

3C1 / 3P1 2015 Crib

1. (a) For plane strain,  $d\varepsilon_2 = 0$ . Hence in order for the flow rule to have a finite, non-zero value in the middle term:  $(\sigma_2 - \sigma_m) = 0$ .

Hence:  $\sigma_2 = (\sigma_1 + \sigma_2 + \sigma_3)/3$ , and therefore  $\sigma_2 = (\sigma_1 + \sigma_3)/2$ .

Substituting into von Mises yield criterion (from Structures Databook):

$$[(\sigma_1 - \sigma_3)^2 + (\sigma_2 - \sigma_1)^2 + (\sigma_3 - \sigma_2)^2] = 2Y^2$$

$$[(\sigma_1 - \sigma_3)^2 + (\frac{\sigma_1 + \sigma_3}{2} - \sigma_1)^2 + (\sigma_3 - \frac{\sigma_1 + \sigma_3}{2})^2] = 2Y^2$$

$$[(\sigma_1 - \sigma_3)^2 + (\frac{\sigma_3 - \sigma_1}{2})^2 + (\frac{\sigma_3 - \sigma_1}{2})^2] = 2Y^2$$

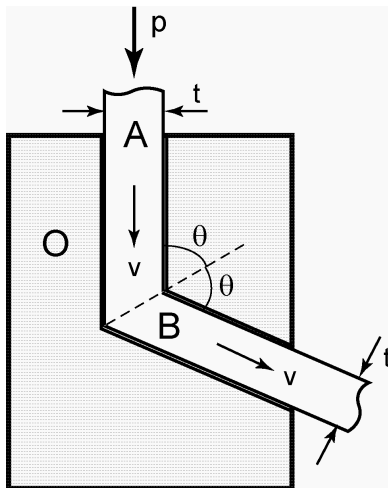
$$6 (\frac{\sigma_1 - \sigma_3}{2})^2 = 2Y^2 \quad \text{and hence} \quad (\sigma_1 - \sigma_3) = \frac{2}{\sqrt{3}}Y .$$

The corresponding Tresca criterion states that:  $(\sigma_1 - \sigma_3) = Y$

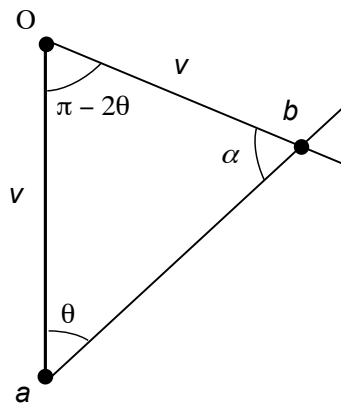
So the von Mises criterion matches Tresca with an apparent "plane strain yield stress" of  $(2/\sqrt{3})Y$ .

(b) (i) The ECAE process introduces plastic deformation into a metal sample. It is unusual in that the geometry of the sample is the same after deformation: most plastic deformation processes are used to introduce a shape change. The process can be used to introduce very high strain (hot or cold deformation) into a metal, so allowing these microstructures to be investigated. It imposes controlled plastic strain, strain-rate and temperature. The ability to control all these parameters allows it to be used to investigate the recrystallization response in terms of these variables (e.g. recrystallised grain size).

ii) Process diagram:



Velocity diagram:



$$\alpha = \pi - (\pi - 2\theta) - \theta = \theta$$

The length of the shear band:  $t / \sin\theta$

$oa = ob = v$  (from continuity), and  $\alpha = \theta$ ,

that is, the velocity diagram triangle is isosceles, so  $v_{ab} = 2v \cos\theta$

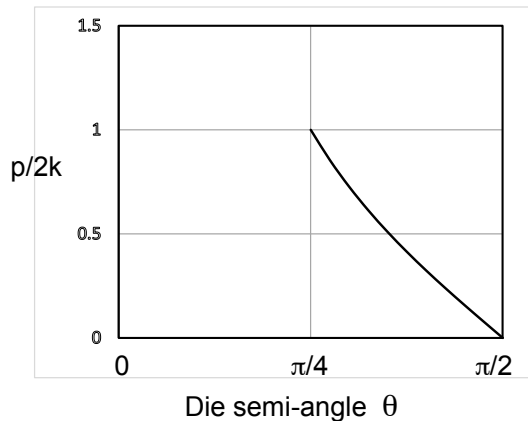
For unit depth into page, extrusion pressure  $p$ , and shear yield stress  $k$ :

external work rate =  $p t v$

internal plastic dissipation =  $k (t/\sin\theta) (2v \cos\theta)$

Hence:  $p = 2k \cot \theta$

Variation with  $\theta$  :



Pressure is lower than the uniaxial yield stress, for channel angles  $2\theta$  greater than  $\pi/2$ .

