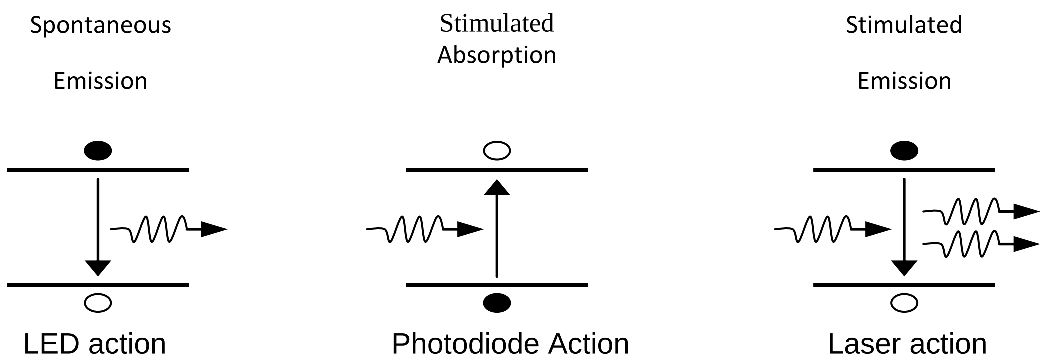
Answers to Examination Paper 3B6

Q.1 (a) This question can be answered by bookwork. A good answer should describe the three major types of electron/photon interactions in materials.



- **Spontaneous Emission**

An electron in a high energy level falls, losing energy which is emitted as a photon – the basis of operation of a light emitting diode.

- **Stimulated Absorption**

An incident photon is absorbed in a material, causing the excitation of an electron to a higher energy level – the basis of operation of a photodiode.

- **Stimulated Emission**

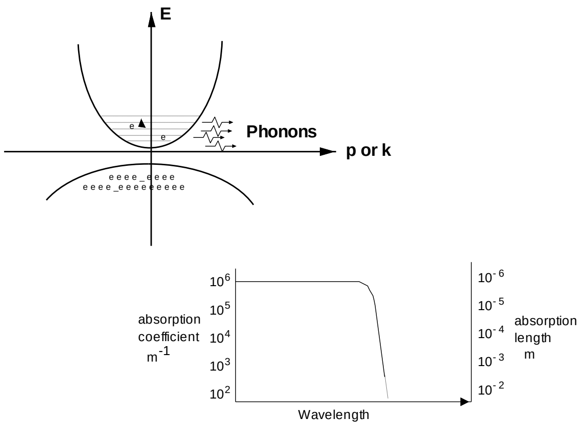
A photon, incident upon an electron in a higher energy level, causes the electron to fall to a lower level thus generating a second photon. This is, therefore, an amplifying action. Two photons are generated from one and, in turn, they can cause the generation of two further photons. Using this method, high optical powers can be generated and this operation is the basis of lasing action. The generated photon has the same frequency and phase as the incident photon and, therefore, very pure monochromatic and coherent light is generated.

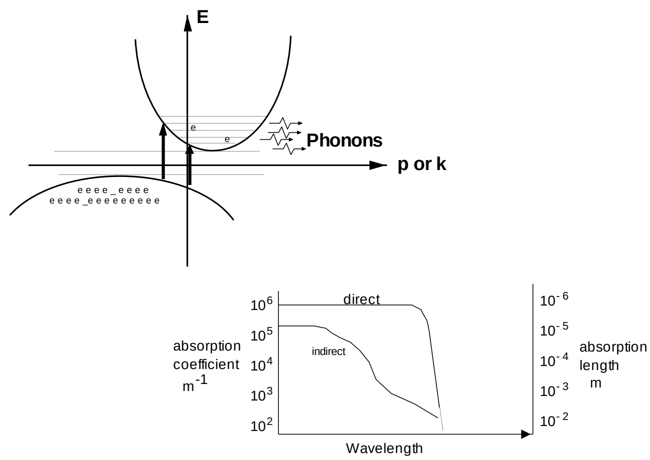
The efficiency of the processes depends much on the nature of the materials used. In the case of semiconductors, for example, absorption or emission within direct bandgap materials can readily occur without requiring momentum changes for the excited carriers. In indirect materials, phonons are frequently involved, allowing substantial momentum changes. This results in different absorption and spontaneous emission properties with reduced efficiency, and lasing in practice is inhibited.

