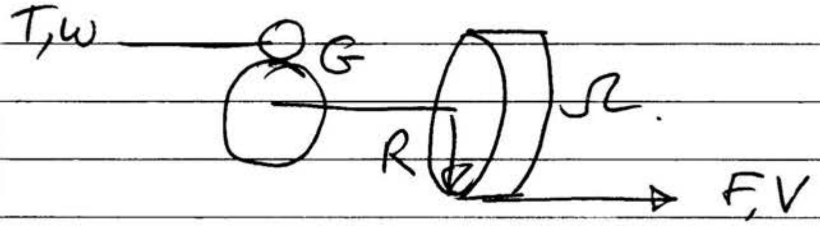
3C8 - 2016 - Solutions

1 a) Determine the load characteristic of the vehicle in terms of the torque and speed at the motor shaft.



$$F = 227 + 0.4V^2$$

power conserved $T\omega = F.V.$

$G = \frac{\omega}{\Omega}$ $\qquad F = \frac{T.\omega}{V} = \frac{T\omega}{\Omega R} = \frac{TG}{R}$

also $V = \Omega R = \frac{\omega R}{G}$

so. $\frac{T.G}{R} = 227 + 0.4\omega^2\left(\frac{R}{G}\right)^2 \qquad G = 5,\ R = 0.3\text{m}$

$$T_{vehicle} = 13.62 + 8.64.10^{-5}\omega^2.$$

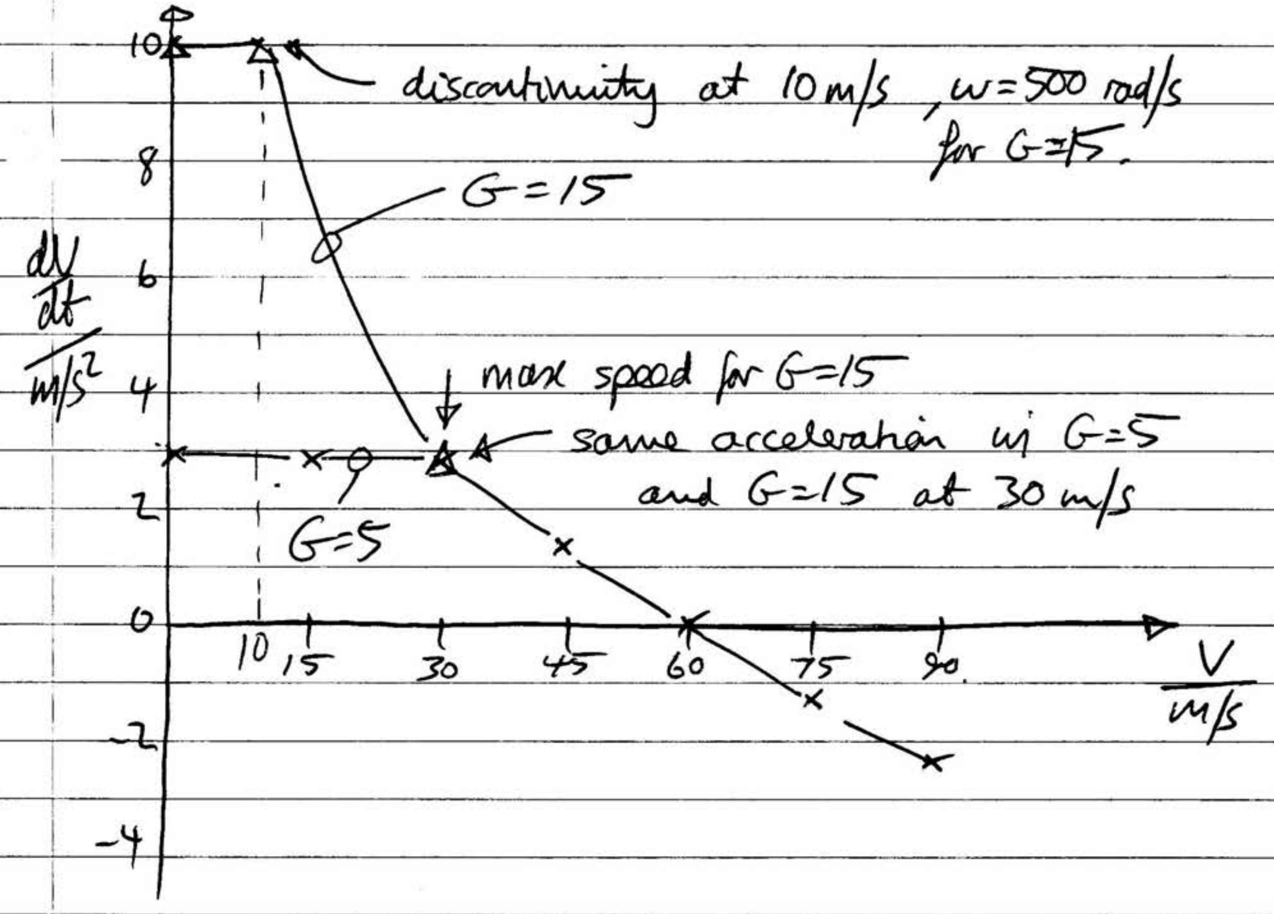
plot this on top of the motor characteristic to find that the operating point at $\omega = 1000$ rad/s

hence max vehicle speed $V = \frac{\omega R}{G} = \frac{1000.0.3}{5} = 60\text{ m/s}$

ω (rad/s)	0	250	500	750	1000	1250	1500
$T_{vehicle}$ (Nm)	14	19	35	61	100	149	208.
T_{motor} (Nm)	200	200	200	133	100	80	67
ΔT (Nm)	186	181	165	72	0	−69	−141
$\frac{dV}{dt}$ (m/s^2)	3.1	3.0	2.8	1.2	0	−1.2	−2.4
V (m/s)	0	15	30	45	60	75	90.

b) Find excess torque $\Delta T = T_{motor} - T_{vehicle}$, then

$$\frac{dV}{dt} = \frac{\Delta T . G}{MR} = \frac{\Delta T . 5}{1000 . 0.3} = \frac{\Delta T}{60}$$, see table



c) Points A & B both lie on the constant power part of the curve, so the max vehicle speed is the same as (a).

