3B5 Exam 2019

Question 1

a) $n_i = 2.1 \times 10^{12}\ m^{-3}$

$N_V = 9.0 \times 10^{24}\ m^{-3}$

$N_C = 4.5 \times 10^{23}\ m^{-3}$

$$np = n_i^2$$

$$n_i^2 = N_c N_v \exp\left(\frac{-E_g}{kT}\right)$$

$$\therefore\ E_g = -kT \ln\left(\frac{n_i^2}{N_c N_v}\right)$$

$$= -0.0258 \ln\left(\frac{(2.1\times10^{12})^2}{9.0\times10^{24} \times 4.5\times10^{23}}\right)$$

$$= 1.4\ eV$$

b) Force on carriers due to magnetic field (Lorentz):

$$\underset{\sim}{F} = e\underset{\sim}{v} \times \underset{\sim}{B}$$

Also $J = Nev$

$$\therefore\ |F| = \frac{J}{N} B$$

Force on carriers due to established Hall voltage:

$$F = eE_H$$

$$= e\frac{V_H}{w}$$

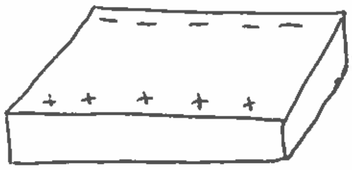
At equilibrium the two forces are equal:

$$\frac{eV_H}{w} = \frac{JB}{N}$$

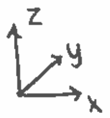
$$\therefore\ V_H = \frac{JBw}{Ne}$$

Assumption: That one type of carrier dominates.

c) (i)



V_H is positive



The carriers are holes. The samples is p-type. Why? Because V_H is positive.

The force on a carrier, $\underset{\sim}{F}_y = q\,\underset{\sim}{v}_x \times \underset{\sim}{B}_z$

Specifically, if the carriers are holes, they experience a force in the negative y-direction. As the holes are deflected towards this surface, they set up an electric field, which builds up until equilibrium is reached. The electric field yields a positive Hall voltage if holes are the majority carriers.

If the carriers were electrons, the Hall voltage would be negative.

(ii)

$$p = \frac{J B w}{V_H e} \quad \text{(majority carrier density)}$$

$$= \frac{100 \times 0.1 \times 0.01}{25 \times 10^{-6} \times 1.602 \times 10^{-19}}$$

$$= 2.5 \times 10^{22} \text{ m}^{-3}$$

Assuming complete ionisation of acceptors,

$$N_A = 2.5 \times 10^{22} \text{ m}^{-3}$$

$$np = n_i^2$$

$$\therefore n = \frac{n_i^2}{p} = \frac{(2.1 \times 10^{12})^2}{2.5 \times 10^{22}} = 176 \text{ m}^{-3}$$

(minority carrier density)

(iii) $$p = N_v \exp\left(\frac{-(E_F - E_v)}{kT}\right)$$

$$\therefore \quad E_F - E_v = -kT \ln\left(\frac{p}{N_v}\right)$$

$$= -0.0258 \ln\left(\frac{2.5 \times 10^{22}}{9.0 \times 10^{24}}\right)$$

$$= 0.15 \text{ eV}$$

$$= 150 \text{ meV}$$

i.e. Fermi level is 150 meV above the valence band edge.

(iv) Laser pulse generates hole density of $1 \times 10^{23} \text{ m}^{-3}$

An electron density of $1 \times 10^{23} \text{m}^{-3}$ must also be generated.

$$\sigma = n e \mu_n + p e \mu_p$$

$$= (1 \times 10^{23}) \times (1.602 \times 10^{-19}) \times 0.9$$

$$+ (1 \times 10^{23} + 2.5 \times 10^{22})(1.602 \times 10^{-19}) \times 0.04$$

$$= 1.52 \times 10^{4} \ \Omega^{-1} \text{m}^{-1}$$

(v) 1600 nm corresponds to a photon energy of 0.78 eV, which is less than the bandgap of 1.42 eV. Therefore, 1600 nm photoexcitation does not change the concentration of holes & electrons.

