At 50% of rated torque and a torque angle of $90^0$ the phase current is 25 A. The maximum speed of the drive will again be either limited by the inverter voltage or frequency. Assume again that the speed is voltage-limited, and repeat the calculations above but with I = 25 A gives

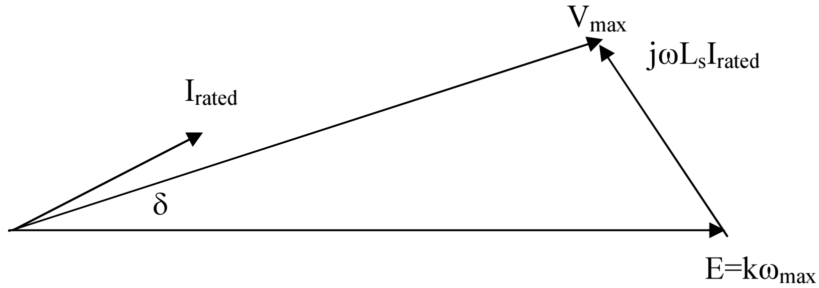
$$\left(\frac{415}{\sqrt{3}}\right)^2 = (0.8\omega)^2 + (3\omega \times 4 \times 10^{-3} \times 25)^2$$

giving ω = 280 rads$^{-1}$ = 2678 rpm which is below the 3000 rpm limit due to the maximum inverter frequency.

[15%]

(iii) Once the inverter voltage has reached its limit, by considering the phasor diagram below it is seen that because the emf induced in the stator winding by the rotor will continue to increase that the torque angle can no longer remain at 900. In turn, this means that a component of the input current produces air-gap flux which directly opposes the rotor flux, thereby partly cancelling the rotor flux. Hence the name field-weakening. [10%]

(iv) The torque angle is no longer equal to $90^0$, but for maximum speed at 50% rated torque the current will be at its rated value of 50 A. The phasor diagram below shows the situation.




The torque angle may be found from $T = 3kI\sin\beta$ giving $60 = 3\times 0.8\times 50\times\sin\beta$ and so $\beta=30^0$.

Applying the cosine rule to the phasor diagram and solving for $\omega_s$ gives 584 rads$^{-1}$ = 5577 rpm. Therefore the speed is now limited by the inverter frequency limit of 150 Hz to 60×150/3 = 3000 rpm. [25%]

**Assessor's comments**

This question was a popular one, attempted by all but one candidate. It concerned sinusoidal brushless DC motors and there were some excellent answers to it. Most candidates knew the bookwork parts of the question, but struggled more with the quantitative parts of the question in (b). Particularly troublesome was (b) part (iv) which involved field-weakening calculations, and then spotting that the speed is in fact limited by the inverter frequency.

2 (a) Distributing the winding means that the conductors are distributed in many slots around the stator periphery. This makes better use of the space available and reduces the harmonic content of the airgap flux density. Short-pitching the coils reduces the harmonic content of the airgap flux density further, which in turn means smoother torque production, reduction/elimination of cogging torques. It also reduces the length of the endwinding and hence coil resistance and losses for a given load current. [10%]

(b) 4 pole in 48 slots means p = 2, m = 48/6p = 48/12 = 4. Thus, showing two poles of the winding only:

RRRR-Y-Y-Y-YBBB-R-R-R-RYYYY-B-B-B-B
RR-Y-Y-Y-YBBB-R-R-R-RYYYY-B-B-B-BRR

$\beta = 360/48 = 7.5^0$ and short-pitching by two stator slots means that $\alpha = 2\beta = 15^0$. Thus the winding factor $k_w$ may be found as:

$k_w = \sin(mp\beta/2)/(m\sin(p\beta/2))\cos(p\alpha/2) = \sin(4\times2\times7.5/2)/(4\sin(2\times7.5/2) \times \cos(2 \times 15/2) = 0.925$

Assume $E_{ph} = V_{ph} = 6600/\sqrt{3}$ and use $E_{rms} = l(\omega/p)dN_{ph}k_wB_{rms}$ Putting in the numbers gives $N_{ph}$ = 163.9.

