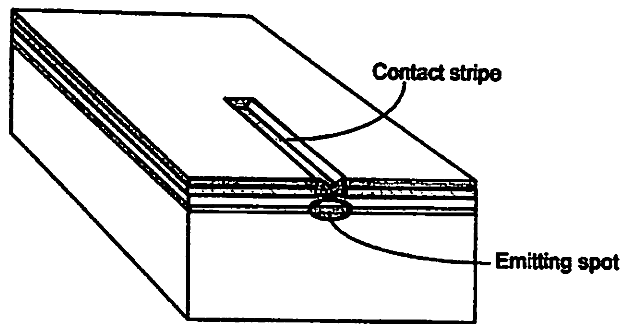
Answers to 2017 Examination Paper 3B6

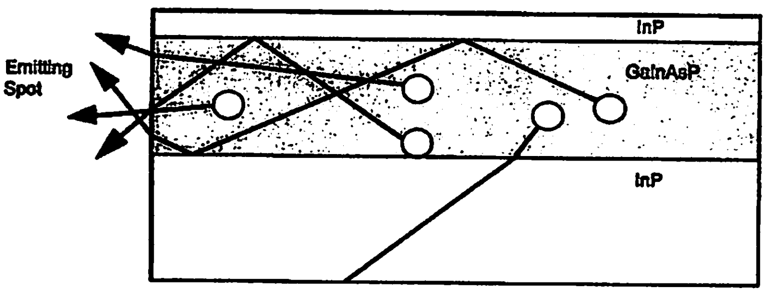
Q1 a) This answer should include the following points:

Edge emitting LEDs generate light within a heterostructure and encourage it to be guided along the device and be output at one facet.

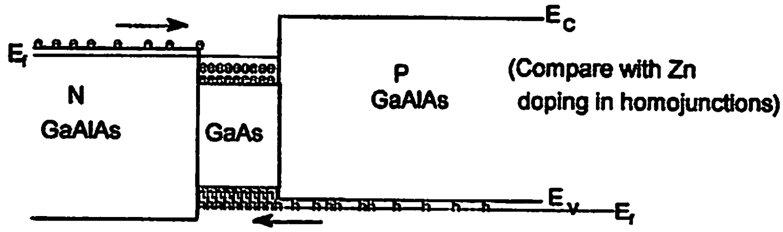
Edge emitting LED



Side view of an edge emitting LED



Double Heterostructure



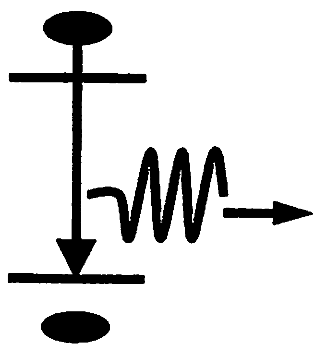
Edge Emitting LEDs rely on the use of double heterostructures to confine carriers and the light that they generate. Double heterostructures allow one to confine carriers very tightly in an LED in that both electrons and holes gather at the lowest or highest energy levels respectively which are in the central region (the GaAs layer in this case). The double heterostructure also has the advantage that photons are not re-absorbed by the wider bandgap material when travelling to the surface.This increases the speed of the LED (because of the strong overlap between holes and electrons) and also its brightness in that the generated photons sees the heterostructure layers (GaAlAs in this case) as transparent and are not absorbed before leaving the LED as may occur in conventional LED structures. The index of the central layer is larger than the others and hence can be used as an optical waveguide, as in an edge emitting LED or indeed laser diode.

As in other LEDs, the edge emitting LED generates light at a pn junction at which electrons and holes recombine to generate photons by spontaneous emission. The relation linking the energy lost by the charged particles in the recombination process E_g and the frequency of light is given by $E_g = hv = hc/\lambda$ where h is Planck's constant, c is the speed of light and λ is the wavelength.