(iii) Assuming adiabatic heating, the external power input is equated to volumetric heat capacity  $\times$  volume throughput rate  $\times$  temperature rise:  $p v t = (\rho C_p) \times (v t) \times \Delta T$

(iv) Friction will add terms to the internal dissipation, e.g. for sticking friction the additional dissipation per unit depth is  $k \times v \times$  (length of channel). Hence the pressure increases significantly, with the added contribution scaling with the length of the channel. The same goes for the temperature rise – though note that the temperature rise will also be non-uniform, as the frictional heating is at the surface of the material (whereas the shear band is internal).

*Examiner's comments:*

(a) This should have been a straightforward derivation, but a significant number of people did not write down the von Mises yield criterion from the databook, and floundered around trying to produce the required result using just the Levy-Mises rule.

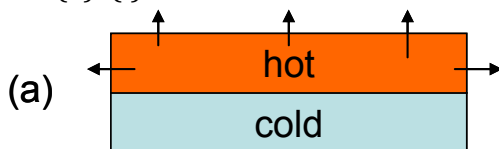
(b) (i) Some had absolutely no idea what the process does, and thought it was for making a bent section.

(ii) Most people knew what they were trying to achieve and got the velocity diagram correct, but some struggled with the geometry producing pages of algebra (and often the wrong answer).

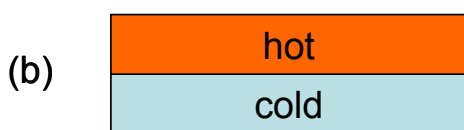
(iii) Most wrote down an equation (which was preferable) or said in words what was required.

(iv) Few realized that the shear band introduces uniform heating whilst the friction from the walls gives heating only at the surface.

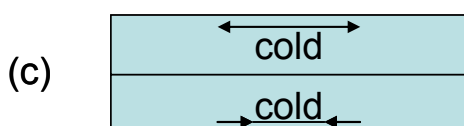
2 (a) (i) Thermal residual stress



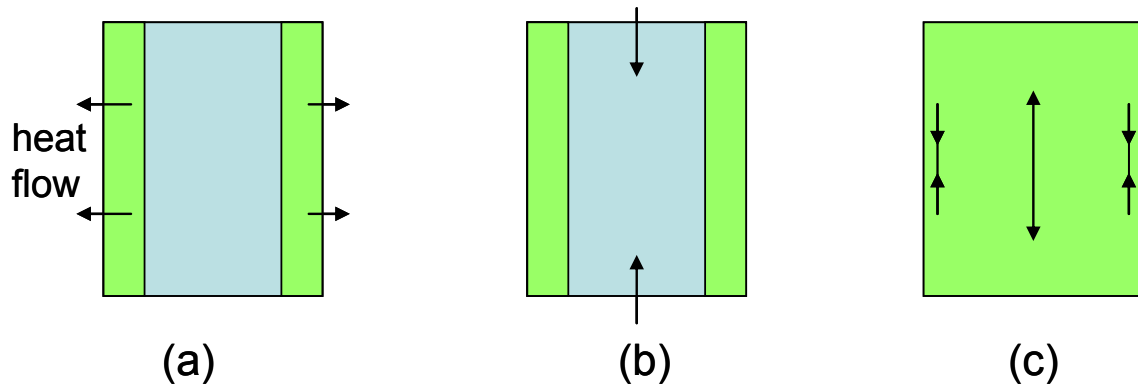
The top surface of a slab is heated. The hot region expands (a). Because its yield stress is reduced when it is hot, it can deform plastically to accommodate the expansion, and the stress in the hot region is small (b).