Double layer winding means that the number of coils is the same as the number of slots ie 48 and so there are 48/3 = 16 coils per phase giving the number of turns per coil, $N_{coil}$ = 163.9/16 = 10.2. This should be rounded up to the nearest integer to reduce the magnetic loading slightly ie $N_{coil}$ = 11 giving $N_{ph}$ = 16×11 = 176. [40%]

(c) VVVF control means that the applied voltage is increased in proportion to the frequency, which in turn determines the drive speed. Because this means that the flux is virtually constant, this results in the motor operating at its rated specific magnetic loading, meaning that the torque and power density are as high as possible, and gives the greatest drive efficiency. [10%]

(d) $T = 3I_2^2R_2/s\omega_s$ and $\omega_s = \omega/p$ giving $T = 3pI_2^2R_2/s\omega$

$I_2^2 = V_1^2/((R_2/s)^2 + \omega^2L_2^2) = s^2V_1^2/(R_2^2 + s^2\omega^2L_2^2)$ assuming that the voltage across the rotor branch, E, is equal to the stator applied voltage, $V_1$. Substituting into the expression for torque and replacing $V_1$ with $k\omega$ gives:

$T = 3pk^2s\omega R_2/(R_2^2 + s^2\omega^2L_2^2)$

For small slip, $s\omega L_2 << R_2$ and so the denominator may be simplified to $R_2^2$ giving:

$T = 3pk^2s\omega/R_2$ [20%]

(e) Ignore $R_1 + jX_1$ and $jX_2$. This is justified because the motor will be operating at a small value of slip.

$I_1 = V_{ph}/jX_m + I_2 = -j415/\sqrt{3}/90 + I_2 = -j2.67 + I_2$

Since $R_2/s >> X_2$, $I_2$ is approximately in phase with $V_1$ and so at the stator current limit of 15 A:

$I_2 = \sqrt{(15^2 - 2.67^2)} = 14.76$ A

$k = V_b/\omega_b = 415/\sqrt{3}/(2\pi\times50) = 0.763$

$I_2 = V_1/(R_2/s) = V_1s\omega/(\omega R_2) = s\omega k/R_2 = 14.76$ so $s\omega = 14.76 \times 1.4/0.763 = 27.1$ and this is fixed for rated torque and rated rotor (and hence stator) current.

$T = 3pk^2s\omega/R_2 = 3 \times 2 \times 0.763^2 \times 27.1/1.4 = 67.6$ Nm

$V_1$ is at its maximum when f = 50 Hz and delivering rated torque. The slip at this frequency is given by $2\pi\times50s = 27.1$ so $s = 0.0863$.

Maximum speed = $(1 - s) \times 60f/p = (1 - 0.0863) \times 1500 = 1370$ rpm.

Maximum unloaded speed = $(120/50) \times 1500 = 3600$ rpm [20%]

**Assessor's comments**

This was another popular question which also received many good attempts. It concerned three-phase induction motor drives. Most students were able to apply the formulae in (b) to obtain the number of turns/phase, but some struggled to draw the winding diagram, frequently getting the number of coils/phase band wrong, and over-pitching the windings. Parts (a), (c) and (d) were generally well-answered, with part (e) causing the most trouble.

