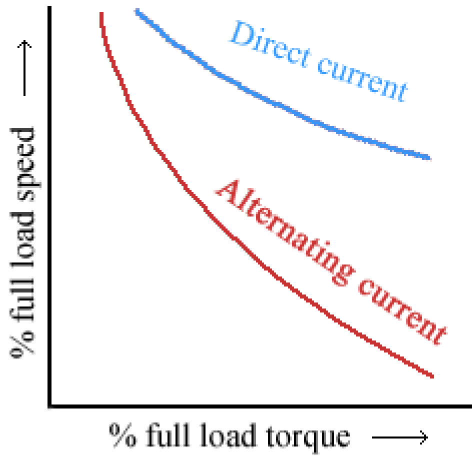
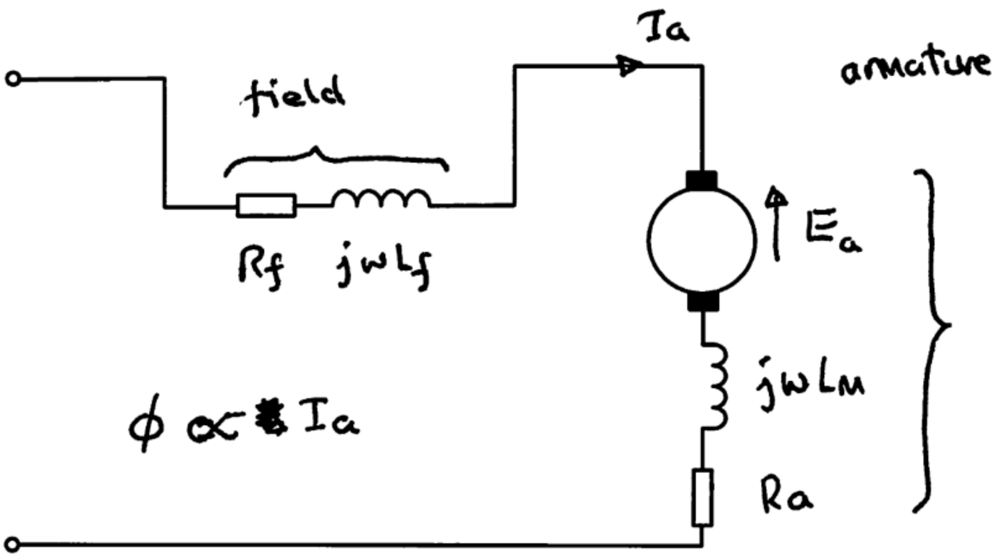
3B4 2017 Crib

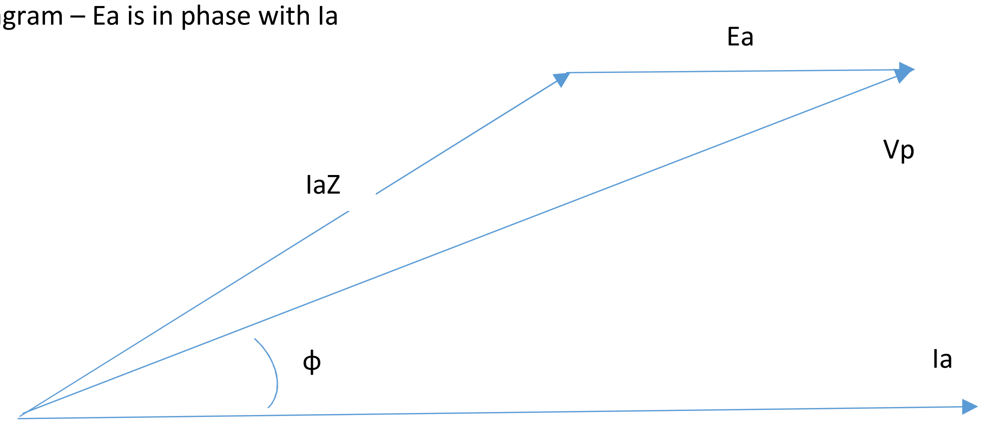
1. a) Torque speed characteristic (note the plot below was taken from - http://www.electricaleasy.com/2014/02/universal-motor-construction-working.html)



b) Equivalent circuit is:



Phasor diagram – Ea is in phase with Ia



$$X = 2 * \pi * 50 * 30 * 10^{-3} = 9.4\Omega$$

$$R = 3\Omega$$

$$V_p{}^2 = ((Ea + IR)^2 + (IX)^2)$$

$$Ea = (240 * 240 - (9.4 * 9)^2)^{1/2} - 3 * 9 = 197.6V$$

$$cos\phi = \frac{Ea + IR}{Vp} = \frac{197.6 + 9 * 3}{240} = 0.94 lagging$$

Power developed = EI=197.6*9=1.78 kW

Input power = VpI=240*9*cos(phi)=2.02 kW

Efficiency = 1.78/2.02=0.88

c) duty cycle question

governing equation is: $P\delta t = C\Delta\theta + k\theta\delta t$

From this the differential equation is: $\frac{d\theta}{dt} + \frac{k}{c}\theta = \frac{P}{c}$

General solution is: $\theta = Ae^{-t/\tau} + P/k$ $\tau = c/k$

However since we have the background temperature we can use

$$C\frac{d\theta}{dt} = \frac{P}{c} - k(\theta - \theta_0)$$

$$-C\int_{\theta start}^{\theta finish} \frac{1}{-P + k(\theta - 20)} d\theta = time$$

We know the power loss from the previous part hence we have P= 243 W

For the two parts of the cycle we have the two equations:

$$\theta_{max} = \theta_{min} + (\theta_1 - \theta_{min})\left(1 + e^{-t_1/\tau}\right)$$

$$\theta_{min} = \theta_{max} + (\theta_2 - \theta_{max})\left(1 + e^{-t_2/\tau}\right)$$

Where

$$\theta_{max} = 93$$

$$\theta_{min} = 80$$

$$t_1 = 120$$

$$t_2 = 60$$

$$\theta_2 = 0\ (ambient)$$

From the second equation, we can calculate that

$$\tau = 398.5 = c/k$$

We can then calculate from the first equation that $\theta_1 = \theta_\infty = 130 = P/k$

We know that P from the previous part and hence k = 1.869 W/k

And finally c =745 J/K

Alternatives for overload protection

None: motor cheap enough to replace with little penalty.

- No direct temperature sensing but motor control gear includes some sort of **thermal model** of the motor, e.g.

  a) **bimetallic strip** (carries or is heated by input current) plus relay

  b) **microprocessor** or other VLSI chip on-line with appropriate temperature estimating software (based on sensed input current and possibly speed).

- **Temperature sensor** in motor.

- **'Current foldback'** (i reduction) implemented by power electronic feedback after a certain time.

Part d)

Brushed motor Advantages:

Simplified wiring: Brush motors can be wired directly to DC power and control can be a simple as a switch.

Low cost:

Brushed Motor Disadvantages:

Less efficient

Electrically noisy: The switching action of the commutators constantly creating and breaking inductive circuits creates a great deal of electrical and electromagnetic noise.

Lifespan: As they are in perpetual physical contact with the shaft, brushes and commutators wear out.

Brushless Motor Advantages:

Long lifespan: No brushes to wear out
low maintenance: No brushes to replace
High efficiency

Brushless Motor disadvantages:

High initial cost: Need for commutating device like an encoder and a drive or controller

Examiner's comments

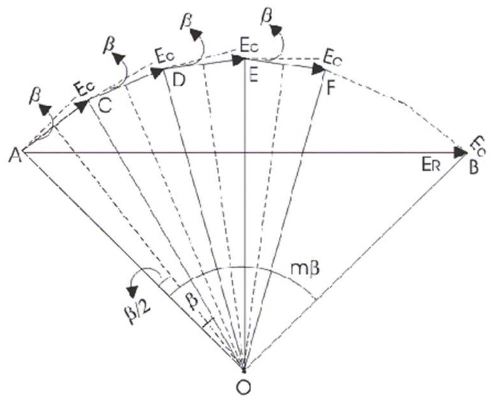
This question was the least popular question, concerning DC and universal machines, and duty cycling operation. It attracted some very good answers.