All LEDs must have a junction, and therefore be compatible with P and N doping. The probability of a recombination leading to the generation of a photon depends greatly on the form of the energy levels in the LED material. Given that in indirect bandgap materials, the electrons at the bottom of the conduction band will have different momenta from holes at the top of the valence band, any electron hole recombination must result in the overall loss of momentum as well as energy. This surplus momentum therefore must be absorbed by, for example, a high-energy lattice vibration (a phonon) being launched during recombination. Two disadvantages result from this process. First, the generation of a phonon will not only absorb the surplus momentum, but also take up a significant amount of the available energy (up to 0.05 eV). Second, and more importantly, as two separate events will be involved in a single recombination process (ie the generation of a phonon and the generation of a photon), the probability of a radiative recombination event occurring is unlikely when compared to the equivalent process in a direct bandgap material.

The light output can be shown to be proportional to the current for the device. The optical output power, P, from the LED may be determined as a function of the current by considering the emission process. In a perfect system, one photon is generated by one electron. As a result N_p photons will be generated per second due to an injected current I into a perfect system so that

$$N_p = I/e$$



where e is the electronic charge. The optical power generated is thus the energy generated per second, ie the energy of a single photon times the number of photons generated per second, so that

$$P = hfN_p \text{ or } hvN_p$$

In a typical LED however only a fraction of the electrons which recombine generate a photon. Hence an efficiency, η is defined as the ratio of the number of photons generated per unit time to the number of electrons injected into the LED per unit time. Hence

P = photon energy × quantum efficiency × number of injected electrons

$$= \left(\frac{hc}{\lambda}\right)\eta\frac{I}{e}$$

Edge emitting LEDs operate at similar current densities and currents to surface emitters, but the emitting spot is much smaller and the use of optical waveguiding increases the maximum brightness available. They are, therefore, often used where the small spot is useful, particularly in high performance fibre optic transmitters, but only when a laser is inappropriate. They are *much* better at coupling light into single mode fibres than surface emitters. However surface emitters allow the generation of circular beams and call allow larger emission areas.

b)

i) $E = \frac{hc}{\lambda} = 2.33 \times 10^{-19}\, Joule = 1.46\, eV \rightarrow$ GaAs

ii) $\frac{1}{\tau_s} = \frac{1}{\tau_{rr}} + \frac{1}{\tau_{nr}} \Rightarrow \tau_s = \frac{\tau_{rr} \times \tau_{nr}}{\tau_{rr} + \tau_{nr}} \Rightarrow \tau_s = 1.2\, nsec$

$$\eta_{int} = \frac{\frac{1}{\tau_{rr}}}{\frac{1}{\tau_{rr}} + \frac{1}{\tau_{nr}}} \Rightarrow \eta_{int} = 0.6 \Rightarrow 60\,\%$$

iii) $P_{out} = \eta_{int}\eta_{ext}\frac{hc}{\lambda}\frac{I}{e} \Rightarrow \eta_{ext} = \frac{P_{out}}{\eta_{int}}\frac{\lambda\, e}{hc\, I} \Rightarrow \eta_{ext} = 0.022 \Rightarrow 2.2\,\%$

iv) Assume that $\frac{dP}{dt} = \frac{P}{\tau_s}$, then following integration

$P(t) = P_o\left(1 - e^{-\frac{t}{\tau_s}}\right)$ where Po is the steady state optical output, $P_o = \eta_{int}\eta_{ext}\frac{hc}{\lambda}\frac{I}{e}$

$$\Rightarrow P(1\, ns) = 1 - e^{-\frac{1}{1.2}}\, mW = 0.57\, mW$$

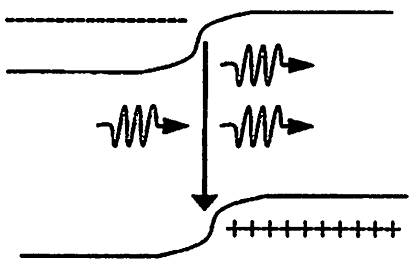
Q2 a) This answer should include the following points:

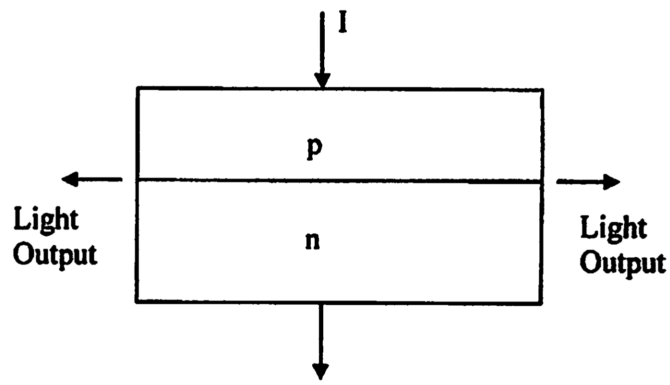
For diode laser operation, the following situations must be produced:

- A state must be obtained whereby more electrons exist in the higher electron energy level than the lower (population inversion). Such a condition may be achieved by driving a p-n junction at voltage greater than the bandgap. Here electrons will be injected directly into the conduction band and holes into the valence band so that at the junction, an incident photon is more likely to cause recombination rather than the excitation of a valence electron into the conduction band. Driving a p-n junction at such a high forward bias level can lead to very high power dissipation (10 MW/cm^3) and hence steps are generally taken to ensure that the active light generating region is kept very small.

- A method is required to ensure that the photon density in the junction is maintained at a high level. This is generally done by cleaving the p-n diode in chip form so that dielectric mirrors are formed to reflect light back into the device. Typical reflectivities are 30%.

Having a combination of a high concentration of electrons in the conduction band, holes in the valence band and photons in the diode cavity, stimulated emission is more likely than absorption and hence laser action occurs.

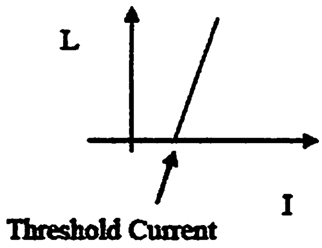




At long wavelengths, no absorption or gain occurs, and at short wavelengths the material always absorbs, but near the wavelength corresponding to the bandgap increasing, forward bias will take the material from loss to gain. As stimulated emission only occurs when the diode is driven at or above bandgap voltage, no lasing light is generated at low drive. The light current characteristic thus shows very little generated light (solely spontaneous emission, as in LED action) until a threshold current is reached when stimulated emission becomes more likely than spontaneous absorption and the light output due to lasing rises rapidly.

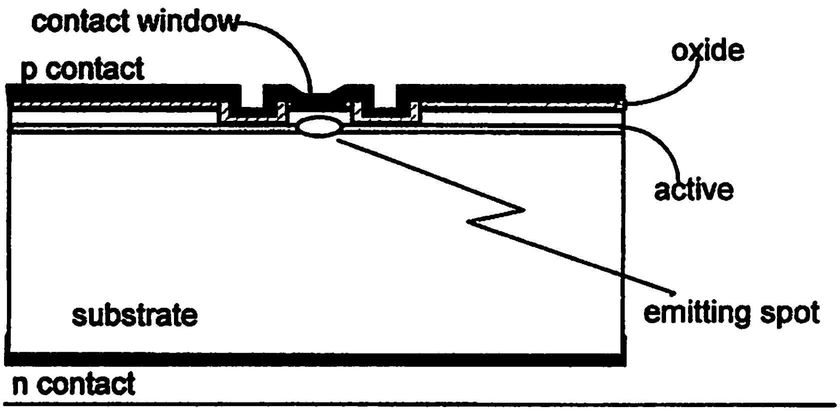
In the region above the threshold current, one photon is generated through the recombination of one hole and electron, and it is possible to write the change in output optical power (δ P) in terms of the change in current *above* threshold (δ I) as δP = energy of one photon x differential quantum efficiency x no. electrons injected/sec = $hc/\lambda \times \eta_D \times \delta I/e$

η_D is a differential quantum efficiency indicating the efficiency of the laser emission process (and is much larger than that observed in LED emission). It must be stressed that lasing action only occurs when there are more free electrons in the conduction band than the valence band, and hence as the threshold current itself is required to achieve this, the above expression solely applies to the component of current above threshold.



