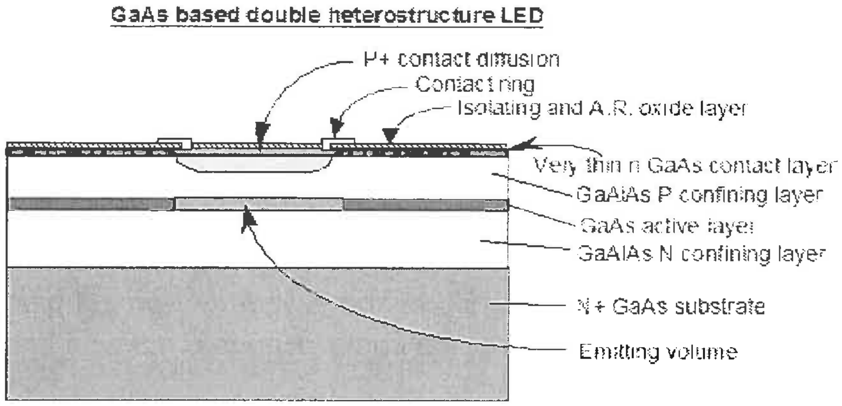
Answers to Examination Questions, Photonics Technology 2015

Q.1 (a) This question should involve a detailed qualitative answer explaining how spontaneous emission is generated at a pn junction diode, commenting on how the output power varies with drive current, how the wavelength is set by the bandgap, and how the response is limited by the recombination properties.

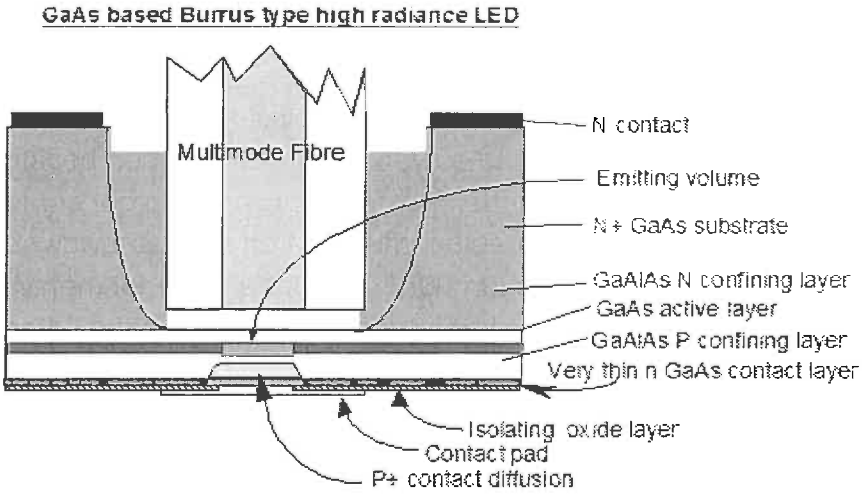
It should then seek to describe the different surface emitting LED structures, illustrations of which are below:

Surface Emitting LED Structures



This is a simple LED structure where the pn junction from which the light is emitted is placed near the top of the device so that light, once generated can leave device out of the top quickly. However, light that which after generation travels to the sides or downwards may be lost. Almost identical structures are used in the InP/GainAsP system for wavelengths of 1300 nm and 1500 nm. In this case the substrate and both confining layers are InP, and the active layer and thin top contact layer are GainAsP. In fact, the thin contact layer is often composed of GainAs, since this is easy to grow and, although this will absorb the generated light, its thickness is very small, so the total light absorbed is small.

GaAs based Burrus type high radiance LED



The Burrus diode has much of the substrate etched away: this allows high coupling into a multimode fibre. Sometimes a spherical micro-lens is introduced as well. In addition, the heat generation is close to the p surface which can be bonded directly to a heatsink and the contact metal also reflects some light back upwards into the fibre. Similar devices are made using the InP/GainAsP materials system at longer wavelengths.

(b) A good answer should discuss the importance of direct bandgap materials in light emitting diodes and, as the wavelength is determined to a large extent by the material bandgap, how this has proved to be challenging at some wavelengths. In addition some discussion should concern the use of compound alloys to allow materials to be selected to generate light at a specifically targeted wavelength. InGaN, GaAsP and GaAlAs materials should be discussed for operation at wavelengths of 400 nm, 650 nm and 800 nm respectively.