2

a) 1)

Distributed windings more efficient use of space better distribution of flux. Short pitching cuts down on the harmonics

ii) straight from lectures



$$E_c = 2 * OA * sin\frac{p\beta}{2}$$

Hence if we have m slots per phase the arithmetic sum would be:

$$E_{rc} = 2 * OA * m * sin\frac{p\beta}{2}$$

But the phasor sum is given by length AD

$$E_{rd} = 2 * OA * sin\frac{mp\beta}{2}$$

Divide one by the other and you get the Kd:

Kp comes from the fact that the flux cut by the coil is reduced by a factor kp where kp is dependent on the reduction in area of the coil hence cos (alpha * p/2) where alpha is equal to the number of slots short pitched by times beta the angle between the slots

Kw = kd*kp

P = number of pole pairs

M = number of slots per phase band

Beta = angle between slots

Alpha – short pitching angle – short pitched by one slot hence is equal to beta

b)

$$B_{rms} = \frac{V_{rms} p}{l \omega d N_{eff}}$$

*l = 95 *10^-3*

*d = 112 *10^-3*

*omega = 2*pi*50*

delta connected hence

Vrms = 415 (delta connected)

P = 2

*Brms =Bbar*pi/(2*2^0.5)=0.5 *pi/(2*2^0.5)=0.555*

*Hence Neff = V*p/(Brms*l*d*ω) = 447.4*

Beta = 2pi/36

One slot short pitching hence alpha = beta

*M = 36 / (3*4) = slots/(phases*poles) = 3*

*Winding factor is kw*kp*

$$k_w = \frac{\sin\left(\frac{mp\beta}{2}\right)}{(m * \sin\left(\frac{p\beta}{2}\right))} \cos\left(\frac{\alpha p}{2}\right) = 0.945$$

$$\frac{N_{eff}}{k_w} = N_{ph} = 473$$

*Turns per coil = Nph/(p*m*2)= 473/12=39.4 ---we need an integer result hence round up to 40 and Nph becomes 480*

Last part – terminal voltages are in the following ratios

*1:kd:kd*kp 1:0.96:0.95*

c) Highest starting torque uses a tapered slot but this leads to a roll off in torque at higher speeds . Good compromise is a Boucherot slot which evens out the torque. At start the slip is 1 and the frequency is high. This means that the current is confined by the skin effect and hence the resistance is high leading to a high torque. As the motor speeds up the current can spread out to occupy the whole bar.

Examiner's comments

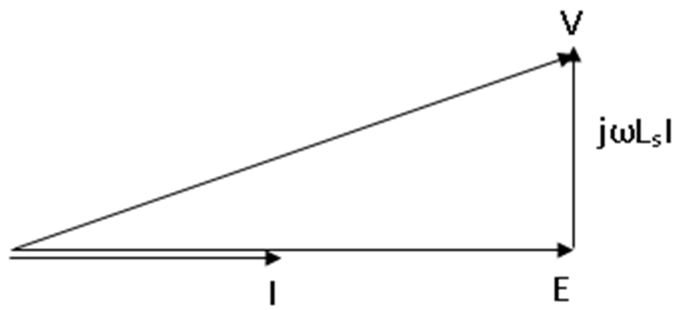
This was a popular question which also received many good attempts. It concerned design calculations of three-phase induction motors.

3 (a) The drive for an all-electric vehicle needs to have high power and torque density, minimal torque ripple and be highly controllable (both torque and speed). The sinusoidal BLDCM satisfies these requirements. Also, no brushes and so good reliability/low maintenance.