In order to get a low threshold current, we need to confine the current to as small an area as possible across the width of the chip, eg a typical laser with the whole chip area pumped, might have a threshold current density of 1000 A/cm^2 (10 A/mm^2) on a chip 0.5mm long and 0.2 mm wide. In other words a current of 1 Amp. However, typically we might confine the current to a region between 1 and 2 μm wide, which will reduce the operating threshold current to 10 mA: much more compatible with high speed drive electronics.

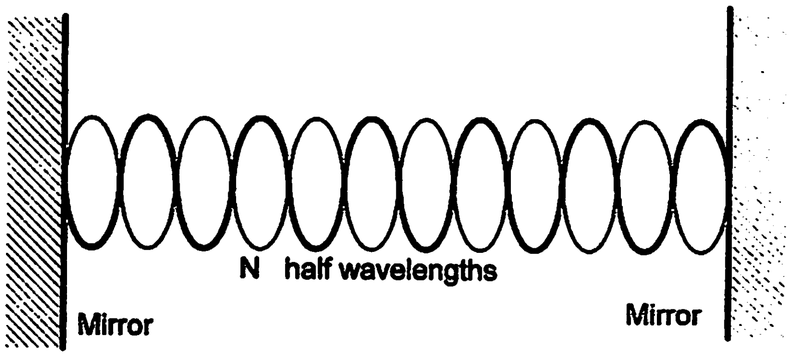
Ridge laser schematic end view



The ridge laser has reasonable current confinement, and the shaped surface provides a weak but adequate lateral waveguide. This design is very widely used for telecommunications.

b) Consider a Fabry Perot cavity. If the laser cavity is formed by two mirrors with power reflection set a distance, L, apart, the optical filament will oscillate at such a wavelength that nodes occur at both reflectors.

Fabry Perot Modes



As a result, a series of different wavelengths λ_m can be supported by such a cavity where $\lambda_m = 2nL/m$ where m is an integer and n is the refractive index. In essence, a series of optical modes that can be generated are given by $\nu = mc/(2L)$ so that the spacing in frequency between adjacent modes, $\Delta\nu_m$, is c/2L.

As $\nu = c/\lambda$, $|\delta\nu| = (c/\lambda^2)|\delta\lambda|$ => $|\delta\lambda_m| = (\lambda^2/c)|\delta\nu_m| = \lambda^2/c.c/2nL = \lambda^2/2nL$

$$\Rightarrow L = \frac{\lambda^2}{2n\Delta\lambda} \Rightarrow L = 333.7\ \mu m$$

c) Consider the Fabry Perot laser cavity below. Assume that stimulated emission encounters a gain per unit length (due to stimulated amplification), G, and a loss per unit length due to scattering and absorption, α, as it passes along the laser. The gain G in practice creates extra photons to compensate for those photons lost as the signal travels over a distance of unit length. Therefore the stimulated light A starting at one facet will be incident on the opposite facet with an optical power: $B = \exp\{(G - \alpha)L\}\, A$

At that point part of the signal is reflected with a coefficient R and the signal then passes back amplified by 1 the same amount as above and again reflected by the initial facet. Lasing action will occur in the net round trip gain of the signal is unity i.e. if $A.\exp\{(G - \alpha)L\}.R_1\exp\{(G - \alpha)L\}.R_2 = A$

=> $G = \alpha + (1/2L)\ln(1/(R_1R_2))$ *[N.B. Gain/unit length]*

The output power from the laser may be determined readily in terms of the laser cavity photon density, P, noting that the light output from the laser is that equivalent to a loss per unit length of $(1/2L)\ln(1/(R_1R_2))$. As a result the proportion of photons leaving the cavity per unit time is given by $(1/2L)\ln(1/(R_1R_2))v_g$