3 (a) Two advantages: no $I^2R$ field winding losses; greater torque and hence power density. Disadvantage: the field cannot be altered as it can for field-wound DC motors, so field-weakening cannot be used as a means of extending the speed range. [10%]

(b) Ampere's Law: $2H_ml_m + 2H_gl_g = 0$

$H_g = B_g/\mu_0$ giving $0.8/(4\pi\times10^{-7}) = 637$ kAm$^{-1}$

The NdFeB characteristic is linear in the second quadrant and so can be expressed as:

$B_m = \alpha H + B_r$

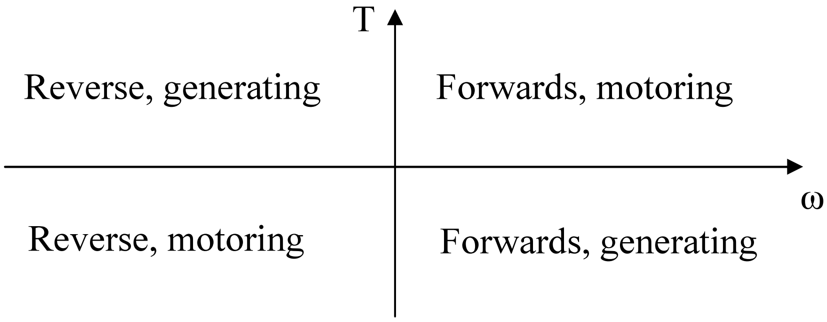
since $B = B_r$ when $H = 0$. To find $\alpha$, set $B_m$ to zero, at which $H = H_{co} = -940$ kAm$^{-1}$:

$0 = \alpha H_{co} + 1.25$ giving $\alpha = 1.33\times10^{-6}$

$0.8 = 1.33\times10^{-6}H_m + 1.25$

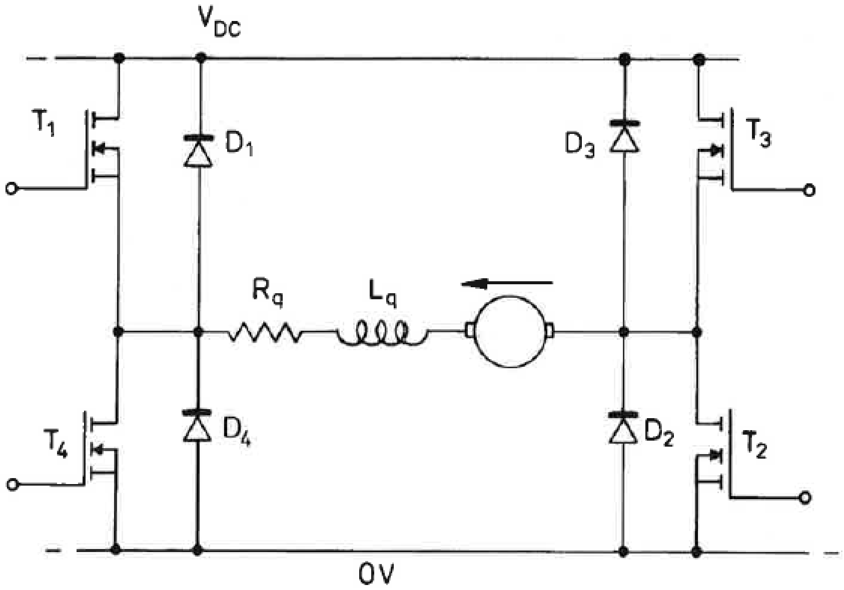
giving $H_m = -338$ kAm$^{-1}$. Plugging this value into the Ampere's Law equation gives $l_m = 5.65$ mm Note that a small correction can be made to account for the curvature of the pole-pieces, but this was not required (although some candidates took the trouble to make the calculation). [30%]

(c) Four quadrant drive control means that the drive machine can be driven in the forwards and reverse directions, either as a motor or as a generator as shown in the figure below.




An application where 4 quadrant drive control is desirable is the electric vehicle. Clearly the vehicle needs to be able to be driven both forwards and backwards. It is desirable if the drive can generate power back to the battery when it is braked, as this improves efficiency. Regenerative braking requires 4 quadrant control if it is to be used to brake the car when it is travelling forwards or backwards.

The full H-bridge drive circuit is shown in the figure below. Transistors T1 and T2 are switched in pairs to produce a positive armature voltage, so that the motor will rotate in the forwards direction. To reverse the motor, transistors T3 and T4 are switched as a pair. In order to vary the mean applied armature voltage, the pairs of transistors can be switched with a variable Ton:T duty ratio. In turn this will vary the speed of the drive motor.