The torque produced by this sort of motor is proportional to $I\sin\beta$ in which β is the torque angle. For maximum torque/amp β should be 90^0. In turn this minimises motor losses and hence maximises the drive efficiency.

(b) (i) Rated torque = $3kI_{rated} = 3\times0.8\times450/\sqrt{3} = 624$ Nm

Rated speed is the maximum speed that rated torque can still be supplied so $I= I_{rated} = 450/\sqrt{3}$ and V = 240 V (delta-connected) and with a torque angle of 90^0 the phasor diagram is:



Applying trigonometry to the phasor diagram of Fig. 3.2:

$$V_{rated}^2 = E^2 + (\omega L_s I_{rated})^2 = (k\omega/p)^2 + (\omega L_s I_{rated})^2$$

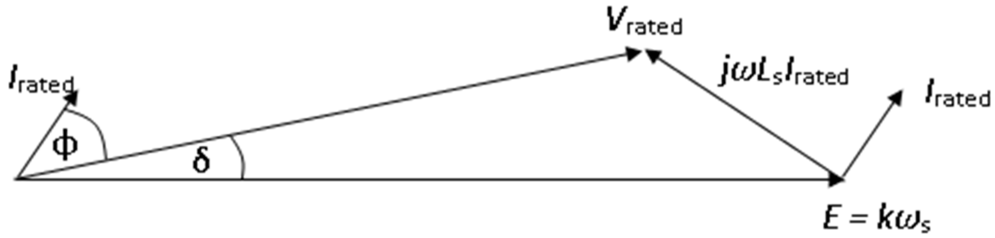
Putting in the numbers: $$240^2 = (0.8\omega)^2 + (\omega \times 2\times10^{-3} \times 450/\sqrt{3})^2$$

giving $\omega = 252$ rads^{-1} = 2402 rpm.

(ii) 75% rated torque means that $I = 0.75I_{rated} = 195$ A

Method is otherwise the same as (i) giving $\omega = 270$ rads^{-1} = 2575 rpm.

(iii) Here the speed is >> greater than rated speed and so it is no longer possible to maintain a 90 degree torque angle i.e. field weakening is required. The resulting phasor diagram is shown below.



$$P_{out} = \frac{3VE\sin\delta}{\omega L_s}$$

and substituting for V as V_{rated} and E as $k\omega/p$:

$$P_{out} = \frac{3V_{rated}k\omega\sin\delta}{p\omega L_s} = \frac{3V_{rated}k\sin\delta}{pL_s}$$

To maximise P_{out} (and hence torque for a given speed) we need to make δ as large as possible. Clearly this will occur at $I = I_{rated} = 260$ A as shown in the phasor diagram above. Apply the cosine rule to the phase diagram above:

$$(\omega L_s I_{rated})^2 = V_{rated}^2 + (k\omega_s)^2 - 2V_{rated}k\omega_s\cos\delta$$

The given speed of 4500 rpm must be converted to rads^{-1} and the resulting number will also be the angular supply frequency, ω, since $p = 1$:

$$\omega = \omega_s = \frac{2\pi}{60}N = \frac{2\pi}{60}\times 4500 = 471\ \text{rads}^{-1}$$

Putting in the numbers and solving for δ:

$$(471\times 2\times 10^{-3}\times 260)^2 = 240^2 + (0.8\times 471)^2 - 2\times 240\times 0.8\times 471\cos\delta$$

giving $\cos\delta = 0.772$ and so $\delta = 39.5^0$.

$$P_{out} = \frac{3V_{rated}k\sin\delta}{pL_s} = \frac{3\times 240\times 0.8\sin(39.5^0)}{2\times 10^{-3}} = 183\,\text{kW}$$

$S = 3V_{ph}I_{ph} = 3\times240\times450/\sqrt{3} = 187$ kVA so $\cos\varphi = 0.978$ lagging

The motor torque is then: $T = \frac{P_{out}}{\omega_s} = \frac{183000}{471} = 389\,\text{Nm}$

T = 3kIsinβ giving β = 141.4^0

S = 3$V_{ph}I_{ph}$ = 3×240×450/√3 = 187 kVA so cosφ = 0.978 lagging since δ is greater than 180-β

Field-weakening cancels some of the rotor-driven flux by injecting a component of stator current that produces a component of stator-driven flux that opposes the rotor-driven flux. This current does not contribute to torque production (and hence output power) but does cause extra copper losses, hence reduced efficiency.