(c)

(i) The internal quantum efficiency may be written as $\eta_{int} = (1/\tau_{rr})/((1/\tau_{rr}) + (1/\tau_{nr})) = \tau_{nr}/(\tau_{rr}+\tau_{nr})$

However $\eta_{tot} = \eta_{int}.\eta_{ext}$ => $\eta_{ext} = [\tau_{nr}/((\tau_{rr}+\tau_{nr})\ \eta_{tot})]^{-1}$ = <u>0.075 or 7.5 %</u>

(ii) Considering the circuit, assume that R_s is the series resistance, R_{int} is the electrical supply internal resistance, R_{LED} is the SELED resistance, I is the current, V is the voltage of the supply and V_f is the forward voltage of the SELED. As a result

$R_s = (V-V_f)/I - (R_{int}+R_{LED})$ (1)

However $I = eP\lambda/(hc\eta_{tot}) = (1.6\times10^{-19} \times 2\times10^{-3} \times 650\times10^{-9})/(6.625\times10^{-34} \times 3\times10^{8} \times 0.05)$

$= 20.9$ mA

But $V_f = hc/(e\lambda) = (6.625 \times 10^{-34} \times 3 \times 10^{8})/(1.6 \times 10^{-19} \times 650 \times 10^{-9}) = 1.91$ V

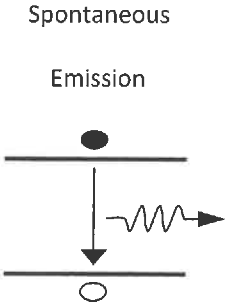
⇨ Substituting V_f into equation (a), <u>$R_s = 49\ \Omega$</u>

(iii) Consider the carrier lifetime τ_s; $1/\tau_s = 1/\tau_{rr} + 1/\tau_{nr}$

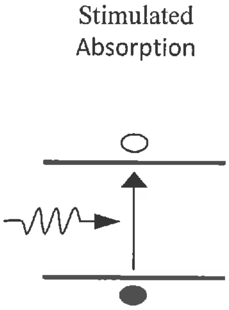
⇨ $\tau_{nr} = [1/\tau_s - 1/\tau_{rr}]^{-1} = 6$ ns
⇨ $\eta_{int} = 0.75$
⇨ $\eta_{tot} = 0.056$

Therefore, using the formulae in section (ii), <u>$R_s = 55\ \Omega$</u>

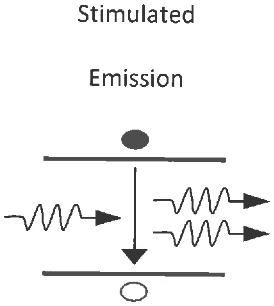
Q.2 (a) A good answer should provide descriptions of the three major types of electron/photon interactions in materials.



LED action



Photodiode Action



Laser action

- **Spontaneous Emission**
  An electron in a high energy level falls, losing energy which is emitted as a photon – the basis of operation of a light emitting diode

- **Stimulated Absorption**
  An incident photon is absorbed in a material, causing the excitation of an electron to a higher energy level – the basis of operation of a photodiode

- **Stimulated Emission**
  A photon, incident upon an electron in a higher energy level, causes the electron to fall to a lower level thus generating a second photon. This is, therefore, an amplifying action. Two photons are generated from one and, in turn, they can cause the generation of two further photons. Using this method, high optical powers can be generated and this operation is the basis of lasing action. The generated photon has the same frequency and phase as the incident photon and, therefore, very pure monochromatic and coherent light can be generated.

In order for these processes to allow lasing action, two main conditions must be achieved, (i) stimulated amplification must be stronger than absorption so that any optical signal is rapidly amplified in power, and (ii) some form of optical feedback must be provided so that the lasing light generated can in part be fed back so that stimulated amplification can continue to occur, thus causing sustained stimulated emission and hence lasing output.

To achieve continual nett stimulated amplification, there must be a larger number of free carriers in the upper level than the lower level so that a photon is more likely to stimulate the emission of another photon rather than be absorbed. This therefore requires a “population inversion” to be created as normally carriers gather at the lowest possible energy levels.

As a result of these requirements, in a typical laser system, much more care must be taken to ensure that the light does not scatter or "leak" out of the lasing region. It is also important to ensure that an optical cavity is bounded by reflectors, so that a lasing filament is formed which oscillates back and forth within the cavity, and that the generated light is confined to cause further stimulated emission. By using partial reflectors, some of the light is emitted from the cavity as the output from the laser.

