EGT2
ENGINEERING TRIPOS PART IIA 2015

Thursday 30 April 2015 9.30 to 11

Module 3B5: SEMICONDUCTOR ENGINEERING

CRIB

Q1 (a) $D = 0$ as the wavefunction must be finite. However, as region II extends infinitely in the *x*-direction, then a divergent function is not a possible solution. [10%]

(b) As the wavefunction must be continuous, at $x = 0$

$$A + B = C$$

Also, the first derivative of the wavefunction must be continuous, so at $x = 0$

$$-jk_I A + jk_I B = -k_{II} C$$

Hence, substituting for *C* gives

$$B = \frac{A(jk_I - k_{II})}{(jk_I + k_{II})}$$

whilst substituting for *B* gives

$$C = \frac{A2\,jk_I}{(jk_I + k_{II})}$$ [30%]

(c) $|A| = |B|$ as any incoming electron (the *A*-term) must ultimately be reflected from the potential step (the *B*-term). As $|B|/|A|$ gives the reflection probability, this must equal unity. [10%]

(d) The probability of the electron existing at a particular location is given by $|\psi|^2$. In region II, $|\psi|^2 = C^2 \exp(-2k_{II}x)$, and so when the probability of finding the electron has dropped by a factor of $\exp(-1)$,

$$\exp(-2k_{II}x) = \exp(-1)$$

$$\therefore x = 1/2k_{II}$$

We therefore need to gain an expression for k_{II}. We can do this by substituting our wavefunction into the TISE

$$\frac{-\hbar^2}{2m}\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = (E-V)\psi$$

$$\frac{\mathrm{d}^2\psi}{\mathrm{d}x^2} = k_{II}^2\psi$$

$$\frac{-\hbar^2}{2m}k_{II}^2\psi = (E-V)\psi$$

$$\begin{aligned} k_{II}^2 &= \frac{2m}{\hbar^2}(V-E) \\ &= \frac{2\times 9.109\times 10^{-31}}{\left(1.055\times 10^{-34}\right)^2}(1-0.62)\times 1.602\times 10^{-19} \\ k_{II}^2 &= 9.97\times 10^{18}\ \mathrm{m}^{-2} \\ k_{II} &= 3.16\times 10^{9}\mathrm{m}^{-1} \end{aligned}$$

Hence

$$\therefore x = 1/2k_{II} = \underline{0.158\,\mathrm{nm}} \qquad [30\%]$$

(e) As device dimensions are scaled uniformly in all dimensions so that vertical and lateral fields remain proportional, it is necessary to decrease the thickness of the gate dielectric in MOSFETs. This results in an exponentially increasing tunnelling current through the gate dielectric leading to unacceptable levels of gate leakage. A thicker dielectric would result in an insufficient capacitance, which is required for charge accumulation in the channel. Therefore, high dielectric constant materials of increased thickness have been used instead.

[Generally, the question was very well answered, with students showing a good appreciation of the physical significance of the wave function solution given to them. The numerical part of the question about decay lengths into the step was particularly well answered. However, only a minority of students then appreciated that this is of significance when considering scaling of the thickness of the gate dielectric in a MOSFET, with some candidates getting confused between this and scaling down lateral dimensions in a MOSFET.]

Q2 (a) (i) As the de Broglie wavelength of the electron is of the same length order as the size of the potential well around an atom, the electron behaves quantum mechanically rather than classically. Furthermore, as electrons are Fermions, no two electrons can exist in identical quantum states, and so these states are filled from the lowest energy upwards in a multi-electron atom. Light emission from excited atoms (such as the Balmer series) is direct evidence of this, although the existence of the Periodic Table may also be considered to be indirect experimental evidence. [15%]

(ii) The electronic configuration of silicon, which has an atomic number of 14, is $1s^2\ 2s^2\ 2p^6\ 3s^2\ 3p^2$. [10%]