$$\sigma = p e \mu_p$$

$$= (2.5 \times 10^{22})(1.602 \times 10^{-19}) \times 0.04$$

$$= 160 \ \Omega^{-1} \text{m}^{-1}$$

2)a) i) From Data book

$$p = N_V \exp\left(\frac{-E_{Fp}}{kT}\right) \qquad \text{where } E_V \text{ is defined as } 0eV$$

$$n = N_C \exp\left(\frac{E_{F_n} - E_F}{kT}\right)$$

assuming all dopants are ionised

$$n = N_D$$
$$p = N_A$$

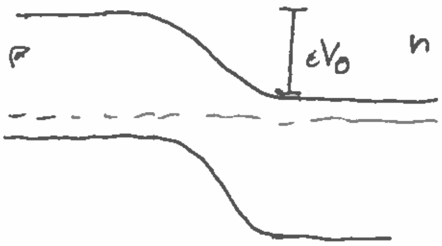
Hence
$$E_{Fp} = kT \ln\left(\frac{N_V}{N_A}\right)$$
$$E_{FN} = E_C - kT \ln\left(\frac{N_C}{N_D}\right)$$

$$\rightarrow \quad eV_0 = E_{FN} - E_{Fp} = E_c - kT\left[\ln\left(\frac{N_c}{N_D}\right) + \ln\left(\frac{N_V}{N_A}\right)\right]$$

from law of mass action

$$n_i^2 = N_c N_V \exp\left(\frac{-E_c}{kT}\right) \longrightarrow \ln\left(N_c N_V\right) = \ln\left(n_i^2\right) + \frac{E_c}{kT}$$

$$\longrightarrow \quad V_0 = \frac{kT}{e} \ln\left(\frac{N_A N_D}{n_i^2}\right)$$



(ii) V_0 can be measured via CV measurements (see 385 lab) or photothermal measurements.

b) for n-side transport dominated by electrons:

$$\frac{1}{S} = e N_D \mu_e \qquad \text{(assuming full dopant ionisation)}$$

$$\longrightarrow \quad N_D = \frac{1}{S e \mu_e} = 2.4 \cdot 10^{23}\ m^{-3}$$

analogue for p-side $\longrightarrow$ $N_A = 3.1 \cdot 10^{22}\ m^{-3}$

2)b) $$V_0 = \frac{kT}{e} \ln \frac{N_A N_D}{n_i^2} = 0.8 \text{ V}$$

for N_D increased to $2.4 \cdot 10^{25} \frac{1}{m^3}$

$V_0' = 0.92$ V → 15 % change

c) i) Take n-region :

$$\varepsilon_0 \varepsilon_r \frac{d^2V}{dx^2} = -eN_D$$

→ Integration gives

$$\frac{dV}{dx} = -\frac{eN_D x}{\varepsilon_0 \varepsilon_r} + C$$

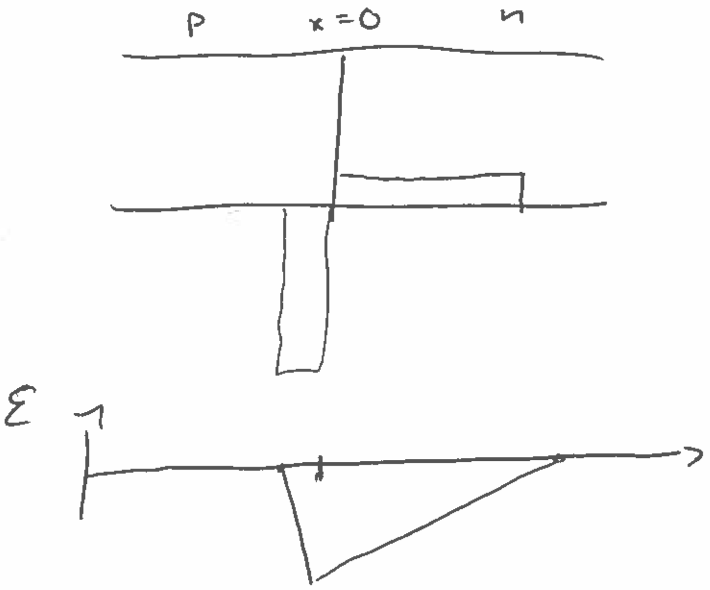
Assume no field outside depletion region, ie. $\mathcal{E} = 0$ at $x = w$

$$\rightarrow \quad \frac{dV}{dx} = -\frac{eN_D(x-w)}{\varepsilon_0 \varepsilon_r}$$