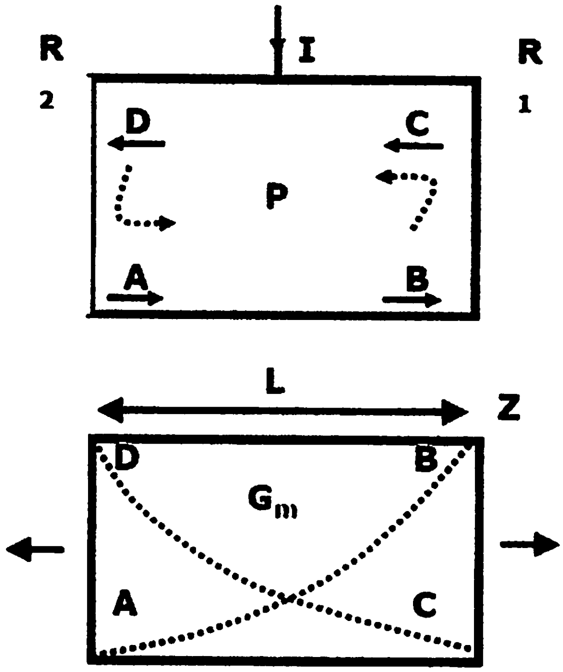
Hence above threshold the differential efficiency is simply the proportion of photons leaving the cavity through the facets over the total number of photons, ie

$$\eta_D = \frac{\ln(1/(R_1R_2))/(2L)}{\alpha + \ln(1/(R_1R_2))/(2L)}$$

$dP/dz = G_m P$

=>

$P(Z) = P(o)\exp(G_m z)$

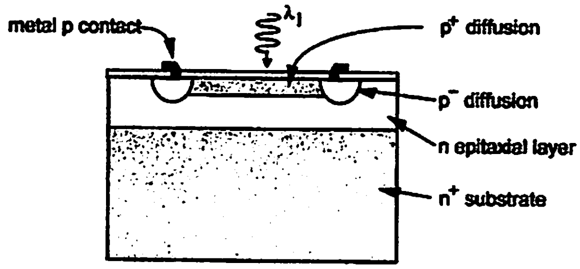


$$\eta_D = \frac{\ln(1/(R_1R_2)/2L}{a+\ln(1/(R_1R_2)/2L} \Rightarrow a = \ln(1/(R_1R_2)/2L \times \frac{1-\eta_D}{\eta_D} \Rightarrow a = 1.46\, mm^{-1}$$

ii) G is equal to the ratio of photons lost as the signal travels a unit length. Hence the proportion of photons lost per unit time is simply the gain G times the speed of light in the laser material, v_g (i.e. gain/length x length/time). As a result the average time for which one photon will remain in the cavity is given by $\tau_p = 1/Gv_g = 1/\{v_g\{\alpha + (1/2L)\ln(1/R_1R_2)\}\}$

$$\Rightarrow \tau_p = \frac{1}{v_g} \times \frac{1}{(a+\ln(1/(R_1R_2)/2L)} = \frac{n}{c} \times \frac{1}{(a+\ln(1/(R_1R_2)/2L)} \Rightarrow \tau_p = 2.46\, psec$$

Q3 a)



Clearly the photodiode material needs to have a bandgap smaller than the photon energy (but not too much smaller). Hence Si or GaAs photodiodes would be good choices. Si has the advantage of being cheap but GaAs has higher carrier mobility and hence drift limited bandwidth is higher so for a 25GHz bandwidth, GaAs may be a good choice.

b) This section can be answered with reference to lecture notes (the below is probably slightly over the top in terms of detail but the main principles should be mentioned). The speed of a photodiode depends on three main factors

