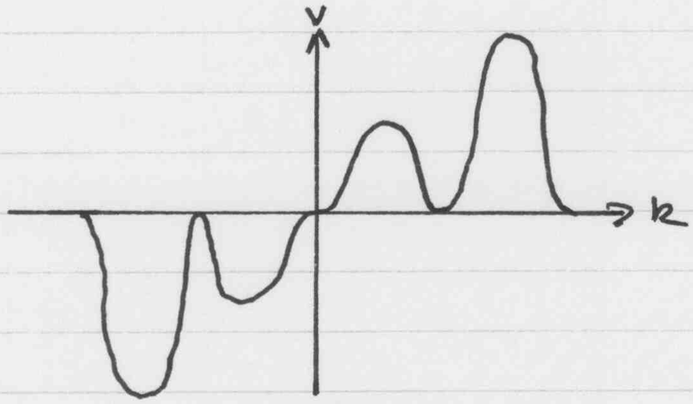
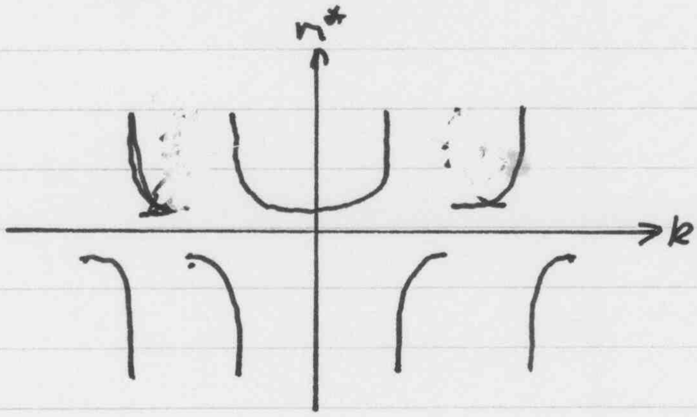
1 a i. When $k = n\pi/a$, the wavefunction of the electron fulfils the Bragg scattering criterion. Therefore, the wave cannot propagate, and so there is a disallowed state.

ii. Close ~~to~~ to the forbidden states, the evolution of $|\psi|^2$ leads to two possibilities: that the electron is close to atoms and so 'sees' the potential well resulting in a lower energy state, or it is between the atoms and 'sees' the potential barrier between atoms resulting in a higher energy state. The electron is no longer sampling an average potential energy of the solid.

b i.



ii.



c i. As the valence band is fully occupied at 0K and only a small number of states become unoccupied by thermal, substitution or electromagnetic effects, it is easier ~~than~~ to consider the effect of removing electrons from the full valence band as being equivalent to adding holes to an empty valence band. We simply need to equate the two effects.

ii. If we remove an electron with a wavenumber k_e from the valence band, then the effect is to change the total wavenumber of all the electrons in this band by $-k_e$. Hence,

$$k_h = -k_e$$

Also, if the energy of the electron before removal was E_e, then the energy of the band is reduced by E_e, so

$$E_h = -E_e$$

Hence

$$v_h = \frac{1}{\hbar}\frac{\partial E_h}{\partial k_h} = \frac{1}{\hbar}\frac{\partial(-E_e)}{\partial(-k_e)} = \frac{1}{\hbar}\frac{\partial E_e}{\partial k_e} = v_e$$

iii. For effective mass,

$$m_h^* = \frac{\hbar^2}{\partial^2 E_h/\partial^2 k_h^2} = \frac{-\hbar^2}{\partial^2 E_e/\partial k_e^2} = -m_e^*$$

iv. The overall shape of the E-k diagram is parabolic. Therefore, the gradient tends to increase with increasing Brillouin Zone number. As the valence band is formed from a lower Brillouin zone, it will have a lower $\partial^2 E/\partial k^2$ and hence a greater effective mass.