A very good answer will provide more detailed descriptions perhaps with diagrams giving examples of the different materials involved.

Direct bandgap semiconductor (example GaAs and InGaAsP)

Indirect bandgap semiconductor (e.g. Silicon)





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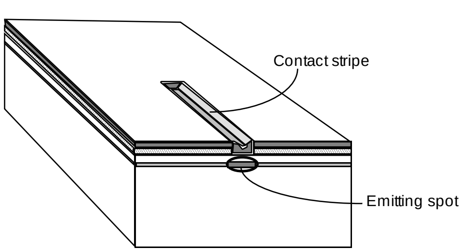
(b) This is also a bookwork question. Some answers may suggest that a surface emitting LED would be acceptable for use, but in practice for very small core narrow diameter single more optical fibres, an edge emitting light emitting diode (EELED) would be more appropriate given its emission dimensions.

A good answer would then describe the EELED, explaining its principle of operation, its structure including a description of how a heterostructure is introduced. EELEDs operate at similar current densities and currents to surface emitters, but the emitting spot is much smaller and the use of optical waveguiding increases the maximum brightness available. A very good answer may also describe related properties of the LED such as light current characteristics and spectra.

Edge emitting LED



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Q.1 (c) (i) $P = hc\eta_{int}\eta_{ext}I/(e\lambda) \Rightarrow \eta_{ext} = Pe\lambda/(hc\eta_{int}I) = \underline{2.3\%}$

[10%]

(ii) $P(t) = P(0)\exp(-\beta t) \Rightarrow t_l = -(1/\beta)\ln(P(t_l)/P(0))$

⇨ At the lifetime $t_l = -(1/\beta)\ln(1/2)$

However $\beta = \beta_o \exp(-E_a/kT)$

⇨ $t_{lT1} = -\ln(1/2)(\exp(-E_a/kT_1))/\beta_o$ and $t_{lT2} = -\ln(1/2)(\exp(-E_a/kT_2))/\beta_o$

⇨ if the lifetime at T1 is 50% that at T2

⇨ $0.5 = t_{lT1}/t_{lT2} = \exp(-E_a/kT_1)/\exp(-E_a/kT_2)$

⇨ $(1/T_1) - (1/T_2) = k\ln(1/2)/E_a$

⇨ If $T_2 = 293$ K => $\underline{T_1 = 310\text{ K}}$

⇨ $P_{T1} = P_{T2}\exp(-(T_1 - T_2)/T_o) \Rightarrow P_{T1} = \underline{0.84\text{ mW}}$

[25%]

(iii) The linewidth in energy is $\delta E = h\delta f = 2kT$

However $|\delta f| = (c/\lambda)|\delta\lambda|$

⇨ $\delta\lambda = 2\lambda^2 kT/(hc)$

⇨ The change in linewidth $= 2\lambda^2 k\delta T/(hc) = \underline{1.75\text{ nm}}$

[15%]

Q.2 (a) This section can be answered as a result of knowledge of the lecture notes. A good answer will state that the laser rate equations are used to describe the characteristics of the electrical and optical interaction within the laser. To allow a simplified analysis, assume that (1) the carrier, photon and current densities are constant in the diode laser throughout its volume, (2) that the laser generates purely monochromatic light in one mode, (3) that the amplification of light by stimulated emission is linear with carrier concentration and, (4) that temperature effects are negligible. The rate equations may therefore be written as:

$$\frac{dn}{dt} = -\frac{n}{\tau_s} + \frac{I}{eV} - g(n-n_o)P$$

- $\frac{dn}{dt}$: Nett change of carrier concentration in laser/s
- $-\frac{n}{\tau_s}$: Depletion of carriers through generation of photons by spontaneous emission
- $\frac{I}{eV}$: Rate of increase of carrier cone by current injection
- $g(n-n_o)P$: Depletion of carriers through generation of photons by stimulated emission