As the hot region then cools, it contracts, but as its yield stress is now higher, it cannot deform plastically and thus exerts a force on the neighbouring cold region. The final tensile residual stress in the heated region can be up to the material's yield stress, and is balanced by compressive stress elsewhere (c).



## (ii) polymer injection moulding



In polymer injection moulding, the material close to the mould wall solidifies quickly and contracts first, with material in the centre still molten (a). This material then cools and shrinks (b), and the slower cooling rate allows more rearrangement of the molecules so it will contract more than the polymer on the mould walls. The effect is particularly marked if the polymer can crystallize, because the crystals are denser than the amorphous polymer on the walls. This all has the effect of putting the surface regions into compression and leaving the core under tension (c).

Residual stresses may be undesirable if they cause warping and distortion of the part (common in welding, injection moulding of polymers and rapid cooling after hot forming of metals). They can add to applied stresses to cause failure at a lower applied load than if they are not present. They can lead to failure even in the absence of applied stress – e.g. stress corrosion cracking (including hydrogen cracking) of metals, environmental stress cracking of polymers, ‘static fatigue’ of glass.

Residual stresses in the welded material can be reduced by pre-heating the whole slab, or by post-weld treatment to allow stresses to dissipate.

With the injection moulding, increasing the hold-on time and the injection pressure will pack more polymer into the mould and hold it in place as it cools, so will reduce shrinkage and so reduce stresses. Use of an amorphous polymer will also help.

(b) For fast fracture to occur, initiated at an edge crack of length  $a$ ,

$$K_{IC} = Y\sigma\sqrt{\pi a}$$

where  $Y \approx 1$  and  $K_{IC}$  is the plane strain fracture toughness (from Materials Data Book p. 6).

In this case  $\sigma \approx 300$  MPa and  $a = 5$  mm.

So  $K_{IC} \approx 300 \sqrt{\pi \times 5 \times 10^{-3}} \approx 37.6$  MPa m<sup>1/2</sup>.

$K_{IC}$  falls linearly from 50 MPa m<sup>1/2</sup> at 20 °C to 30 MPa m<sup>1/2</sup> at –20 °C i.e. by 0.5 MPa m<sup>1/2</sup> for 1 °C. So temperature  $T$  at which fracture occurs is given by  $(50 - 37.6) = 0.5 (20 - T)$ .

$$\text{so } (50 - 37.6) / 0.5 = 20 - T$$

$$\text{therefore } T = 20 - (50 - 37.6) / 0.5 = -4.8 \text{ °C.}$$

Failure results from the combination of high tensile residual stress (150 MPa) and 5 mm deep cracks resulting from pitting corrosion, combined with a high applied stress (also 150 MPa). So failure could be avoided (i) by reducing the applied stress through changing the design of the bridge – e.g. use of thicker sections etc., (ii) by ensuring that residual stresses in the beams are lower – e.g. though better control of the way in which the beams are cooled after hot rolling, and (iii) by better maintenance to avoid pitting

corrosion – e.g. use and maintenance of a high quality paint film. In some designs of bridge it might be feasible to galvanize the beams to avoid corrosion.

*Examiner's comments:*

*A very popular question, answered by nearly all.*

*(a) (i) Thermal expansion and contraction are reversible, so the important point here (missed by many) is that the hot metal must have changed its shape for there to be any residual stress when it cools down again, otherwise it would just go back to its original unstressed state.*

*(ii) Lots of confusion between metals and polymers, with a number of people talking about pressure die casting (a metal process). Others referred to extrusion of polymers, and melt swell (not relevant here).*

*(b) Most people managed the calculation correctly.*

*Rather few people pointed to the simplest ways of avoiding failure: by changing the design to reduce stresses, and in service by painting the bridge regularly, and inspecting for obvious damage. Despite the link to the first part of the question, few recommended reducing residual stresses in rolled beams.*

3. (a) Al-12%Si chosen (eutectic alloy so particularly suitable for casting because of abrupt liquid-solid transformation). Where mechanical properties are important the modified alloy (add 1% sodium) should be used to change shape of Si precipitates, making them smaller and more globular rather than crack-like plates. Pressure die casting will be needed for the first application to fill the section. Liquid alloy squeezed under pressure into metal die. The second application could also use this process, but with this thicker section gravity die casting (lower cost) could also be used.