(iii) As electrons are Fermions, no two electrons can exist in the same quantum state. When atoms are brought into close proximity with each other, as in a solid, the more spatially extensive electron states overlap. As the overlapping states cannot be identical, they split in energy. Therefore, in a real solid with very large numbers of atoms, it is simpler to consider these well-defined energy levels in the solid as having split into bands of states. In the case of silicon, the 3s and 3p states hybridise into a band of completely filled bonding states, and a band of completely empty (at 0 K) anti-bonding states, separated by an energy gap of 1.1 eV. As this gap is relatively small, at normal temperatures, there is sufficient energy for a small number of the electrons in the bonding states to be excited into the anti-bonding states. This is the condition for a semiconductor. [20%]

(b) The Free Electron Theory shows that the density of electron states in a solid of volume V is given by

$$g(E)dE = \frac{V}{2\pi^2\hbar^3}(2m)^{3/2}E^{1/2}dE$$

and the Fermi function is

$$f(E) = \frac{1}{\exp\left(\frac{E - E_F}{kT}\right) + 1}$$

(i) If n is the number density of free electrons in the conduction band, then

$$n = \frac{1}{V}\int_{E_C}^{\infty} g(E)f(E)\mathrm{d}E$$

We know that for $E - E_F >> kT$, the occupancy of states is low, so that Boltzmann statistics can be used instead of the Fermi function, so

$$n = \frac{1}{V}\int_{E_C}^{\infty} \frac{V}{2\pi^2\hbar^3}\left(2m_e^*\right)^{3/2}\left(E - E_C\right)^{1/2}\exp\left(\frac{-\left(E-E_F\right)}{kT}\right)\mathrm{d}E$$

To solve this, we substitute $x = \left(E - E_C\right)^{1/2}$, so

$$n = \frac{\left(2m_e^*\right)^{3/2}}{2\pi^2\hbar^3}\exp\left(\frac{E_F - E_C}{kT}\right)\int_0^{\infty} x\exp\left(\frac{-x^2}{kT}\right)2x\mathrm{d}x$$

Using the standard integral

$$\int_0^{\infty} x^2\exp\left(-\alpha x^2\right)dx = \frac{1}{4}\sqrt{\frac{\pi}{\alpha^3}}$$

this becomes

$$n = N_C\exp\left(\frac{E_F - E_C}{kT}\right)$$

where

$$N_C = 2\left(\frac{m_e^* kT}{2\pi\hbar^2}\right)^{3/2}$$

$$= \frac{2\left(2\pi\times 1.09\times 9.109\times 10^{-31}\times 298\times 1.38\times 10^{-23}\right)^{3/2}}{\left(6.626\times 10^{-34}\right)^3} \qquad [30\%]$$

$$N_C = 2.825\times 10^{25}\ \mathrm{m}^{-3}$$

(ii)
$$N_D = 2.825\times 10^{25}\exp\left(\frac{-0.1}{0.862\times 10^{-4}\times 298}\right) \qquad [15\%]$$

$$N_D = 5.76\times 10^{23}\ \mathrm{m}^{-3}$$

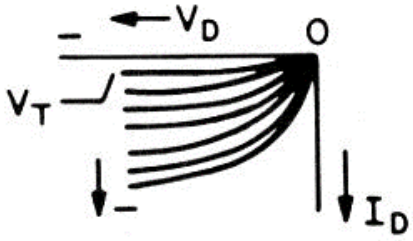
(iii) In practice, a combination of the bandgap and reasonable doping levels limits the maximum built-in potential that can be achieved. [10%]

[Whilst most students appreciated why electrons exist in energy bands rather than discrete levels in a solid, they failed to appreciate that it is the length scale of localisation relative to the electron wavelength in an atom which means that quantum mechanics has to be applied, and this leads to discrete energy levels. The second part of the question

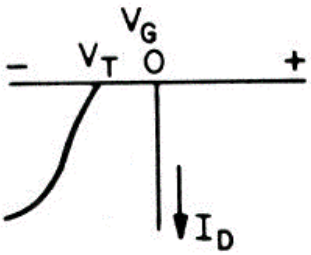
required the derivation of N_C for the conduction band of a material. This was very poorly done, despite being a bookwork derivation from the notes. Many students did not approximate the Fermi function to a Boltzmann distribution, and did not take the bottom of the conduction band to be at an energy E_C, but rather just at 0. Also, many students forgot that the expression for N_C is in the Data Book and therefore, having not derived Nc, were unable to answer the following numerical part of the question.]