$$\frac{dP}{dt} = g(n-n_o)P + \beta\frac{n}{\tau_s} - \frac{P}{\tau_p}$$

- $\frac{dP}{dt}$: Nett change in photon density of lasing mode/s
- $g(n-n_o)P$: Increase in photon density by stimulated emission
- $\beta\frac{n}{\tau_s}$: Increase in photon density of lasing mode by spontaneous emission
- $\frac{P}{\tau_p}$: Decrease due to losses in laser and from facets

where: n is the carrier concentration in the laser, P is the photon density in the lasing mode, g is a gain constant, n_o is the transparency carrier density (where gain = loss), τ_s is the spontaneous recombination time of carriers, τ_p is the photon lifetime in the cavity (ie the effective time for which the photon remains in the cavity after generation before either leaving or being reabsorbed), β is the coupling coefficient (the ratio of spontaneous emission at the lasing wavelength to that generated totally, typically small ~ 10^{-4} or 10^{-5}), V is the laser active region volume, e is the electronic charge and I is the laser current.

[20%]

(b) Assume a steady state situation, i.e. dn/dt = dP/dt = 0 and that β is very small. Rewriting the photon rate equation,

$$0 = g(n-n_o)P - P/\tau_p$$

=> $$P\{g(n-n_o) - 1/\tau_p\} = 0$$

As P may have values greater than 0 (and not less!),

$$g(n-n_o) - 1/\tau_p = 0$$

=>

$$n = n_o + 1/(g\tau_p)$$

However all the terms on the right hand side of the equation are constants. Maintaining a steady state for all values of lasing photon density greater than zero, the carrier constant in the laser is constant. Let this value be called the threshold carrier density, n_{th}.

Considering the electron rate equation,

$$0 = -g(n - n_o)P - n/\tau_s + I/eV$$

But n = n_{th} for all P>0, so in this regime,

$$P = \frac{\{I/eV - n_{th}/\tau_s\}}{g(n - n_o)}$$

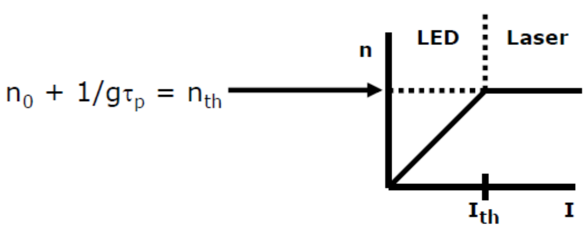
Let $I_{th} = eVn_{th}/\tau_s$,

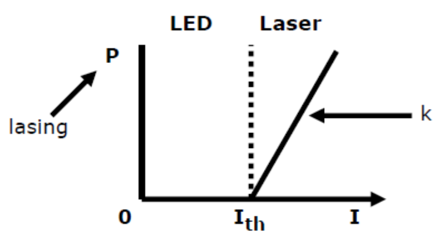
$$=> \quad P = \frac{\{I - I_{th}\}}{eV\, g(n_{th} - n_o)} = k(I - I_{th})$$

As a result the optical power generated by the laser may be shown to be proportional to the current above the threshold current. Below threshold, when P = 0 (there is no lasing light generated), the electron rate equation becomes simply

$n = I\tau_s/eV$

so that the overall operation of the laser can be understood. The carrier concentration in the laser increases linearly with current until threshold, when it saturates to a constant value. Below lasing, no stimulated emission is emitted but above, the light increases linearly with current.