[20%]

(d) (i) E = V under open-circuit conditions and so the emf (and hence torque) constant may be found:

$E = k\omega$ and so $k = 180/(1000\times2\pi/60) = 1.72$ Vsrad$^{-1}$

$T_{rated} = kI_{rated} = 1.72\times20 = 34.4$ Nm

Armature voltage is at a maximum when the duty ratio is 100%. Neglecting voltage drops across the pair of switching devices, $V_{amax} = 400$ V (DC supply of the H bridge)

$400 = k\omega_{rated} + I_{rated} R_a$ giving $\omega_{rated} = 140$ rads$^{-1}$ = 1332 rpm [15%]

(ii) At 50% rated torque $I_a = 10$ A giving $Va = 10\times8 + 1.72\times500\times2\pi/60 = 170$ V

The duty ratio is therefore 170/400 = 0.425 [15%]

(e) Ferrite has a much lower $B_r$ and $H_{co}$ than NdFeB and so if all the motor dimensions remain the same as before the airgap flux density will be much lower, perhaps around 3 times smaller. Therefore the torque and emf constant will also be around 3 times smaller, and so at rated current the torque will be one third of what it was for the motor with NdFeB magnets. However, the back-emf induced in the armature will be reduced by the same factor, and so with the same drive as before the rated speed will be increased by around a factor of three.

**Assessor's comments**

This question on permanent magnet brushed DC motor drives and magnetic circuits was the least popular question, but it did receive some excellent attempts. Most candidates attempted parts (a) and (b) well, although a common error in (b) was to forget that the flux has to cross the airgap twice, but there are two magnets also, meaning that many candidates got either half or double the required magnet length. Parts (c) and (d) were generally done well, and to my surprise most candidates realised what the effect of replacing the NdFeB with ferrite would do (the so-called 'sting in the tail' part !).

4. (a) Three advantages: single-phase induction motors are cheap to manufacture, especially with cast rotors; there is little to go wrong with them (no bushe or slip rings) so they are robust and reliable; they operate from a single-phase ac supply which is typically available in the domestic environment. The main pump motor in a dishwasher would usually be a single-phase induction motor. [15%]

(b) The rotor will 'see' the two mmf waves, and if the rotor speed is $\omega_r$, then defining slip with respect to the rotating mmf wave which is in the same direction of rotation as the rotor as $s_f$ we have:

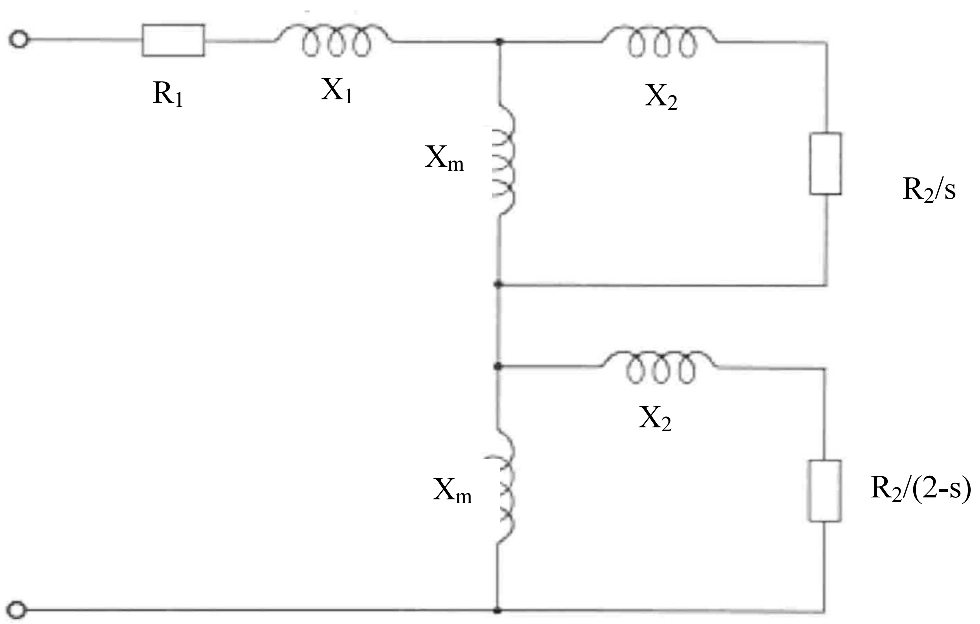
$s = s_f = (\omega_s - \omega_r)/\omega_s$