→ $\mathcal{E}$ - field at peak value when $x = 0$ (at junction)

$$\mathcal{E}\Big|_{x=0} = -\frac{eN_D w}{\varepsilon_0 \varepsilon_r}$$

ii)



Note: one-sided junction

$\mathcal{E}_{peak} = 45 \frac{MV}{m}$

beyond breakdown field

(w assumed too large)

2)d) Junction capacitance relates to depletion region

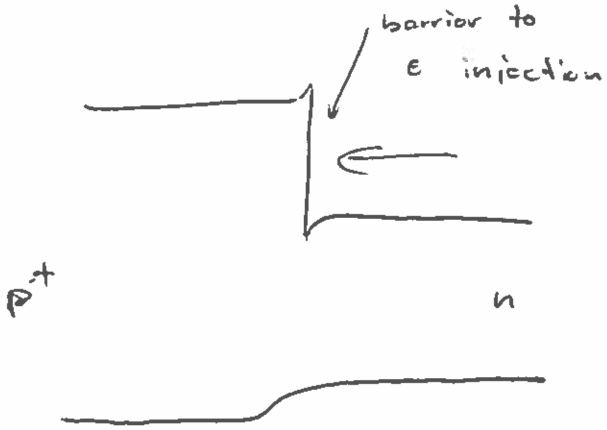
$$\rightarrow C_{Junction} = \frac{\varepsilon_0 \varepsilon_r}{w}$$

Charge storage capacitance relates to the fact that p-n junction is bipolar device and relies on injection of minority charge carriers across the junction.

Charge storage C dominates under forward bias.

e) In order to prevent electron injection into p^+ region a heterojunction can be used where a semiconductor like GaAlAs with a larger band gap is used for p^+ compared to GaAs based n region.

Based on band offsets, band diagram will look like:



(under forward bias)

Use GaAlAs / GaAs as this is one of the few systems where E_{gap} can be varied and still maintaining good epitaxial fit / interface.

3 (a.)

- The wavefunction Ψ describes everything that can be known about the electron, including its

  total energy
  kinetic energy
  $$\left.\begin{array}{l}\text{momentum}\\ \text{wavelength}\end{array}\right\}\text{ related by de Broglie relationship}$$

- The wavefunction satisfies the Schrödinger equation at all times.

- The wavefunction is complex, single-valued and continuous, as are $\frac{\partial\Psi}{\partial x}$, $\frac{\partial\Psi}{\partial y}$ and $\frac{\partial\Psi}{\partial z}$.

- The probability of finding the particle is
  $$P = |\Psi|^2\, dx\, dy\, dz.$$

(b.) To derive α and E, substitute Ψ and V into the Schrödinger equation:

$$\underbrace{-\frac{\hbar^2}{2m}\nabla^2\Psi}_{\text{kinetic energy}} + \underbrace{V\Psi}_{\text{potential energy}} = \underbrace{E\Psi}_{\text{total energy}}$$

let $A = \frac{1}{\sqrt{\pi}}\left(\frac{1}{a}\right)^{3/2}$

Potential energy term: $V\Psi = \frac{-e^2}{4\pi\varepsilon_0 r}\Psi$

Kinetic energy term: $-\frac{\hbar^2}{2m}\nabla^2\Psi = \frac{-\hbar^2 A}{2m r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\Psi}{\partial r}\right)$

$$= \frac{-\hbar A}{2mr^2}\frac{\partial}{\partial r}\left(r^2\left(-\frac{1}{a}\exp\left(-\frac{r}{a}\right)\right)\right)$$

$$= +\frac{\hbar A}{2mr^2 a}\frac{\partial}{\partial r}\left(r^2\exp\left(\frac{-r}{a}\right)\right)$$

$$= +\frac{\hbar}{2mr^2 a}\left(2r\exp\left(\frac{-r}{a}\right) - \frac{r^2}{a}\exp\left(\frac{-r}{a}\right)\right)$$

$$= +\frac{\hbar A}{2ma}\exp\left(-\frac{r}{a}\right)\left(\frac{2}{r} - \frac{1}{a}\right)$$

$$= -\frac{\hbar}{2ma}\left(\frac{1}{a} - \frac{2}{r}\right)\Psi$$

Kinetic + Potential = Total

$$\therefore -\frac{\hbar^2}{2ma}\left(\frac{1}{a}-\frac{2}{r}\right) - \frac{e^2}{4\pi\varepsilon_0 r} = E$$