Q3 (a) p-channel enhancement MOSFET (see lecture notes):

Output characteristic



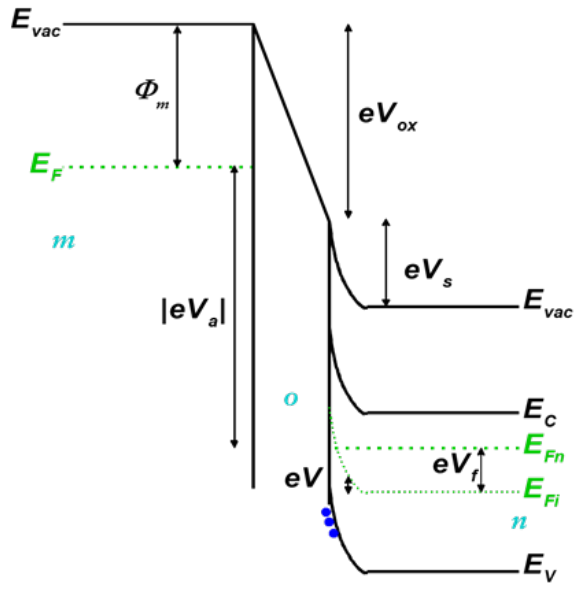
Transfer characteristic



Threshold voltage required to get MOS gate capacitor into strong inversion, ie open a conductive channel between source and drain. At the pinch-off point there is an insufficient voltage drop across the oxide at the drain end to maintain the inversion layer, with the effect that eventually further increase in drain-source voltage has no significant effect anymore on drain-source current, which then remains at saturated level.

[15%]

(b) (i) Band diagram at strong inversion:



[10%]

(ii) Surface potential at onset of strong inversion (see band diagram above):

$$V_s = \frac{2(E_F - E_i)}{e} = 2V_f$$

in general: $n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

assume all dopants are ionised: $N_D = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

thus $E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right)$

and $V_s = 2V_f = \frac{2kT}{e} \ln\left(\frac{N_D}{n_i}\right) = 0.69V$

Hence $w = \sqrt{\frac{2\varepsilon_0 \varepsilon_r V_S}{eN_D}} = 300\ nm$ [25%]

(iii) Lowest capacitance at point of strong inversion, as w is maximum then.

Oxide capacitance per unit area:

$$C_{Ox} = \frac{\varepsilon_0 \varepsilon_{r-SiO2}}{d_{Ox}} = \frac{3.9 \times 8.85 \times 10^{-12}}{20 \times 10^{-9}} \frac{F}{m^2} = 1.7 \times 10^{-3} \frac{F}{m^2}$$

Capacitance per unit area of depletion region:

$$C_{Dep} = \frac{\varepsilon_0 \varepsilon_{r-Si}}{w} = \frac{12 \times 8.85 \times 10^{-12}}{300 \times 10^{-9}} \frac{F}{m^2} = 3.5 \times 10^{-4} \frac{F}{m^2}$$

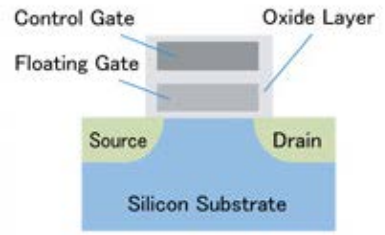
Hence the minimum capacitance of the MOS structure is

$$C_{\min} = \frac{C_{ox} C_{dep}}{C_{ox} + C_{dep}} = 2.9 \times 10^{-4} \frac{F}{m^2}$$