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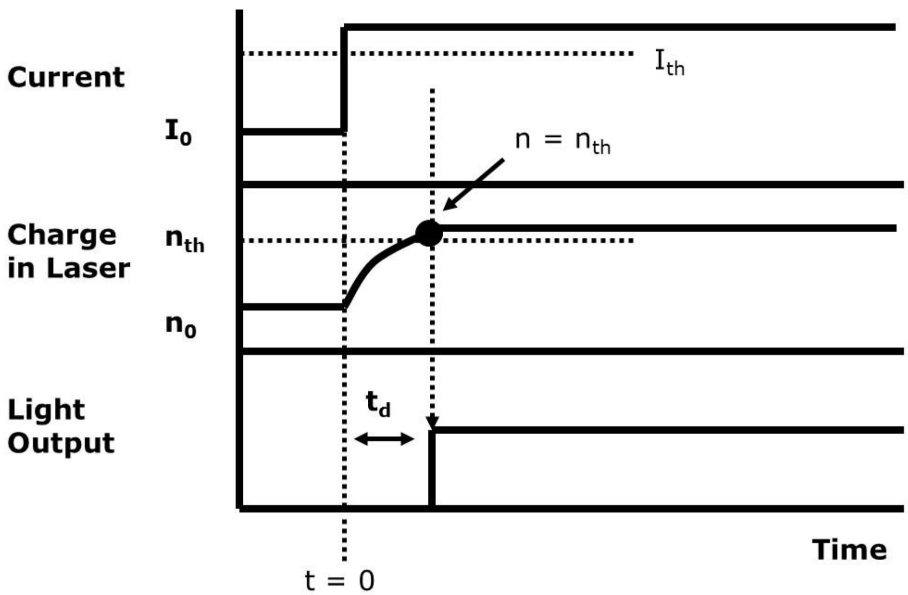
Q2 (c) (i) $I_{th} = (eV/\tau_s)(n_o + 1/(g\tau_p))$ = <u>33.7 mA</u>

[15%]

(c) (ii)



From the electronic rate equation

$dn/dt = I/(eV) - n/\tau_s$ => $n = A \exp\{-t/\tau_s\} + B$

as $I_o = 0$ and $n = 0$ at $t = 0$, $A = I\tau_s/(eV)$ where I is the value of the current after the step.

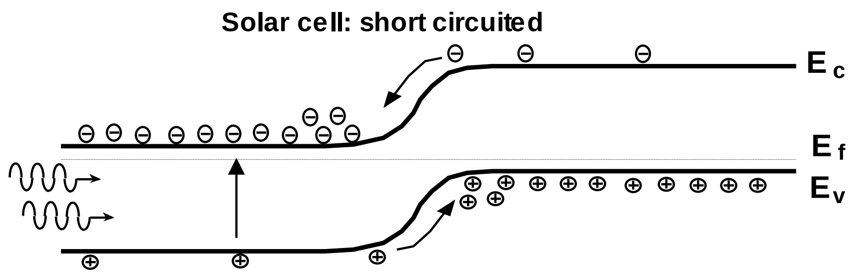
- ⇨ $n = (I\tau_s/(eV))(1 - \exp\{-t/\tau_s\}) = n_1(1 - \exp\{-t/\tau_s\})$

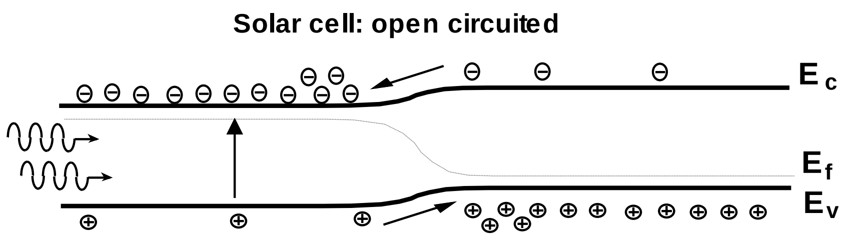
  But $n_{th} = n_1(1 - \exp\{-t_d/\tau_s\})$ => $\exp\{-t_d/\tau_s\} = 1 - n_{th}/n_1$

- ⇨ $t_d = -\tau_s \ln(1 - n_{th}/n_1)$ = <u>2ns</u> as $n_1 = 2n_{th}$ (as $I = 2I_{th}$)