Consider the terms without r - dependency

$$-\frac{\hbar^2}{2m}\frac{1}{a^2} = E \qquad ①$$

Consider the terms with r - dependency

$$+\frac{\hbar^2}{2ma}\left(\frac{2}{r}\right) - \frac{e^2}{4\pi\varepsilon_0 r} = 0$$

$$\therefore \quad a = \frac{4\pi\varepsilon_0\hbar^2}{me^2} \qquad ②$$

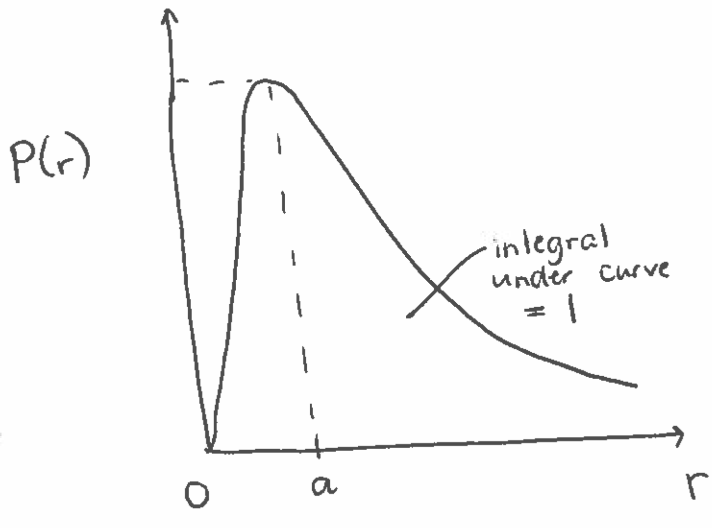
Substitute ② into ①

$$E = -\frac{\hbar^2}{2m}\left(\frac{me^2}{4\pi\varepsilon_0\hbar^2}\right)^2$$

$$= \frac{-me^4}{32\pi\varepsilon_0^2\hbar^2}$$

$$= \frac{-me^4}{8\varepsilon_0^2 h^2}$$

Ⓒ



Probability

$$= \int_{r_1}^{r_2} 4\pi r^2 |\Psi|^2 \, dr$$

Maximum probability when $\frac{dP}{dr} = 0$ (i.e. turning point)

$$\frac{dP}{dr} = A \frac{d}{dr}\left(4\pi r^2 \exp\left(\frac{-2r}{a}\right)\right)$$

$$= 4\pi A \left[2r \exp\left(\frac{-2r}{a}\right) - \frac{2r^2}{a} \exp\left(\frac{-r}{a}\right)\right]$$

$$= 0$$

$$\therefore \quad 2r = \frac{2r^2}{a}$$

$$\therefore \quad r = a$$ ← maximum occurs here.

(d.) Kinetic energy $= \dfrac{\hbar^2}{2ma^2}$

$$= \frac{p^2}{2m}$$

$$\therefore \quad p^2 = \frac{\hbar^2}{a^2}$$

$$\therefore \quad p = \frac{\hbar}{a}$$

Now $\lambda = \dfrac{h}{p}$

$$= \frac{h}{\hbar/a}$$

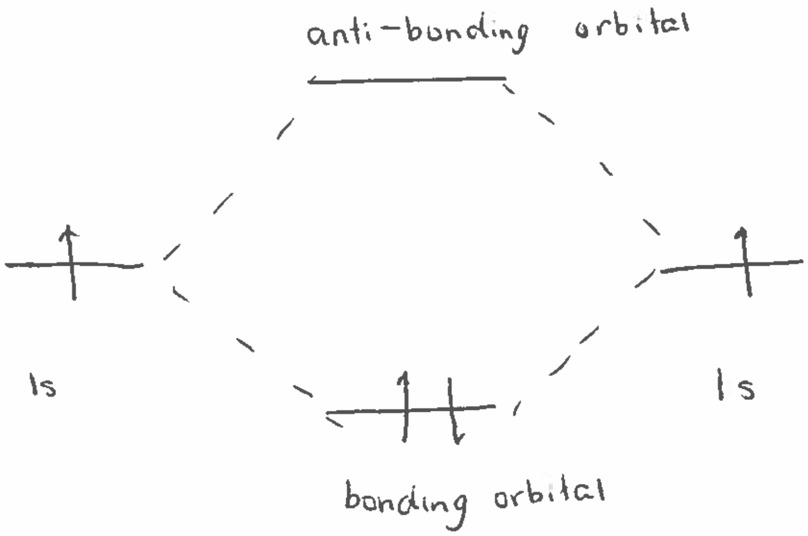
$$= 2\pi a$$

The wavelength is $2\pi a$, that is, equal to the circumference of an orbit with $r = a$. A standing wave is formed.