2 a i. The Einstein postulate is demonstrated by the photoelectric effect. The maximum energy of electrons emitted from a metal surface in vacuum is linearly increases with photon ~~energy~~ frequency suggesting that $E = h\nu$.

ii. Electron diffraction from a single crystal metal sample is evidence of the de Broglie postulate. When a beam of electrons is incident on such a sample, a peak in the reflected electron signal is measured at an angle corresponding to constructive interference of a wave where $\lambda = h/p$.

b $\frac{\partial\psi}{\partial t} = -j\omega\psi$ $\frac{\partial\psi}{\partial x} = jk\psi$

$$\frac{\partial^2\psi}{\partial x^2} = -k^2\psi$$

∴ Substitution into the SE gives:

$$\frac{-\hbar^2}{2m}.(-k^2\psi) + V\psi = j\hbar.(-j\omega\psi)$$

$$\therefore \frac{\hbar^2 k^2}{2m} + V = \hbar\omega$$

Now. ~~E~~ $\hbar\omega = h\nu = E$ from Einstein. Also

$$\frac{\hbar^2 k^2}{2m} = \frac{\hbar^2.(2\pi)^2}{\lambda^2.2m}$$

$$= \frac{h^2}{\lambda^2}.\frac{1}{2m}$$

From de Broglie

$$\frac{\hbar^2 k^2}{2m} = \frac{p^2}{2m} = \text{Kinetic energy.}$$

Hence

Kinetic Energy + Potential Energy = Total Energy.

c iii We have time independence, so

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} + V\psi = E\psi$$

Outside the well the only valid solution is $\psi = 0$.
Inside the well,

$$-\frac{\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$$

which has the general solution

$$\psi = A\sin(kx) + B\cos(kx)$$

but the boundary conditions mean $\psi(0) = \psi(L) = 0$
so $B = 0$ and

$$\psi = A\sin(n\pi x/L)$$

Also

$$\int_0^L A^2 \sin^2\left(\frac{n\pi x}{L}\right) dx = 1$$

$$\therefore A = \sqrt{2/L}$$

$$\therefore \psi = \sqrt{2/L}\,\sin(n\pi x/L)$$

ii. Now

$$\frac{-\hbar^2}{2m}\frac{\partial^2\psi}{\partial x^2} = E\psi$$

$$\frac{-\hbar^2}{2m}\left\{\frac{-n^2\pi^2\psi}{L^2}\right\} = E\psi$$

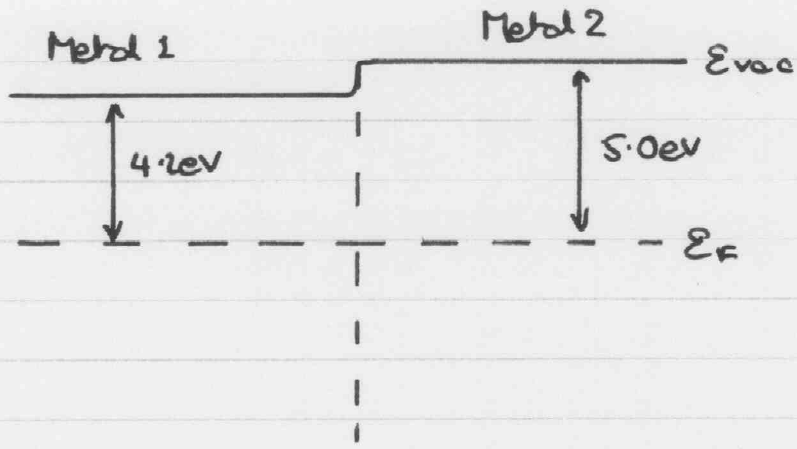
$$\therefore\ E = \frac{n^2\pi^2\hbar^2}{2mL^2}$$

Hence, if $L = 5\times10^{-9}$ m then for the ground state ($n=1$)