(b) Low cost: use chopped fibre spray-layup, with a polyester matrix. Relatively low fibre volume fraction; cheap matrix. Maximised bending stiffness: vacuum bagging of prepreg sheets of uniaxial long fibres in epoxy matrix. To avoid out-of-plane distortion use balanced layup with fibres parallel to the edges of the plate (e.g. 0-90-90-0).

(c) High volume of plates so an automated process with high set-up cost may be suitable: Resin transfer moulding (mechanical properties better than required; long fibre mats with vinyl ester matrix). Spray-layup is the most likely option (materials as in (b), although it is less easily automated.

Degradation will be expected in the following ways: Matrix absorbs water and swells; plate may distort; polymer can crack. Cracks can allow water to 'wick' into plate interior so degradation happens through the section. Glass fibres can suffer leaching so properties degrade. Temperature range means water may freeze inside composite so increasing cracking. Degradation is more severe with chopped-fibre composites.

(d) Complex shape so either injection moulding (polymer injected under pressure into metal mould), or extrusion if the geometry allows. HDPE crystallises and shrinks; this will happen non-uniformly because of the variable section (thicker sections cool more slowly, crystallise more, shrink more). Sink marks will be visible on the top surface of an injection moulding. To reduce: increase injection pressure and hold-on time. An extruded section may also suffer melt swell, which can be reduced by using a long die land (to allow polymer chains to coil up again before leaving die) or by quenching immediately after extrusion. Post-extrusion draw-down is not likely to be an appropriate process for a complex section.

*Examiner's comments:*

*(a) People had forgotten that casting alloys are, by choice, eutectics (cheaper processing costs because of minimum temperature; reduced defect rates because of minimum*

*shrinkage; high fluidity makes it easier to fill moulds and cope with thin sections). Several people bizarrely stated that Si content should be minimized to reduce cost. This led them to recommend using Al-2% Si for the thin plate (where strength is unimportant), not grasping that the high fluidity of the eutectic will be particularly important to allow mould filling here.*

*Disappointingly few remembered that the main way to increase the strength of this alloy is to add a poison (0.1% sodium) to modify the shape of the Si precipitates.*

*Many didn't understand that casting alloys can't be rolled or otherwise plastically deformed. A frequent mistake was to ignore that a particular casting alloy had been specified (aluminium silicon) and to write instead about steel.*

*(b) The question referred to GFRP and defined it as glass fibre reinforced plastic, but many answers wrote about other materials instead, especially carbon fibre.*

*(c) Reasonably well done.*

*(d) Generally, very poor understanding of manufacturing processes was revealed. Common suggestions were to put reinforcing bars into the injection mould die before injecting the HDPE to form the plate, or to make the bars and the plate separately and glue them together. Despite the question stating that only HDPE was to be used, many answers referred to other materials (random metals and composites).*

4. (a) The equivalent diameter of a component is the diameter of an infinitely long circular cylinder which, if subjected to the same cooling conditions as the component, would have a cooling rate on its axis equal to that at the position of slowest cooling in the component.

(b) (i) Section at C longer than at A, so this will have the slower cooling rate of the two.

Find  $D_e$  for B and C:

At B, L is at least 150mm (due to end pieces), and  $D = 35\text{mm}$ , so  $L/D = 3.4$   
From curves for solid cylinder,  $f = 1$  (effectively infinite length), so  $D_e = 35\text{mm}$ .

At C, assume  $z \approx 130\text{mm}$ ;  $x = 50\text{mm}$  and  $y = 25\text{mm}$ . Hence  $x/y = 2$ ,  $y/z \approx 0.19$ .  
From curves for tubes,  $f = 1.5$  (effectively infinite length, and thin-walled), so  $D_e = 37.5\text{mm}$ .

Hence the slowest cooling rate will be at C – the ruling diameter is 37.5mm (where the hardness will be minimum).

(ii) From the tempered hardness curves, interpolating to a diameter of 37.5mm, hardness after temper at 550°C is approx. 350, and at 500°C it is approx. 400, so both satisfy the target range of 340-420. The tempered hardness is too low using 600°C, and too high using 400°C.