Examiner's comments

This question on sinusoidal brushless DC motor drives was the most popular question, attempted by all candidates. Many excellent answers, with most candidates knowing the 'bookwork' parts of the question and being able to perform the more straightforward calculations of rated torque and speed. The final part concerning field-weakening was less well-attempted.

4 (a) Hybrid stepper motors step from one well-defined position to the next as the phases are sequentially excited. Therefore open loop position control is possible, so no sensors, no feedback loops.

They can provide a holding torque even at zero current.

They offer a high torque density.

They are mass produced and cheap, yet very reliable.

Full-stepping: phases are sequentially excited A, B, -A, -B, A or alternatively two phases at a time full stepping is $AB, B\overline{A}, \overline{A}\,\overline{B}, \overline{B}A, AB....$ for more torque.

Rotor moves by ¼ tooth pitch at each change in excitation.

Half-stepping: Phase excited as $A, AB, B, B\overline{A}, \overline{A}, \overline{A}\,\overline{B}, \overline{B}, \overline{B}A, A, AB....$ and the rotor moves 1/8 of a tooth pitch at each step i.e. half of a full step, hence the name.

Micro-stepping: Currents in the two windings are now modulated so that even more intermediate steps are available.

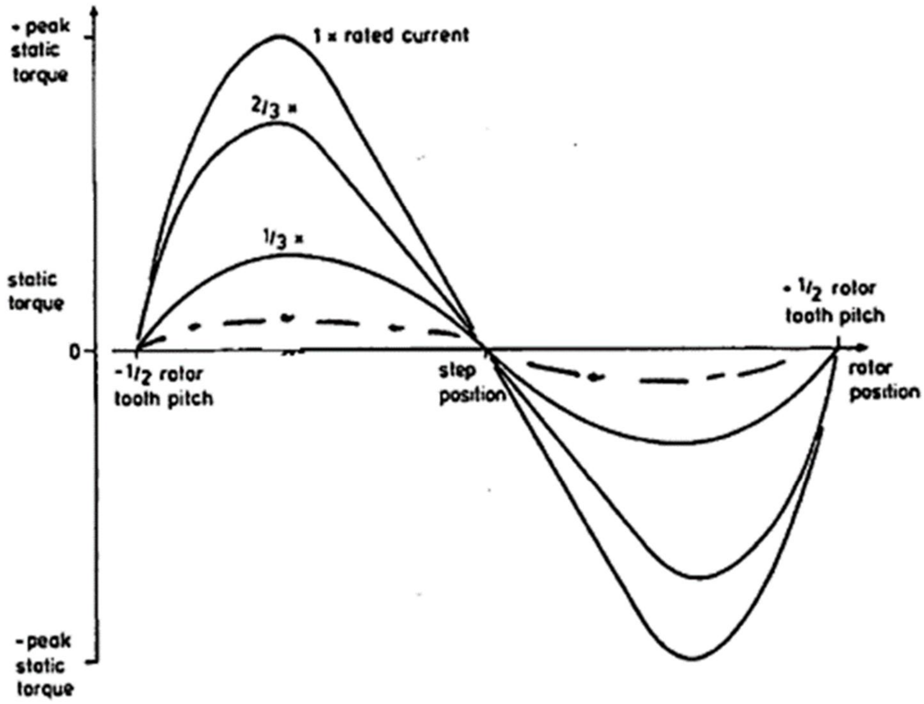
Advantages: Full stepping is simple and easy to implement and can produce more torque temporarily with the two-phases on at a time scheme. The rotor position is very well defined.