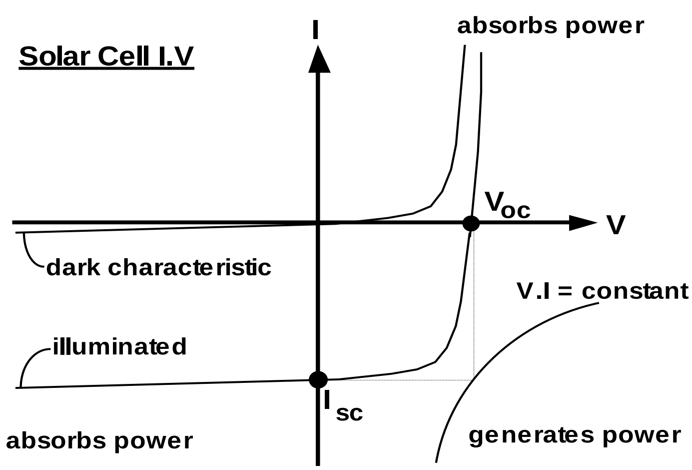
[30%]

Q3a) This section can be answered with reference to lecture notes. From the p side of a junction, diffusing electrons which reach the junction will be accelerated across it to the n side, where they will tend to accumulate, causing it to become more negative. Exactly the same thing happens for holes from the n side, crossing to the p side and making it more positive.





If we regard the solar cell as an ordinary photodiode, we can plot its I.V characteristics, both in the dark condition, and under illumination, which basically means that the whole curve is shifted downwards by the observed photocurrent. For power generation the solar cell must operate in the lower right quadrant, where lines of constant output power are hyperbolae as shown. The maximum available power is where the I.V curve touches the hyperbola with greater power. Photodiode operation is in the bottom left quadrant where the device absorbs power.



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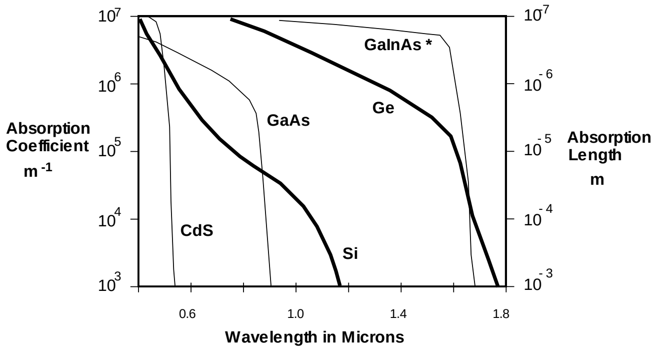
b) If the solar cell has bandgap energy E_g , any photons absorbed with energies > E_g will have their excess energy wasted to heat due to phonon decay, and of course any photons with energies < E_g will not generate holes and electrons at all. The optimum bandgap for a solar cell therefore depends on the wavelength-intensity curve of solar radiation. In this case it would be best to have the bandgap wavelength as within the band of the irradiation and closer to the peak to maximize efficiency. CdS has a bandgap wavelength of around 550nm but it has a low mobility and quantum efficiency. GaInP is probably better from this perspective but large cells made from this material would also be expensive. Si is probably the best option since it is cheap though its relatively long bandgap energy around 1 micron would mean it would be very inefficient (more than half of the energy would be lost as the peak of the irradiance at 400nm has a photon energy 2.5 x the bandgap energy.)



[25%]

c) i)

$$I_{ph} = \eta_q \frac{P\lambda e}{hc} \Rightarrow \; I_{ph} = 4.586\, mA = 4.6mA\; (2\, S.F.)$$

[10%]

ii)

Open Circuit:

$$I_{ph} = I_0\left(exp\frac{eV_{oc}}{nkT} - 1\right)$$

or

$$\frac{I_{ph} + I_0}{I_0} = exp\frac{eV_{oc}}{nkT}$$

or taking logs

$$V_{oc} = \frac{nkT}{e}\ln(1 + \frac{I_{ph}}{I_o}) \Rightarrow \; V_{oc} = 0.2896\, V = 0.29V(2\, S.F.)$$

$$I_{sc} = \; I_{ph}$$

[20%]

iii)