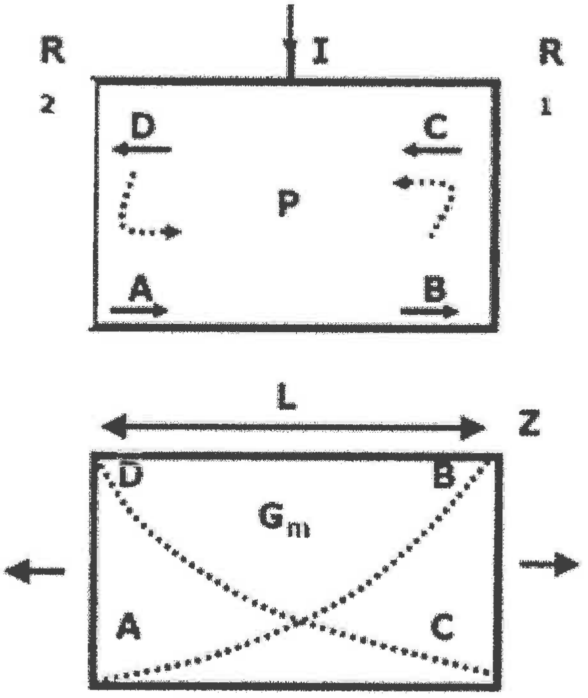
A good answer should describe how the formation of such a cavity, however, has a major effect on the form of optical spectrum generated, namely creating modes with spacings in wavelength of $\lambda^2/(2nL)$. The use of other forms of feedback causing different types of emission may also be discussed.

(b) Consider the cavity as in the following diagram:

$$dP/dz = G_m P$$

$$\Rightarrow$$

$$P(Z) = P(o)\exp(G_m z)$$



The photon lifetime of the laser cavity can be readily determined by considering the amplification of laser light as it propagates along the laser cavity. Assume that stimulated emission encounters a gain per unit length (due to stimulated amplification), G, and a loss per unit length due to scattering, α, as it passes along the laser. The gain G in practice creates extra photons to compensate for those photons lost as the signal travels over a distance of unit length.

Therefore the stimulated light A starting at one facet will be incident on the opposite facet with an optical power

$$B = \exp\{(G - \alpha)L\}\ A$$

At that point part of the signal is reflected with a coefficient R_1 and the signal then passes back amplified by 1 the same amount as above and again reflected by the initial facet. Lasing action will occur in the net round trip gain of the signal is unity i.e. if

$$A.\ \exp\{(G - \alpha)L\}.R_1\exp\{(G - \alpha)L\}.R_2 = A$$

$$=> \quad G = \alpha + (1/2L)\ln(1/(R_1R_2)) \qquad \textit{[N.B. Gain/unit length]}$$

This value of G is equal to the ratio of photons lost as the signal travels a unit length. Hence the proportion of photons lost per unit time is simply the gain G times the speed of light in the laser material, v_g (i.e. gain/length x length/time). As a result the average time for which one photon will remain in the cavity is given by

$$\tau_p = 1/Gv_g \ = 1/\{v_g\{\alpha + (1/2L)\ln(1/R_1R_2)\}\}$$

If $R_1 = R_2 = R$ then,

$$\tau_p = = 1/\{v_g\{\alpha + (1/L)\ln(1/R)\}\}$$

(c) The differential quantum efficiency of the laser may be determined noting that the light output from the laser is the equivalent to a loss per unit length of:

$$(1/2L)\ln(1/(R_1R_2)).$$

As a result the proportion of photons leaving the cavity per unit time is given by

$$(1/2L)\ln(1/(R_1R_2)).v_g$$

Whereas the proportion leaving the cavity in total is:

$$\{v_g\{\alpha + (1/L)\ln(1/R)\}\}$$

Hence above threshold the differential efficiency, η_D, is the proportion of photons leaving the cavity through the facets over the total number of photons, i.e.

$$\eta_D = \frac{\ln(1/(R_1R_2))/(2L)}{\alpha + \ln(1/(R_1R_2))/(2L)}$$

(d) Consider the optical power, P, generated at a current δI above threshold.