[25%]

(iv) There is still some drain conduction below threshold voltage. This is known as sub-threshold conduction and is due to weak inversion in the channel. The slope of ln I_{DS} as a function of V_{GS} is known as subthreshold slope S. The smaller S, the better the transistor as a switch. [10%]

(c) Flash memory allows non-volatile information storage via the addition of a so-called floating gate. A memory cell resembles a standard MOSFET except with two gates:



《Flash Memory》

'Hot electrons' can be pushed through the oxide from the channel into the floating gate where they will remain. This charge shifts the threshold voltage of the device, and a '1' or a '0' stored can be read via the control gate without affecting the charge stored.

[15%]

[The most common source of lost marks in parts (a), b(iv) and (c) that involved conceptual understanding was the lack of detail and clarity. Quite a few candidates did not draw the transfer characteristics [part(a)] and offered no answer to part (c). Parts (b)(i)-(iii) were generally answered well, in particular the calculation of the maximum depletion width [(b)(ii)]. In part (b)(iii), weaker candidates tended to forget about the oxide capacitance.]

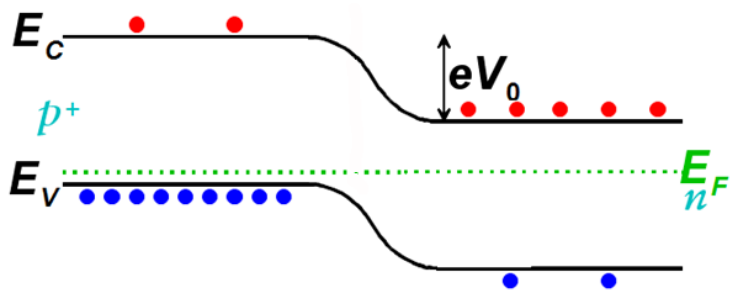
Q4 (a) (i) For n region:

$$E_F - E_i = kT \ln\left(\frac{N_D}{n_i}\right) = 0.28eV$$

For p region:

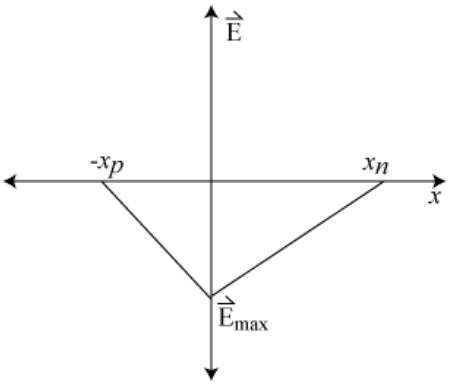
$$E_i - E_F = kT \ln\left(\frac{N_A}{n_i}\right) = 0.46eV$$ [10%]

(ii) Equilibrium band diagram for pn junction



Built-in potential eV_0 = 0.28 eV + 0.46 eV = 0.74 eV [10%]

(iii) Electric field across unbiased junction



Starting from the Poisson equation, and looking at the p-side:

$$\nabla^2 V = \frac{-\rho}{\varepsilon_0 \varepsilon_r} = \frac{eN_A}{\varepsilon_0 \varepsilon_r}$$

For the given variation in only x-direction:

$$\varepsilon_0 \varepsilon_r \frac{\mathrm{d}^2 V}{\mathrm{d}x^2} = eN_A$$

Integration gives

$$\frac{dV}{dx} = \frac{eN_A x}{\varepsilon_0 \varepsilon_r} + C$$

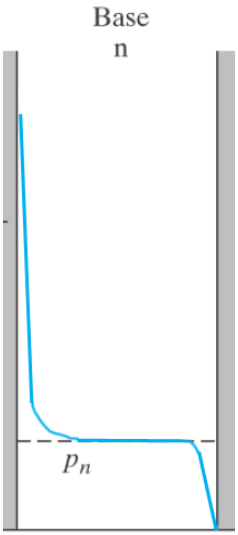
Known boundary condition that E-field is 0 at x=-x_p and hence

$$\frac{dV}{dx} = \frac{eN_A}{\varepsilon_0 \varepsilon_r}\left(x + x_p\right)$$

Maximum E-field at x=0:

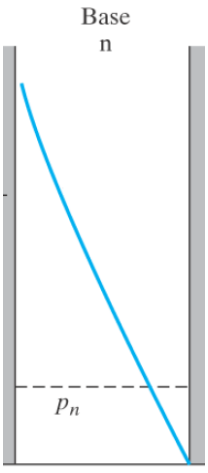
$$\varepsilon\big|_{x=0} = -\frac{eN_A x_p}{\varepsilon_0 \varepsilon_r}$$ [25%]

b) (i) Distribution of holes across the shared n-type for W >> L_h



[10%]

(ii) Distribution of holes across the shared n-type for W << L_h



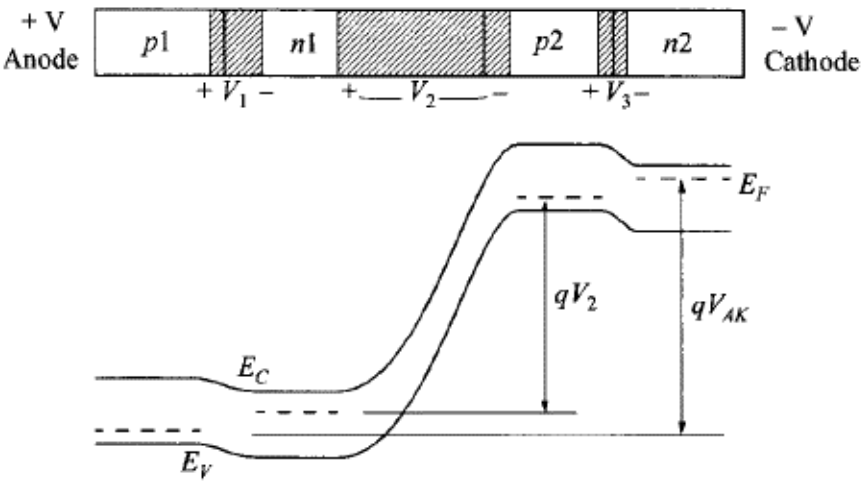
[10%]

(iii) The basic principle of a BJT relies on two coupled pn diodes as in (ii). In the active mode of operation, the emitter junction thereby serves to inject minority carriers into the base region. These excess minority carriers get extracted via the collector junction. It is a small base current that is compensating for recombining carriers which controls the much larger current out of the collector. It takes τ_{base} for the average hole to recombine with an electron in n-base region. The average transit time of an hole $\tau_{transit}$ from emitter to collector is << τ_{base} because W << L_h. For each electron entering the base contact, $\tau_{base}/\tau_{transit}$ holes can pass from emitter to collector while maintaining space charge neutrality. Hence the ratio of collector to base current, expressed as β, can be approximated as

$$\beta \approx \frac{\tau_{base}}{\tau_{transit}}$$

[20%]

(c) Band diagram of Shockley diode in forward-off state:



[15%]

[The majority of candidates could draw an equilibrium band diagram in part (a), but failed to recognise the appropriate relations for the carrier statistics in part (i) and unnecessarily derived the built-in potential in part (iii). Part (b) demanded understanding of minority carrier distributions, and while most candidates could sketch the distribution correctly for W<<L_h, their sketch for W>>L_h would erroneously still assume some coupling. In part (b)(iii) the BJT operation was generally explained well, but in many cases just replicated lecture notes and did not use the steer given to use carrier life and transit times. In part (c) some candidates wrongly sketched 3 non-coupled pn diodes, but candidates in general understood the band diagram concept.]