ⓔ The two atoms, if spatially separated, can host one electron each in the same ground state, with the same spin.

e.g. A spin up electron can exist in the 1s state of hydrogen atom A. Another spin up electron can exist in the 1s state of hydrogen atom B.

If the two hydrogen atoms A and B are brought closer together, their 1s wavefunctions overlap. The Pauli exclusion principle is invoked, so that the two electrons cannot exist in the same 1s state. The two wavefunctions become perturbed and their energy levels split to form two distinct energy levels : bonding and anti-bonding.



The two electrons form a bond and lie in the "bonding molecular orbital".

4) a) (i) The Si is p-doped, and assuming full acceptor ionisation:
(B is group III)

$$p = N_A = N_V \exp\left(\frac{-E_F}{kT}\right)$$

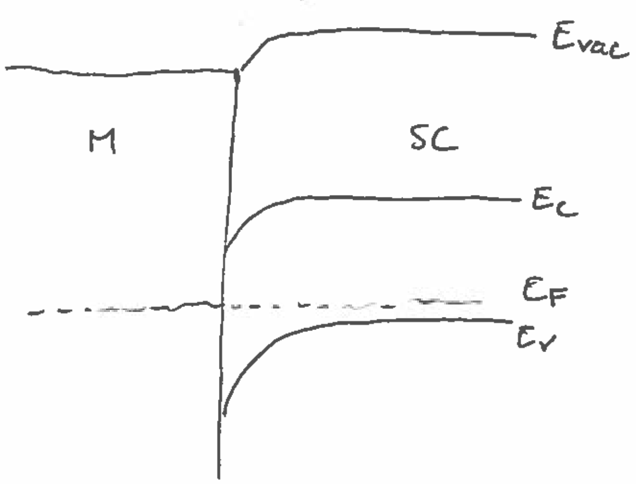
assuming E_V is at 0 eV.

Hence $$E_F = -kT \ln\left(\frac{N_A}{N_V}\right)$$

$$= -k\,300K \ln\left(\frac{10^{24}}{1.04 \cdot 10^{25}}\right) = 0.06 \text{ eV}$$

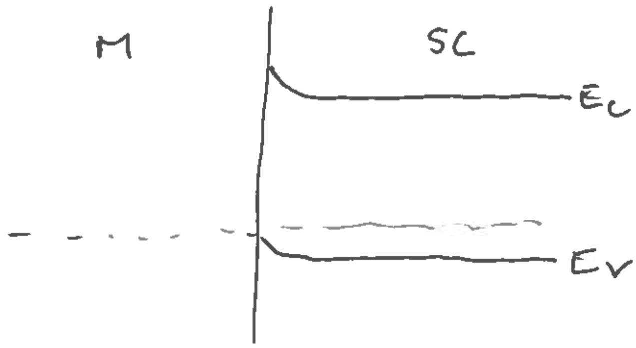
Hence the workfunction $e\phi_{sc} = \chi_e + E_g - E_F = 5.11$ eV

$\phi_{sc} > \phi_M$ which means its a Schottky contact.