Thus, the backwards slip is

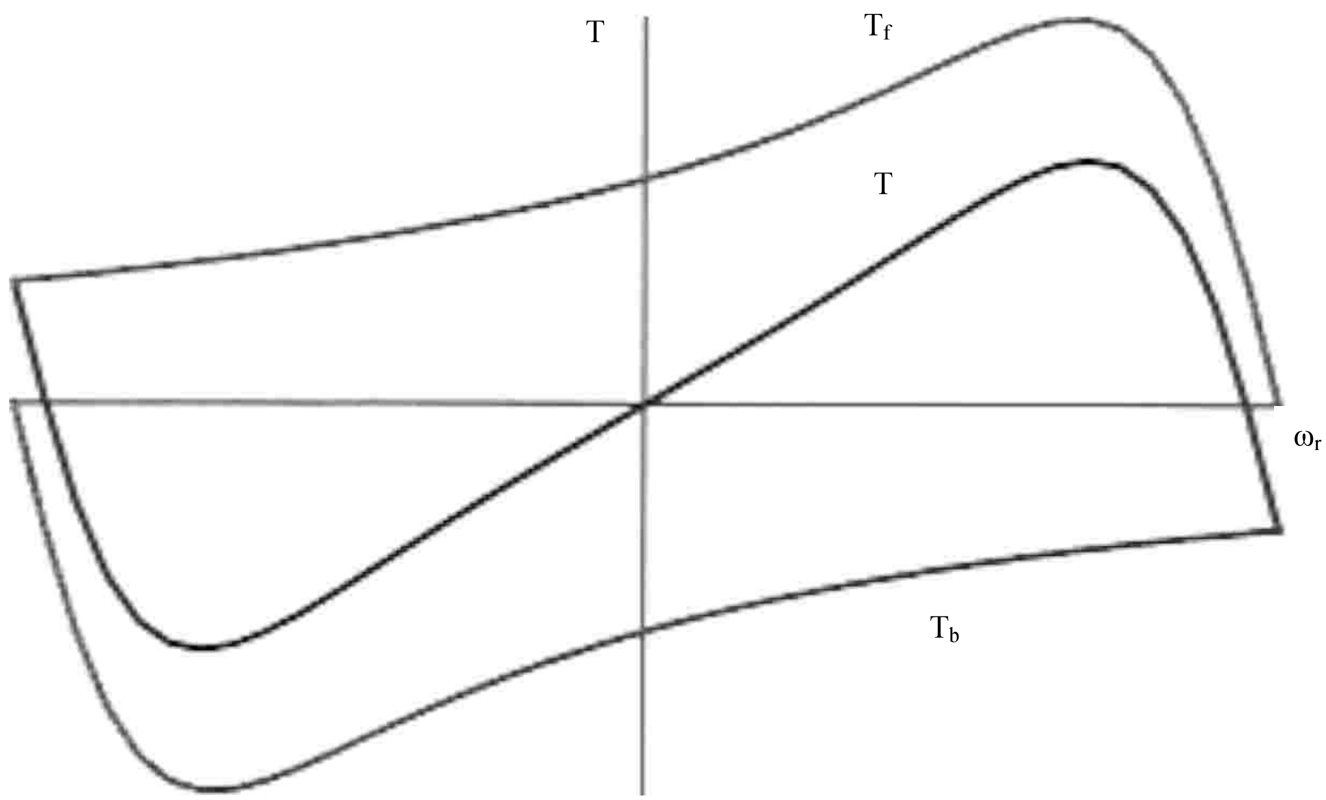
$s_b = (-\omega_s - \omega_r)/-\omega_s$

$= 2 - s$

Thus the equivalent circuit is as shown below




The torque-speed curve is obtained by superimposing the torque-speed curves of two three-phase induction motors, one forwards rotating, the other backwards rotating, but both producing torque in their directions of rotation.