1. The drift velocity of photogenerated carriers

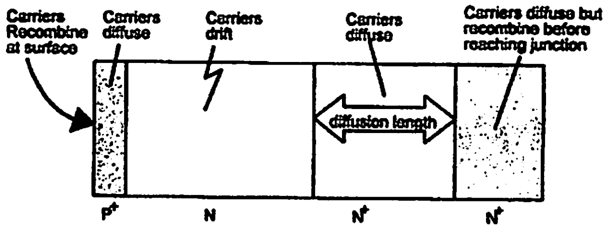
2. Diffusion processes

3. The capacitance of the photodiode.

The first two of these processes are carrier transport mechanisms. Diffusion is a slow process and so, for a fast photodiode, needs to be avoided by design. Thus a high bandwidth photodiode will be limited by a combination of drift velocity saturation and capacitance.

Key Regions in a Photodiode

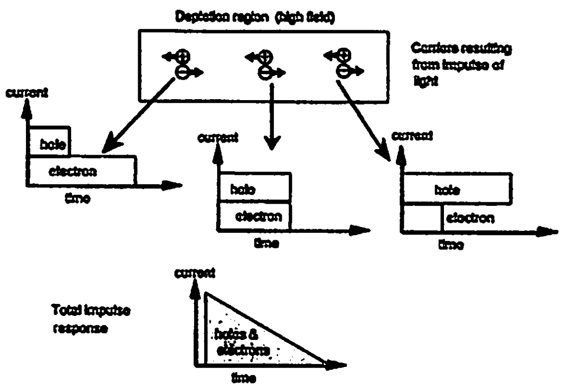
Clearly the depletion region has a high field in it and so any photogenerated carriers are quickly accelerated to the saturated drift velocity in this region. In the P^+ and the N^+ regions, the field is low, so the dominant transport process is diffusion. The P^+ is relatively thin, so this is not a great problem. However, the N^+ is wide and so this can severly limit the transport in the photodiode.



The overall time response can be broken down into two separate regions of operation, the response of the depletion region and the response of the diffusion region. These responses are shown schematically below.

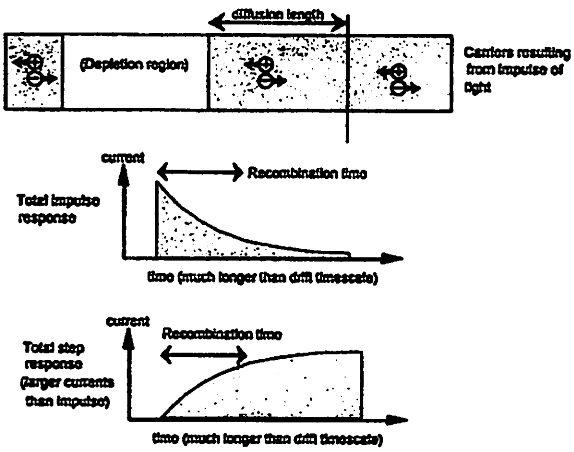
Time response: Depletion Region

If an impulse of light is input into the depletion region, then the positive and negative photo-generated charges immediately begin to separate under the action of the high electric field. Depending on how far from the edges of the region the carriers are, they will generate a high, then linearly reducing current against time. The timescale is quite short – typically of the order of ~100ps.



Time response: Diffusion regions

Carriers generated in the diffusion region obey the diffusion equations described above. Solutions to these equations show exponential like behaviour though the time constants are much longer than those for carrier drift in the depletion region, with 10-100ns time constants being common.



From the p side of a junction, diffusing electrons which reach the junction will be accelerated across it to the n side, where they will tend to accumulate, causing it to become more negative. Exactly the same thing happens for holes from the n side, crossing to the p side and making it more positive.

Thus, it is desirable, in a p^+-n photodiode to ensure that the depletion region is wide enough absorb all of the light – this can be done by ensuring a very highly absorbing material or by having a high reverse bias to make sure that the depletion width is very wide.