$P = \eta_D \cdot \delta I \cdot (hc/(e\lambda)) \Rightarrow \eta_D = P\lambda e/(hc.\delta I) = 0.68$

But $\eta_D = \dfrac{\ln(1/(R_1 R_2))/(2L)}{\alpha + \ln(1/(R_1 R_2))/(2L)}$

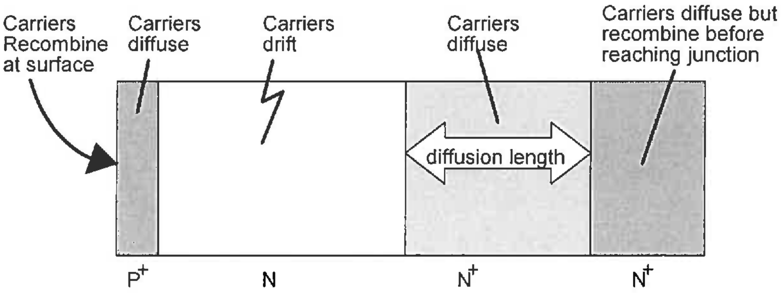
⇨ $\alpha = (1 - \eta_D).\ln(1/R)/(L\,\eta_D) = \underline{11\ \text{cm}^{-1}}$

3. a) E_g (in eV) < $hc/e\lambda = 0.97$ eV

Suitable material is InGaAs, though Ge would be acceptable (but have higher dark current)

b) **Key Regions in a Photodiode**

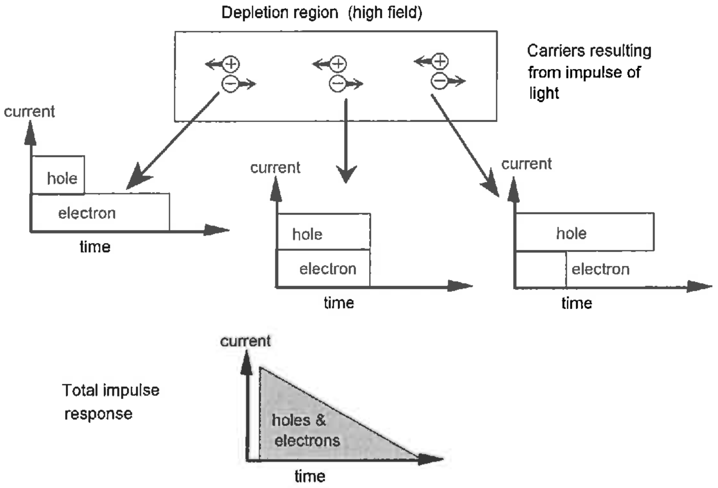
Clearly the depletion region has a high field in it and so any photogenerated carriers are quickly accelerated to the saturated drift velocity in this region. In the P^+ and the N^+ regions, the field is low, so the dominant transport process is diffusion. The P^+ is relatively thin, so this is not a great problem. However, the N^+ is wide and so this can severly limit the transport in the photodiode.



The overall time response can be broken down into two separate regions of operation, the response of the depletion region and the response of the diffusion region. These responses are shown schematically below.

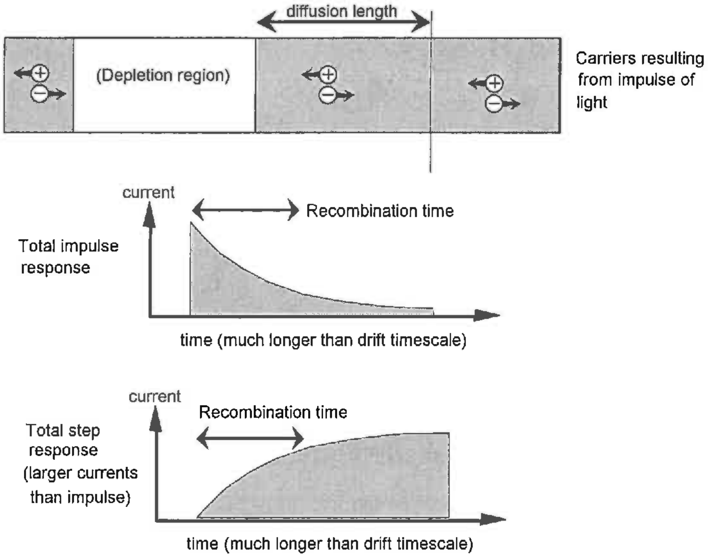
Time response: Depletion Region

If an impulse of light is input into the depletion region, then the positive and negative photo-generated charges immediately begin to separate under the action of the high electric field. Depending on how far from the edges of the region the carriers are, they will generate a high, then linearly reducing current against time. The timescale is quite short – typically of the order of ~100ps.