[15%]

(c) 8 pole at 50 Hz means a synchronous speed of 750 rpm. Thus, 700 rpm corresponds to a forward slip of (750 – 700)/750 = 0.0667.

The synchronous speed is $2\pi f/p = 78.5$ rads$^{-1}$.

The total impedance of the motor is:

$Z = 2 + 2.5/0.0667 + 2.5/(2 – 0.0667) + j(3 + 3 + 3) = (40.8 + j9)\ \Omega$

Thus the input current $I = 240/(40.8^2 + 9^2)^{1/2} = 5.74$ A

The forwards torque $T_f = I^2R_2/(s\omega_s) = 15.7$ Nm

The backwards torque $T_b = I^2R_2/((2\text{-}s)\omega_s) = 0.54$ Nm giving a total torque of $T_f – T_b = 15.2$ Nm

Stator loss = $I^2R_1 = 5.74^2\times2 = 65.9$ W

Rotor loss = $I^2R_2 = 5.74^2\times 2.5\times2 = 164.7$ W

$P_{out} = T\omega_r = 15.2 \times (1\text{-}0.0667) \times 78.5 = 1113$ W

$P_{in} = P_{out} + P_{loss} = 1113 + 65.9 + 164.7 = 1343$ W

$\eta = P_{out}/P_{in} = 1113/1343 = 82.8$ % [35%]

(c) (i) Denoting θ as the temperature wrt to ambient the governing equation when the motor is switched on is:

$$\theta = \frac{P}{k}\left(1 - e^{-t/\tau}\right)$$

in which the thermal time constant $\tau = C/k$

At steady state $\theta_{ss} = P/k$ giving $k = 230.6/60 = 3.84$ W/K

When the motor is switched off and is cooling down the governing equation is:

$\theta = \theta_{ss} e^{-t/\tau}$ giving $30 = 60e^{-600/\tau}$ and so $\tau = 866$ secs and so $C = k\tau = 3328$ $JK^{-1}$ [15%]

(ii) When the motor is operating at rated load the temperature will vary according to:

$$\theta = \theta_{min} + (\theta_{ss} - \theta_{min})(1 - e^{-t/\tau})$$

At t = 20 minutes = 1200 s $\theta = \theta_{max}$ so

$$\theta_{max} = \theta_{min} + (60 - \theta_{min})(1 - e^{-1200/866})$$

When the motor is switched off:

$\theta = \theta_{max} e^{-t/\tau}$ and at t = 5 minutes = 300 s $\theta = \theta_{min}$:

$$\theta_{min} = \theta_{max} e^{-300/866}$$

Eliminating $\theta_{min}$ between these equations gives $\theta_{max} = 54.7$ $^0$C and so $\theta_{min} = 38.6$ $^0$C. Remembering that $\theta$ means temperature wrt to ambient gives the maximum and minimum motor temperatures as 74.7 $^0$C and 58.6 $^0$C respectively. [20%]

**Assessor's comments**

This question on single-phase induction motors was less well-attempted than other questions, possibly as a result of being the last question on the paper and candidates running out of time. Whilst many candidates knew the straightforward bookwork parts of (a) and (b), very few did well at the numerical parts of (c) and (d). A common mistake was to forget that these motors are singe-phase and so no factor of 3 is needed in calculations. Most candidates understood the principle of the duty cycle calculations of part (d) but only a handful could actually get the correct answers.