Now checking the surface hardness, this equals 500 for tempering at 500°C, but is around 430 for 550°C.

Hence tempering at 500°C satisfies the hardness criteria.

(c) The question relates to wrought precipitation hardened aluminium alloys, e.g. Al-4%Cu. These are typically shaped by extrusion, as here, immediately followed by ageing to produce the required distribution of precipitates and increase the strength. The temperature-time profile of an extrusion process is shown here:

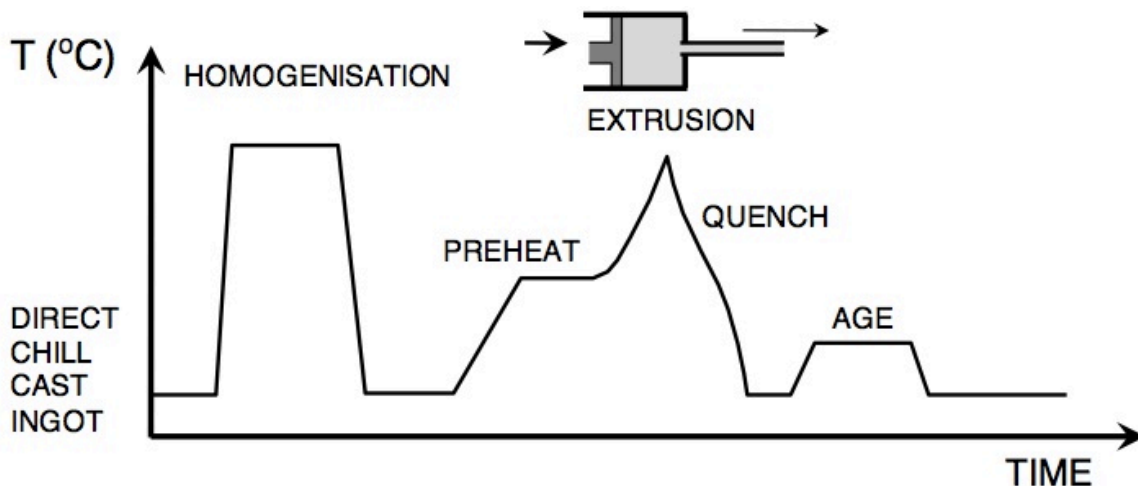
Homogenise the starting material (cast ingot) to chemically equilibrate cast structure, dissolve precipitates.

Preheat and then start extrusion: plastic deformation and die friction cause further heating. Extrusion takes place in single-phase solid solution region of phase diagram.

Dynamic recrystallisation takes place during the hot working.

Quench after extrusion to retain solid solution.

Artificial ageing to form fine distribution of precipitates.



Key parameters and characteristics:

- section geometry: governs the cooling rate, particularly important for the quench stage. i.e. distance from surface to point of slowest cooling (so section thicknesses, position of internal hollows etc).

- process history: billet pre-heat temperature, uniformity of deformation heating and temperature rise, whether fully solutionised, cooling process after extrusion, ageing temperature and time. Recrystallisation occurs during extrusion; grain size is affected by thermomechanical history and presence of dispersoids (particle stimulated nucleation of recrystallisation).

- alloy composition and microstructure: composition of heat-treatable alloy determines artificial ageing response, presence of dispersoids from earlier process steps may lead to quench sensitivity (coarse non-hardening precipitation during cooling, depending on imposed cooling rate)

Overall effect: cooling rate imposed across whole profile needs to avoid C-curves for coarse precipitation, to retain 100% supersaturated solid solution and achieve full potential peak hardening during artificial ageing.

*Examiner's comments:*

*All candidates answered this question.*

*Parts (a) and (b) were straightforward and very well done.*

*Answers to (c) were very poor. Many answers did not mention wrought precipitation hardened aluminium alloys. Candidates often wrote only about steels and aluminium casting alloys; a lot of them seemed to be writing down anything they knew about hardening mechanisms in metals or polymers in the vain hope that it might relate to the question. Some answers referred to residual stresses, as in question 2.*

*A particular area of confusion in answers that did talk about aluminium alloys was to say that dispersoids formed during ageing and were responsible for the hardening as well as for nucleation of recrystallisation.*