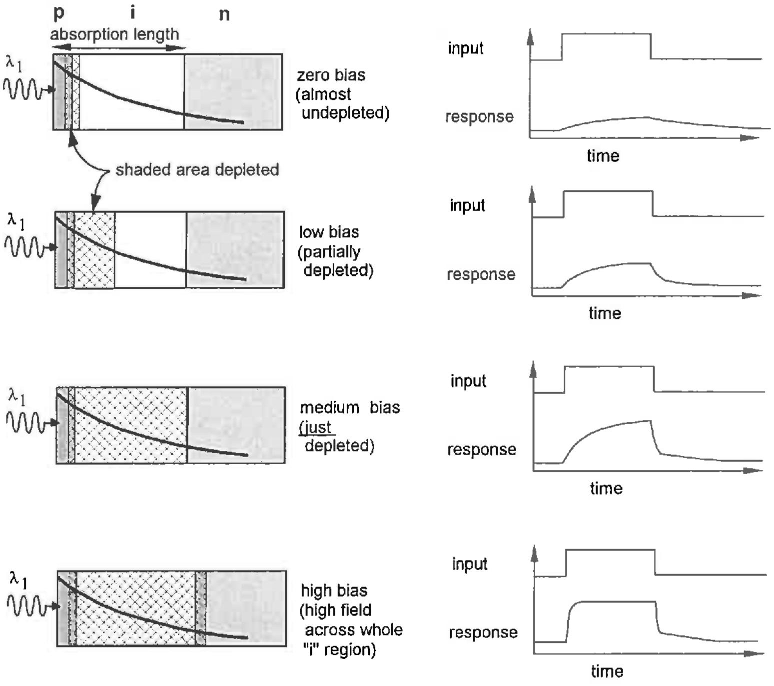


Time response: Diffusion regions

Carriers generated in the diffusion region obey the diffusion equations described above. Solutions to these equations show exponential like behaviour though the time constants are much longer than those for carrier drift in the depletion region, with 10-100ns time constants being common. Since the carrier lifetime is much shorter, then many of the carriers photogenerated in the diffusion region recombine before forming part of the photocurrent (hence reducing the overall responsivity).

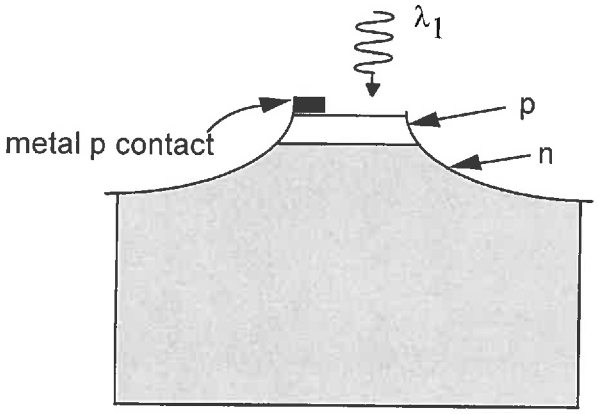


change of response with bias



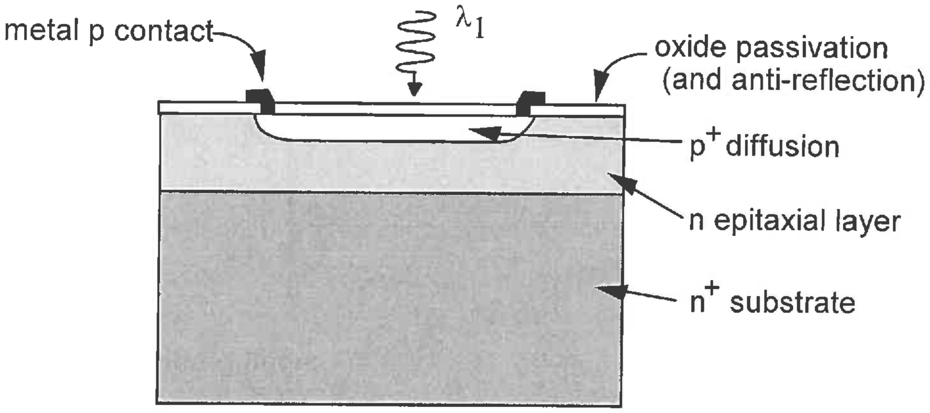
c) Homojunction "Mesa" photodiode

The homojunction mesa photodiode is perhaps the simplest photodiode configuration. A simple etch is used to isolate a section of p doped material and light is coupled through this to be absorbed in the n depletion region.