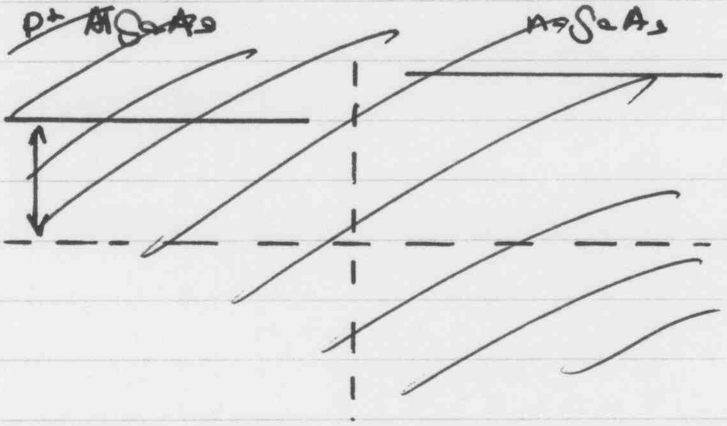
$$E = \frac{\pi^2.(1.055\times10^{-34})^2}{2.9.109\times10^{-31}.(5\times10^{-9})^2}$$

$$\underline{E = 2.41\times10^{-21}\,J = 15\,meV}$$

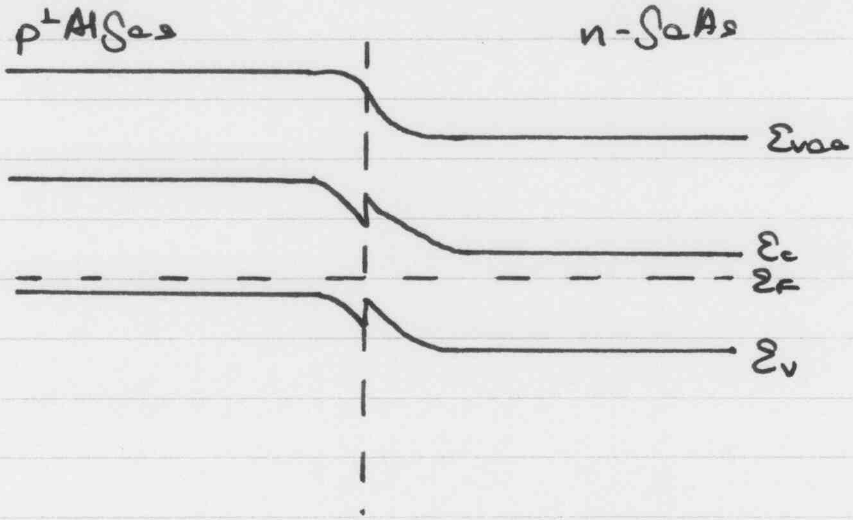
3 a i.



ii.



For p⁺ AlGaAs, the E_F is close to the VB, so $\phi \approx 3.6 + 1.9 \approx 5.5eV$. For the GaAs, ϕ is just a bit bigger than the electron affinity, so ~ 4.2eV



iii. The Sc is p-doped, so

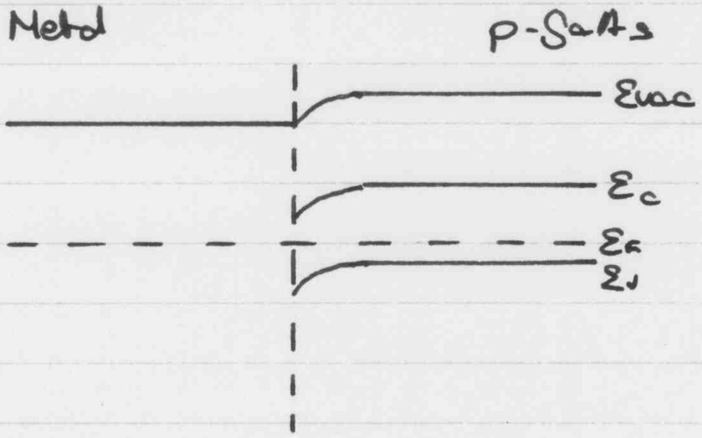
$$p \approx N_A = N_V \exp\left(-\frac{E_F}{kT}\right)$$

$$\therefore E_F = -kT \ln\left(\frac{N_A}{N_V}\right)$$

$$= -0.862\times 10^{-4} \cdot 300 \ln\left(\frac{10^{21}}{6\times 10^{24}}\right)$$