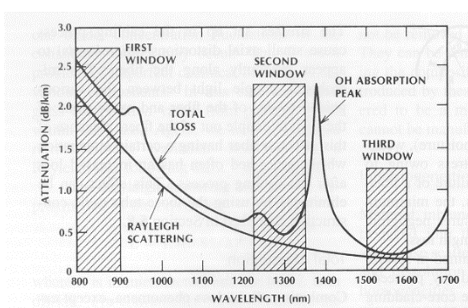
$$P = \eta_{sf}\, I_{sc} V_{oc} = 0.86\, mW$$

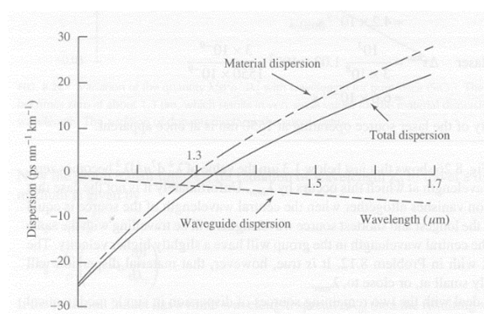
Q4a) From the diagrams of dispersion and attenuation in single mode optical fibres, it is possible to define preferred wavelengths of operation. These are

1550nm: Lowest fibre attenuation, coupled with the availability of the EDFA optical fibre make this wavelength very attractive. Residual dispersion (about 17ps/nm.km) can be removed by use of dispersion compensating fibre. Low spectral linewidth lasers (e.g. DFBs) are needed to minimize dispersion.

1300 nm: This is the next lowest attenuation band and has intrinsically low dispersion. This is usually the wavelength of choice for systems that do not require optical amplifications. Spectral width is not so important as dispersion is low – so FP lasers are used as they are cheaper.

850nm: Losses and dispersion are relatively high but the availability of very cheap optical sources make this wavelength attractive for very short links which are cost sensitive (i.e. computer LANs). Usually MMF is used here since the dispersion does not matter. LEDs are often used for low speed operation, VCSELs for higher speed. Spectral width of the laser is typically not an issue.





[25%]

b) i)

$$n_{cl} = \sqrt{n_{co}^2 - NA^2} = 1.46232 = 1.46\ (2\ S.F.)$$

[5%]

(ii)

$$\theta = \sin^{-1} NA = 8.627° = 8.6°\ (2\ S.F)$$

[10%]

(iii)

$$N \sim \frac{V^2}{2}, V = \frac{2\pi a}{\lambda} NA \Rightarrow N \sim 753$$

[5%]

(iv)

$$\frac{\Delta t}{L} = \frac{n_{co}}{c\, n_{cl}}(n_{co} - n_{cl}) \Rightarrow \frac{\Delta t}{L} = 25.7 \frac{ns}{km}$$

[10%]

(v)

$$\Delta t_{\mathrm{out}} = \sqrt{\Delta t_{\mathrm{in}}^2 + (\frac{\Delta t}{L} \times 2\ km)^2} \Rightarrow \Delta t_{\mathrm{out}} = 55.2\ \mathrm{ns}$$

[10%]

(c)

attenuation limit : $\mathrm{P_{min}^{rec}} = 1 \frac{\mathrm{nW}}{\mathrm{Mb\ s^{-1}}} \times \mathrm{R} \Rightarrow \mathrm{L_{max}^{att}} = \frac{10}{\mathrm{a}} \log \frac{\mathrm{P_{in}}}{\mathrm{P_{min}^{rec}}}$

dispersion limit: $\Delta t_{\mathrm{in}} = R^{-1} \Rightarrow \Delta t_{\mathrm{out.max}} = 1.5\ \Delta t_{\mathrm{in}} \Rightarrow$