If diffusion is eliminated as an effect, then the capacitance of the photodiode may become an issue. The capacitance of a photodiode consists of mainly junction depletion capacitance plus strays (and package lead thro's etc.).

$$C_{dep} = \varepsilon_0 \varepsilon_r A/w_{dep}$$

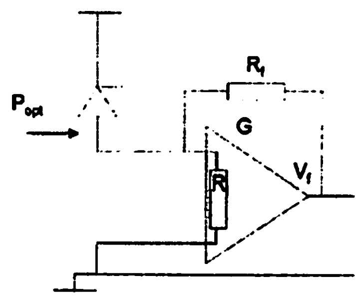
p^+n diode: $W_{dep} \propto \sqrt{[V_{built\text{-}in} + V_{bias}]}$

p-i-n diode: W_{dep} = "i" region width (assuming bias to full depletion)

Capacitance therefore reduces with w_{dep} (and hence increasing bias) for the p^+n diode but the drift limited transit time increases. Hence there is a trade off between the two effects.

The *pin* diode differs from the p^+n in that, assuming there is sufficient bias to fully deplete the intrinsic region, has a depletion width defined by the width of the instrinsic region and hence the capacitance, and thus bandwidth of the diode, can be much better controlled and does not depend on the doping levels in the diode and the reverse bias voltage.

c) i)



ii) It can be shown that the effect of the capacitance of the photodiode is reduced by a factor equivalent to the gain of the amplifer.

Hence the frequency dependent output has a 3dB bandwidth of $\omega = G/CR$ or $f_{3dB} = G/2\pi C_d R_f$

The stray capacitance of the transimpedance resistor does not have its capacitance reduced and hence $f_{3dB} = 1/2\pi(C_d/G + C_R) R_f$

So $0.8 \times 25 \times 10^9 = 1 / 2\pi(10^{-12}/1000 + 50 \times 10^{-15}) R_f$

⇨ $R_f = 1/\ 2\pi(\ 10^{-12}/1000 + 50 \times 10^{-15}\ ) \times 0.8 \times 25 \times 10^9 = 156\Omega$

iii) $V_o = I_{ph}\ R_f = e\eta\lambda/(hc) \times P_{opt} \times R_f$

$V_o\ /\ P_{opt}\ = e\eta\lambda/(hc) \times R_f$

$= 1.602 \times 10^{-19} \times 0.8 \times 0.9 \times 830 \times 10^{-9} \times 156\ /\ 6.62 \times 10^{-34} \times 3 \times 10^8$

$= 75$ V/W

iv) Assumption is that at low input power levels the noise is dominated by thermal noise and therefore shot noise can be ignored. Hence the SNR reduces to

SNR $= (e\eta\lambda/(hc) \times P_{opt})^2\ /\ 4kTB/R_f$ or

$P_{opt}^2\ = SNR \times 4kTB\ /\ R_f \times\ (e\eta\lambda/(hc))^2$

$= 100 \times 4 \times 1.38 \times 10^{-23} \times 303 \times 20 \times 10^9\ /$
$156 \times (1.602 \times 10^{-19} \times 0.8 \times 0.9 \times 830 \times 10^{-9}\ /\ 6.62 \times 10^{-34} \times 3 \times 10^8)^2$

$= 925 \times 10^{-12}\ W^2$

$P_{opt}\ = 21.1 \times 10^{-6} W = 10 \log (1.33 \times 10^{-6}\ /\ 10^{-3}\ ) = -15.2$ dBm

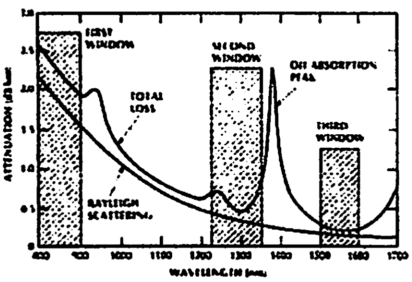
d) Since the bandwidth is limited by the RC time constant of the transimpedance resistor, if the capacitance cannot be reduced, the resistance (and hence the sensitivity) is fixed unless the relative amount of shot v thermal noise can be changed. This can be done with an APD which increases the SNR (at least for low levels of avalanche gain).