$$E_F = 0.22\,\text{eV}$$

Hence, $\phi = \chi + E_g - E_F = 4.0 + 0.67 - 0.22 = 4.45\,\text{eV}$



b i. $n = N_C \exp\left(\frac{E_F - E_C}{kT}\right) \approx N_D$

$$\therefore -E_F + E_C = -kT \ln\left(\frac{N_D}{N_C}\right)$$

$$E_F = -kT \ln\left(\frac{N_A}{N_V}\right)$$

$$\therefore E_C = -kT\left(\ln\left(\frac{N_D}{N_C}\right) + \ln\left(\frac{N_A}{N_V}\right)\right)$$

$$E_C = -0.862\times 10^{-4} \cdot 300 \left\{ \ln\left(\right.\right.$$

b i $n = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$ $p = N_v \exp\left(\frac{-E_F}{kT}\right)$

$$\therefore np = n_i^2 = N_c N_v \exp\left(\frac{-E_c}{kT}\right)$$

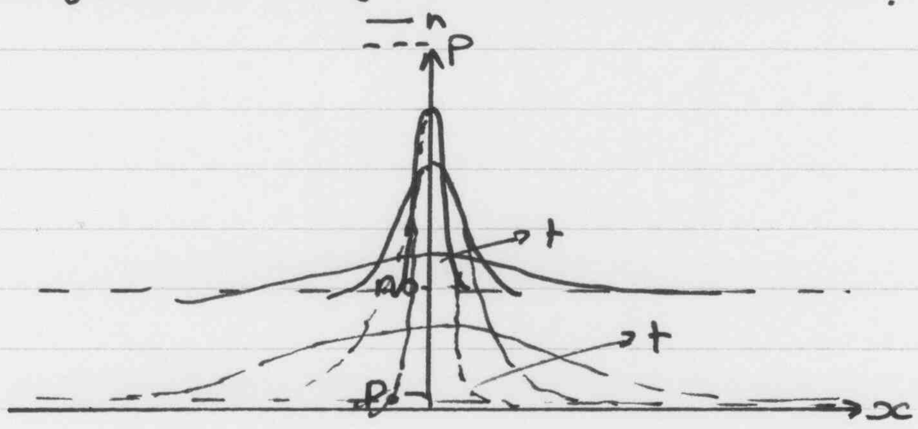
$$\therefore E_c = -kT \ln\left(\frac{n_i^2}{N_c N_v}\right)$$

$$= -0.862\times10^{-4}\cdot 300 \ln\left(\frac{(10^{16})^2}{(10^{25})^2}\right)$$

$$\underline{E_c = 1.07\,\text{eV}}$$

ii. $n \approx N_D = \underline{10^{24}\,m^{-3}}$ $p = \frac{n_i^2}{n} = \frac{(10^{16})^2}{10^{24}} = \underline{10^{8}\,m^{-3}}$

iii. The laser will induce creation of electrons and holes in equal numbers. These will diffuse away from the generation region and recombine.



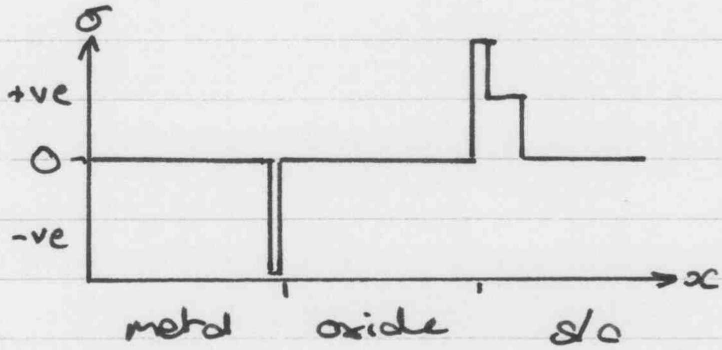
iv. $p \approx N_A - N_D = 10^{24}\,m^{-3}$ $n = \frac{n_i^2}{n} = \underline{10^{8}\,m^{-3}}$