$$\mathrm{L_{max}^{disp}} = (\frac{\Delta t}{L})^{-1} \sqrt{\Delta t_{\mathrm{out.max}}^2 - \Delta t_{\mathrm{in}}^2} = (\frac{\Delta t}{L})^{-1} \times \Delta t_{\mathrm{in}} \times 1.118$$

for $\mathrm{R} = 1\ \mathrm{Mbs^{-1}}$: $\mathrm{P_{min}^{rec}} = 1\ nW \Rightarrow \mathrm{L_{max}^{att}} = 15\ \mathrm{km}$

$$\Delta t_{\mathrm{in}} = 1000\ \mathrm{ns} \Rightarrow \mathrm{maximum}\ \Delta t_{\mathrm{out,max}} = 1500\ \mathrm{ns} \Rightarrow \mathrm{L_{max}^{disp}} = 43.4\ \mathrm{km}$$

so L_{max}= 15 km (attenuation limited)

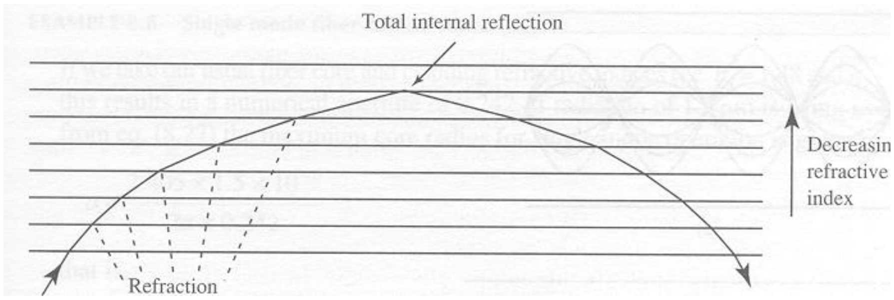
for $\mathrm{R} = 140\ \mathrm{Mbs^{-1}}$: $\mathrm{P_{min}^{rec}} = 140\ nW \Rightarrow \mathrm{L_{max}^{att}} = 9.634\ \mathrm{km}$

$$\Delta t_{\mathrm{in}} = 7.142\ \mathrm{ns} \Rightarrow \mathrm{maximum}\ \Delta t_{\mathrm{out,max}} = 10.713\ \mathrm{ns} \Rightarrow \mathrm{L_{max}^{disp}} = 0.3103\ \mathrm{km}$$

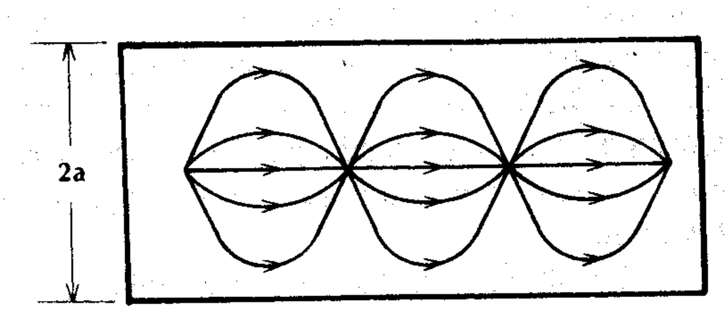
so L_{max}= 310 m (dispersion limited)

[25%]

d) Consider the figure below where a ray is passing through the fibre. Instead of a continuously graded index, assume that the index changes occur as a series of small steps. The ray will then undergo a series of small refractions and the value of ϕ increases. Eventually ϕ will be big enough that the ray undergoes total internal reflection, rather than further refraction and then is directed back towards the centre of the fibre. If we then assume that the refractive index step between layers becomes smaller and the layers become closer together, then the path becomes close to a sinusoidal shape, as shown in the lower figure.



At the optimum grading profile (α~2), then different initial ray angles will pass through the origin with the same frequency, as shown in the figure below. Although the rays which have bigger initial angles travel further, they predominantly travel further away from the centre of the fibre and here the refractive index is lower so they travel faster than rays travelling along the fibre axis. Thus these rays can compensate for the longer paths by having a lower average index.



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