(ii) For a Ω-contact $\phi_M > \phi_{SC}$, so

$\Delta\phi_M$ of $> +0.51$ eV required. i.e. ϕ_M must be 0.51 eV higher



4) b) i) The Si is now n-doped, and assuming full donor ionisation

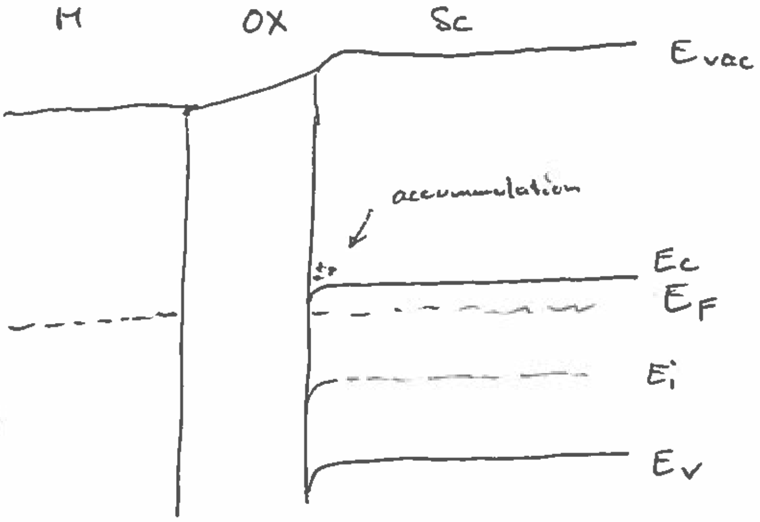
$$n = N_D = N_c \exp\left(\frac{E_F - E_c}{kT}\right)$$

(assuming RT operation)

Hence $E_F - E_c = kT \ln \frac{N_D}{N_C} = -0.15 \text{ eV}$

Hence $e\phi_{sc} = \chi + (E_C - E_F) = (4.05 + 0.15)\text{eV} = 4.2 \text{ eV}$

$e\phi_{sc} > e\phi_M$ → leads to small accumulation of electrons:



ii) The flat band voltage is $V_{FB} = \dfrac{e\phi_M - e\phi_{sc}}{e} = -0.2 \text{ V}$

iii) Surface potential at onset of strong inversion

$$V_S = \frac{2(E_F - E_i)}{e} = \frac{2kT}{e} \ln\left(\frac{N_D}{n_i}\right)$$

noting that $n = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

assuming full dopant ionisation: $N_D = n_i \exp\left(\frac{E_F - E_i}{kT}\right)$

4) b) iii) Hence $V_S = 0.81\ V$ (at RT)

Charge per unit area in depletion region at strong inversion is

$$Q_D = eN_D w_{max} = 16 \cdot 10^{-3} \frac{C}{m^2}$$

where $w_{max} = 100\ nm$

Oxide capacitance per unit area is

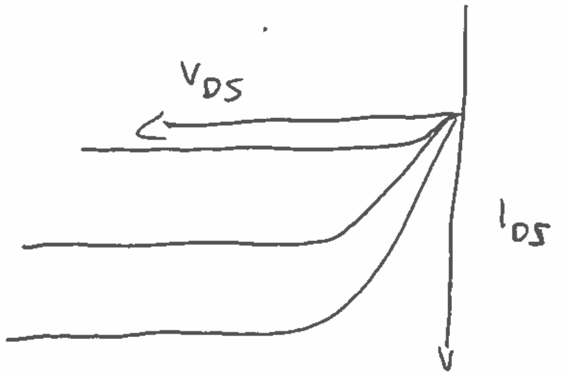
$$C_{ox} = \frac{\varepsilon_0 \varepsilon_r}{d_{ox}} = 2.3 \cdot 10^{-3} \frac{F}{m^2}$$

Hence threshold voltage V_T

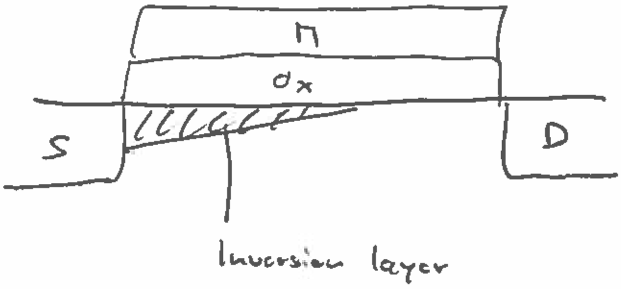
$$V_T = \overbrace{-\frac{Q_D}{C_{ox}}}^{\text{Drop across oxide}} - \overbrace{V_S}^{\text{band bending}} + \overbrace{V_{FB}}^{\text{correction for } \Delta\phi} = (-0.69 - 0.81 - 0.2)$$

$$= -1.7\ V$$

c) i) Channel is n-type hence its p-channel enhancement MOSFET



Beyond pinch-off



4) c) ii) Floating gate structure :