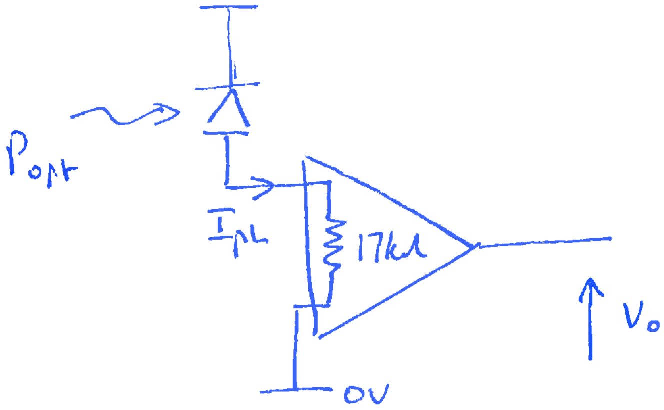


Planar homojunction photodiode

A more sophisticated design is the planar homojunction photodiode. Here the p regiod is in-diffused into the n region to form the p+-n junction. A ring metal contact and oxide passivation layer (which also can act as an anti-reflection layer – see below) are deposited on top.



d)



$$I_{ph} = \eta \frac{e\lambda}{hc} P_{opt}$$

$$= \frac{0.85 \times 1.602\times10^{-19} \times 1278\times10^{-1}}{6.63\times10^{-34} \times 3\times10^{8}} \times P_{opt}$$

$$= 0.875 \; P_{opt}$$

$$SNR = \frac{\left(\eta \frac{e\lambda}{hc} P_{opt}\right)^2}{2e\left(\eta \frac{e\lambda}{hc} P_{opt} + I_d\right)B + \frac{4kTB}{R}}$$

Assumption: thermal noise limited at low optical power

$$SNR = \frac{\left(\eta \frac{e\lambda}{hc} P_{opt}\right)^2}{\frac{4kTB}{R}} = 3dB = 2$$

$$(P_{opt})^2 = \frac{8kTB}{\left(\eta \frac{e\lambda}{hc}\right)^2 R} = \frac{8 \times 1.38\times10^{-23} \times 373 \times \cancel{10^3}}{(0.875)^2 \times 17 \times \cancel{10^3}}$$

$$= 3.16 \times 10^{-21}$$

$$\Rightarrow \quad P_{opt} = 56.2 \text{ pW} = -72.5 \text{ dBm}$$

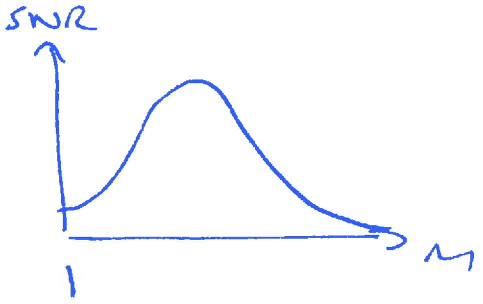
e) The SNR can be increased by the use of an APD which can increase the signal current by an avalanche gain factor M

The SNR becomes

$$\frac{(MgP_{opt})^2}{\underbrace{2M^{2+x}e(gP_{opt}+I_d)B}_{\text{no longer negligible at high } M} + \underbrace{\frac{4kTB}{R}}_{\text{fixed}}}$$

where g is responsivity & x is excess noise factor

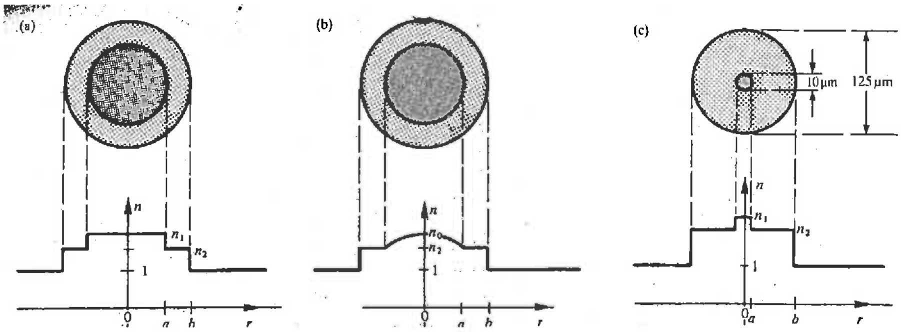
The SNR v. M curve will thus have a peak & ~~hence~~ the sensitivity can therefore be increased