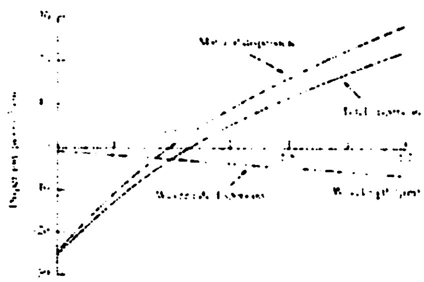
Q4a) From the diagrams of dispersion and attenuation in single mode optical fibres (see below), it is possible to define preferred wavelengths of operation. These are

1550nm: Lowest fibre attenuation, coupled with the availability of the EDFA optical fibre make this wavelength very attractive. Residual dispersion (about 17ps/nm.km) can be removed by use of dispersion compensating fibre. Low spectral linewidth lasers (e.g. DFBs) are needed to minimize dispersion.

1300 nm: This is the next lowest attenuation band and has intrinsically low dispersion. This is usually the wavelength of choice for systems that do not require optical amplifications. Spectral width is not so important as dispersion is low – so FP lasers are used as they are cheaper.

850nm: Losses and dispersion are relatively high but the availability of very cheap optical sources make this wavelength attractive for very short links which are cost sensitive (i.e. computer LANs). Usually MMF is used here since the dispersion does not matter. LEDs are often used for low speed operation, VCSELs for higher speed. Spectral width of the laser is typically not an issue.





b) The principal fundamental loss mechanism is Rayleigh scattering. The fibre attenuation scales with wavelength as a function of $1 / \lambda 4$. If the phonon absorption edge (which for silicate fibre starts being noticeable at 1550nm) can be extended to longer wavelength via the use of low phonon energy glass, then the absorption minimum can be made much smaller.

$$\text{Absorption } (2.0\ \mu m) = \text{Absorption } (1.3\ \mu m) \times (1.3/2.0)^4 = 0.08\ dB/km$$

(c) i)

attenuation limit : power budget

$P_{transmit}$: + 3 dBm

Losses: Fibre: $0.5 \times L_{max}$

Splices: 4 x 0.2 dB = 0.8 dB

Coupling: 2 + 1 = 3 dB

Sensitivity: - 20 dBm

Power budget = $P_{transmit}$ – Sensitivity = 23 dB = Losses = 3.8 + 0.5 x L_{max}

L^{att}_{max} = (23-3.8) x 2 = 38.4 km

dispersion limit: $\Delta t_{in} = R^{-1} \Rightarrow \Delta t_{out\,max} = 3\,\Delta t_{in} \Rightarrow$

$$\Delta t_{out\,max} = 3\,\Delta t_{in} = \sqrt{\Delta t_{in}{}^2 + \Delta t_{disp}{}^2}$$

$$\Delta t_{disp}{}^2 = 8\,\Delta t_{in}{}^2$$

for $R = 10\text{ Gbs}^{-1}: \Delta t_{in} = 100ps, \Delta t_{disp} = \sqrt{80000} = 282.8\,ps$

$$\Delta t_{disp} = D L_{max} \Delta\lambda = 1.5\,x\,3.2\,x\,L_{max} = 4.8 L_{max}$$

$$L^{disp}_{max} = \frac{284.8}{4.8} = 58.9\,km$$

So link is attenuation limited at 38.4 km

ii) To achieve this data rate, it is likely that no single source would have the bandwidth, so some form of multiplexing is necessary. The most common form is WDM. It would be possible to keep the same channel rate, but have 8 wavelength channels. To go the great distance, some amplification would be necessary, resulting in a change of wavelength to the 1550nm regime to be able to use the EDFA. This would result in higher fibre dispersion, and so it is likely (also since it allows the laser wavelength to be fixed) a narrower linewidth laser such as a DFB would be necessary. It might also be necessary to use dispersion compensating fibre.