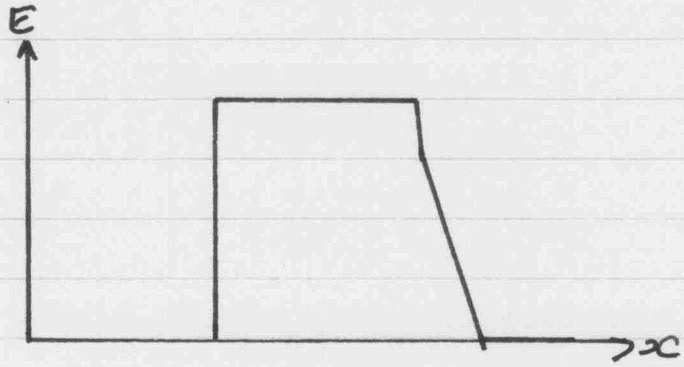
4 a Cmax occurs when there is no depletion region, so

$$C_{max} = \frac{\varepsilon_0 \varepsilon_{rox} A}{t_{ox}}$$

V_T is the point of onset of strong inversion when the opposite carriers to the semiconductor bulk build up on the semiconductor at the dielectric interface. V_T is -ve, so +ve charge is building up so this is an n-type semiconductor.

b





$$\sigma = \frac{C}{A} . V_T$$

$E_T = V_T/t_{ox}$

$$\therefore \sigma = \frac{\varepsilon_0 \varepsilon_{rox}}{t_{ox}} . E_T . t_{ox}$$

$$E_T = \frac{\varepsilon_0 \varepsilon_{rox}}{\sigma}$$

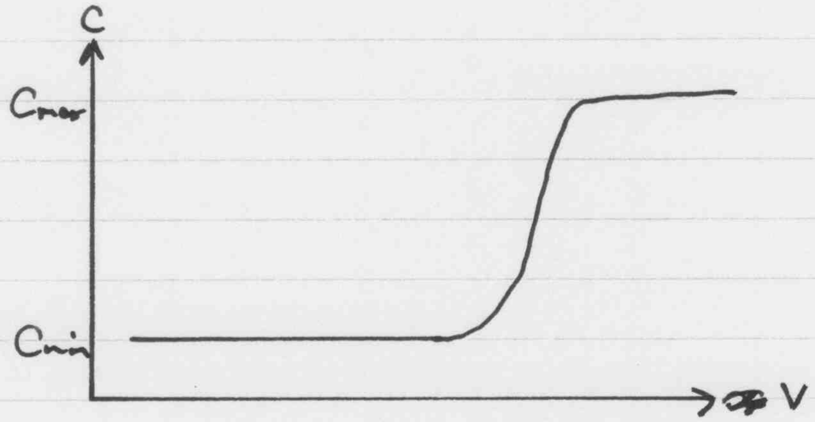
c $\tau_{inv} = \dfrac{e n_o t_{inv}}{p_o e v_s}$

$n_o = N_D = 5\times10^{22}\,m^{-3}$ $p_o = \dfrac{n_i^2}{n_o} = \dfrac{(1.5\times10^{16})^2}{5\times10^{22}}$
$= 4.5\times10^{9}\,m^{-3}$

Take $t_{inv} \approx 5ns$ so

$\tau_{inv} = \dfrac{1.602\times10^{-19}\cdot 5\times10^{22}\cdot 5\times10^{-9}}{4.5\times10^{9}\cdot 1.602\times10^{-19}\cdot 10^{5}} = \underline{0.56\,s}$

d



e ~~$Q_T = C V_T$~~ $\sigma_T = \dfrac{C}{A}\cdot V_T$

$\sigma_T = \dfrac{\varepsilon_o\varepsilon_{rox}}{t_{ox}}\cdot V_T$

$\therefore V_T = \dfrac{\sigma_T t_{ox}}{\varepsilon_o \varepsilon_{rox}}$

$\therefore V_T \propto 1/\varepsilon_{orox} \Rightarrow \varepsilon_{rox}$ can be used to control V_T.

f Subthreshold slope will increase.