Half-stepping: more precise position control, positions are well-defined.

Micro-stepping: more steps and so more precise position control but the positions are not well-defined. Can be used to produce smoother torque and hence avoid resonant behaviour.

(b) (i) $\Delta\theta = \frac{360}{4N_t}$ in which N_t is the number of rotor teeth per wheel = 50 giving a full step angle of 1.8^0.

(ii) Torque-position characteristic shown below.



(iii) Position error is defined as the angular deviation away from the full-step equilibrium position caused by a static load torque. Modelling the motor torque as

$T_m = \hat{T}\sin(N_t\theta)$ and noting that the peak torque will be reduced because the current is 1.5 A and not rated current of 2 A giving 300 mNm gives:

$175 = 300\sin(50\theta_e)$ giving $\theta_e = 0.71^0$.

(iv) $T_m = -\hat{T}\sin(N_t\theta) \approx -\hat{T}N_t\theta$

$$\hat{T}N_t\theta = J\frac{d^2\theta}{dt^2}$$

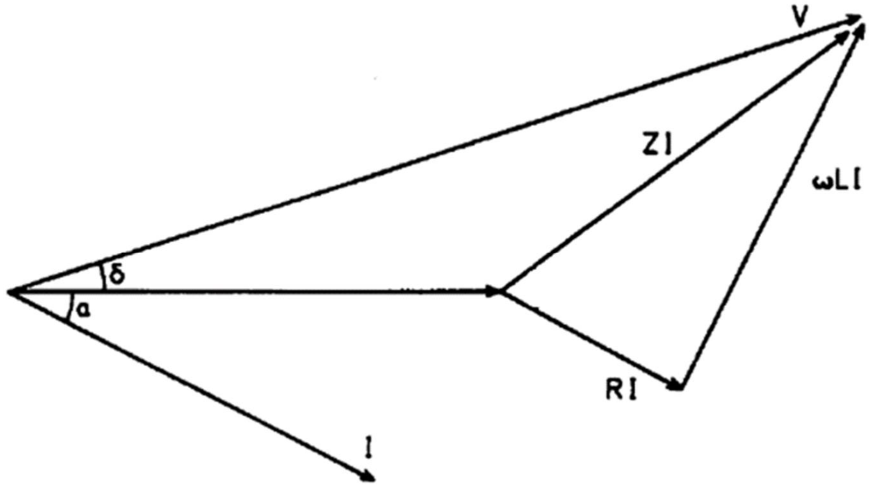
This is the simple harmonic motion equation with

$\omega_0 = \sqrt{\dfrac{\hat{T}N_t}{J}}$ and so

$$f_0 = \frac{1}{2\pi}\sqrt{\frac{\hat{T}N_t}{J}}$$

Putting in the numbers gives $f_0 = 5.03$ Hz and the corresponding speed is $60f_0/200 = 1.51$ rpm.

(c) (i) The phasor diagram is shown below.



(ii) 1 rps requires an excitation frequency of 50 Hz and so 100 rpm = 100/60 rps requires an excitation frequency of (100/60)×50 = 83.3 Hz

$E = k\omega_r = 2.5\times100\times2\pi/60 = 26.2$ V

$\overline{Z} = R + j\omega L = 1.5 + j2\pi\times83.3\times2.0\times10^{-3} = (1.5 + j1.05)\Omega = 2.6\angle46.4^0$

$\overline{V} = \overline{E} + \overline{ZI} = 26.2 + 2.6\angle46.4^0\times2\angle-15^0 = 29.7\ \text{V}\angle2.4^0$

Examiner's comments

This question on hybrid stepper motors was another popular question, and received many good answers, especially to the earlier parts of the question concerning static operation. Around half of the candidates were able to determine the excitation frequency (and associated speed) needed to avoid resonance, and very few were able to determine the applied voltage required to operate at speed.

Dr T J Flack and Dr T A Coombs 26/5/2017