Hence vary the bias voltage to the APD to optimise the sensitivity

4 a) **Types of Optical Fibre**

There are three main types of optical fibre, namely step index (SI) multimode fibre, graded index multimode fibre and step index single mode fibre. The refractive index profiles are shown below. There are two main types of multimode fibre. Each of these has a core diameter typically in excess of 50μm. Step index multimode fibre is relatively cheap, easy to handle and to join together, but it does suffer from high dispersion, and therefore limited bandwidth. Graded index multimode fibre is the most expensive type but it combines the other advantages of SI multimode fibre with greatly reduced dispersion. This is the predominant fibre type used in in-building (up to 550m) applications. Finally, SI single mode fibre has a core diameter <10μm – resulting in only one mode being allowed. This means that dispersion is low but the fibres are quite difficult to handle. Because of the low dispersion, it is the only choice for long distance transmission (>2km – 10,000km).



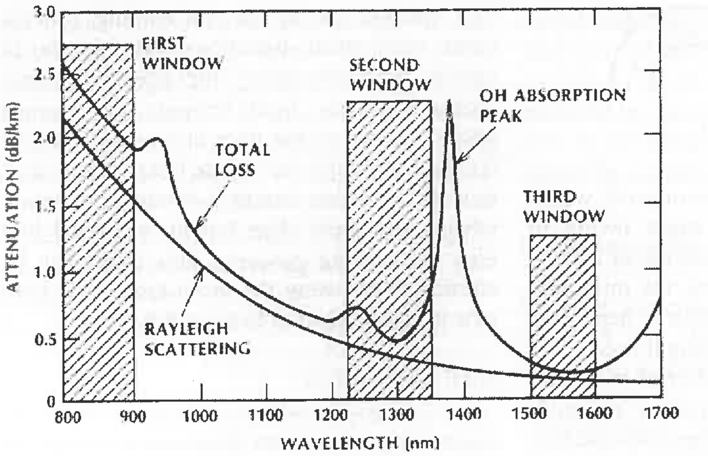
Wavelength Bands for Optical Communications.

From the diagrams of dispersion and attenuation in single mode optical fibres, it is possible to define preferred wavelengths of operation. These are:

1550nm: Lowest fibre attenuation, coupled with the availability of the EDFA optical fibre (see next handout) make this wavelength very attractive. Residual dispersion (about 17ps/nm.km) can be removed by use of dispersion compensating fibre.

1300 nm: This is the next lowest attenuation band and has intrinsically low dispersion. This is usually the wavelength of choice for systems that do not require optical amplifications

850nm: Losses and dispersion are relatively high but the availability of very cheap optical sources make this wavelength attractive for very short links which are cost sensitive (ie computer LANs).

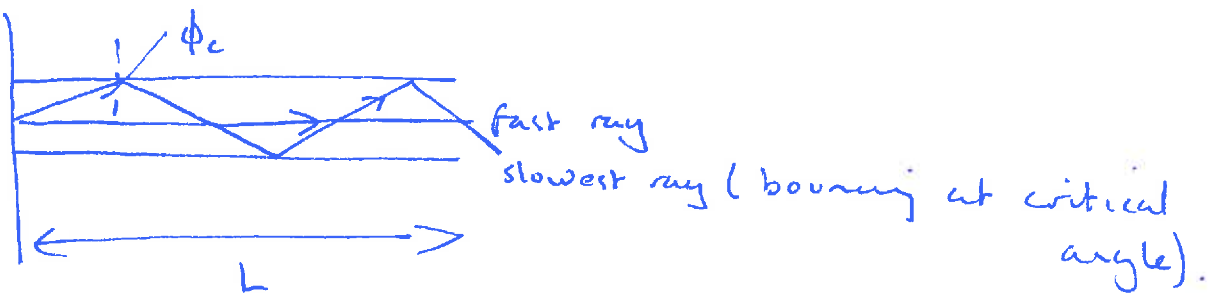


b) i) Bandwidth will be maximised for minimum Δn. Hence use glass 3 as the core and glass 2 as the cladding

ii) $NA = \sqrt{n_{co}^2 - n_{cl}^2} = \sqrt{1.545^2 - 1.53^2}$

$= 0.215$

Intermodal dispersion



Transit times $\quad t_{min} = \dfrac{L\, n_{co}}{c}$

$t_{max} = \dfrac{L\, n_{co}}{c \sin\phi_c}$ but $\sin\phi_c = \dfrac{n_{cl}}{n_{co}}$

$\Rightarrow t_{min} = \dfrac{L\, n_{co}^2}{c\, n_{cl}}$

Dispersion $= \dfrac{\Delta t}{L} = \dfrac{t_{max} - t_{min}}{L} = \dfrac{n_{co}}{c\, n_{cl}}(n_{co} - n_{cl})$

$= \dfrac{1.545}{3\times10^8 \times 1.53}(1.545 - 1.53)$

$= 5.04 \times 10^{-11}$ s/m

or 50.4 ns/km.

c) i) $$\text{Prob error} = 0.5\left(P(0|N) + \underbrace{P(1|0)}_{=0}\right)$$

For Poisson distribution

$$P.E. = 0.5\,\frac{e^{-N}N^0}{0!} = 0.5\,e^{-N}$$

$$= 3\times10^{-4}$$

$$e^{-N} = \frac{3\times10^{-4}}{0.5} = 6\times10^{-4}$$

$$\Rightarrow N = 7.42$$

So we need 7.42 ~~photons~~ electrons but with Q.E. of 0.9 we'd need $\frac{7.42}{0.9} = 8.24$ photons / 1 bit

Assuming equal numbers of 1s + 0s, average no. of photons / bit = 4.12

$$\Rightarrow \text{average energy / bit} = 4.12\times\frac{hc}{\lambda}$$

$$+ \text{ power (at 10 Gb/s data rate)} = 4.12\times\frac{6.62\times10^{-34}\times3\times10^{8}}{1550\times10^{-9}}\times10\times10^{9}$$

$$= 5.28\times10^{-9}\,W$$

$$= -52.8\ dBm$$

ii) Attenuation limit

Power budget $= 2\,\text{dBm} - (-27\,\text{dBm}) = 29\,\text{dB}$

Losses $= 3 + (2 \times 0.25) + \alpha L\ \text{dB}$

$= 3.5 + 0.4L\ \text{dB}\quad (L \text{ in km})$

Budget $\quad 29\,\text{dB} = 3.5\,\text{dB} + 0.4L + 3\,\text{dB} \leftarrow \text{margin}$

$0.4L = 22.5\,\text{dB}$

$$L_{max} = \frac{22.5}{0.4} = 56.3\,\text{km}$$

Dispersion limit

$t_{in} = 100\,\text{ps}$, max $t_{out} = 150\,\text{ps}$

$t_{out}^2 = t_{in}^2 + t_{disp}^2$ (convolution)

$t_{disp} = \sqrt{150^2 - 100^2} = 111.8\,\text{ps}^{2}$

$= D.\Delta\lambda.L_{max}$

$$L_{max} = \frac{111.8}{18 \times 0.18} = 34.5\,\text{km}$$

So link is dispersion limited at 34.5 km

ENGINEERING TRIPOS PART IIA 2015
MODULE 3B6: PHOTONIC TECHNOLOGY

Q1 Surface Emitting LEDs

This popular question began with qualitative questions concerning the structures of surface emitting light emitting diodes, and their materials. A second part of the question was then quantitative, with answers being of a significantly higher standard than in recent years, there being relatively few mathematical errors.

Q2 Fabry Perot Lasers

This question was taken by all students and began with a qualitative section on the interactions between electrons and photons in materials. There followed a question requiring a theoretical derivation of the equation of the photon lifetime and differential efficiency of a laser diode, finishing with a numerical question. As in question 1, the answers of the students were significantly better than in recent years.

Q3 Photodetectors and Photosensor Sensitivity

Quite an unpopular question, though it was really very standard so it's hard to understand why. The background (on photodiode bandgap and p+-n photodiode bias dependence were answered well. Few candidates realised that the simplifying assumption in the sensitivity calculator was that the photosensor would be thermal noise limited at low optical power, and hence shot noise could be ignored. Again few stated that replacing the p+-n diode with and APD would improve the sensitivity.

Q4 Fibre Construction and Link Budget

The background section on the different types of fibre was reasonably well answered, though many people, whilst explaining which wavelength regimes were used and why, did not explain the fundamental reasons for this choice (i.e. the absorption and dispersion wavelength dependence). The majority identified the two materials correctly though some did individual calculations for each combination, rather than realising the intramodal dispersion would be the least for the smallest index step combination. The first part of the quantum limited sensitivity section, namely the calculation of number of photons in an average 1 bit, was done quite well though some candidates mistakenly rounded to the nearest whole number. Translating this into a sensitivity was less well done. The determination that the link was dispersion limited was well done by the majority.