Point A – G = 2.5 (motor speed is half that in (a))
- excess torque and consequent acceleration are lower than in (a)

Point B – G = 7.5 (motor speed is 1.5 times that in (a))
- excess torque and consequent acceleration is higher than in (a)
- motor is at maximum speed, hence no possibility for higher speed downhill.

d) i) for $G = 15$, $T = 4.54 + 3.2 . 10^{-6} \omega^2$

at $\omega = 1500$ rad/s, $T_{vehicle} = 11.74$ Nm
$T_{motor} = 67$ Nm.

$$V = \frac{\omega R}{G} = \frac{1500 . 0.3}{15} = \underline{\underline{30 m/s}}.$$

$$\frac{dV}{dt} = \frac{\Delta T G}{mR} = \frac{(67 - 11.74) 15}{1000 . 0.3} = \underline{\underline{2.8 \ m/s^2}}$$

Hence same acceleration as for $G = 5$ at same vehicle speed (30 m/s).

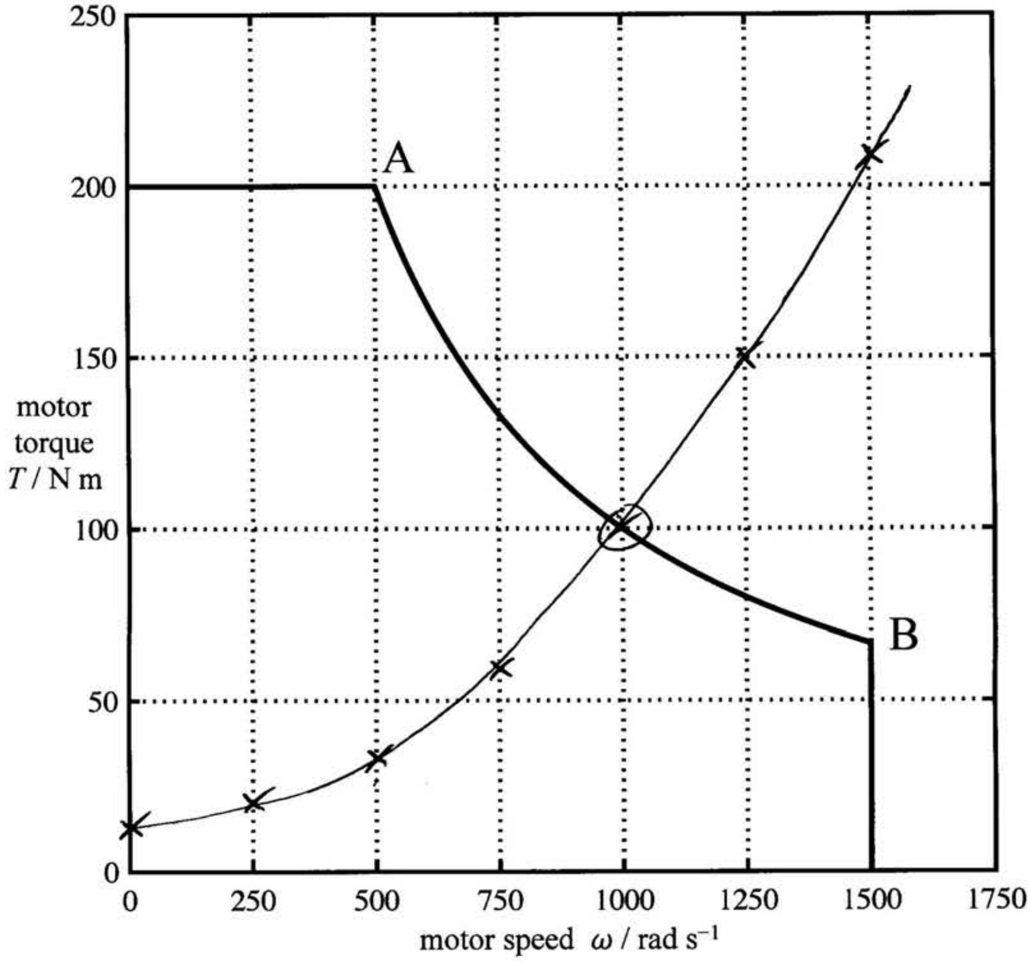
ii) see plot on previous page.
For minimum time to accelerate from rest to max vehicle speed, use $G = 15$ from 0 m/s to 30 m/s, then $G = 5$ from 30 m/s to 60 m/s.

Version DJC/1

EGT2 Candidate Number:

ENGINEERING TRIPOS PART IIA

???day ?? April 2016, Module 3C8, Question 1.



Extra copy of Fig. 1: Motor output characteristic for Question 1.

2 a) from data book.

$$p_0 = \left\{ \frac{P' E^*}{\pi R} \right\}^{\frac{1}{2}}$$

but $\frac{1}{R} = \frac{2}{d} + \frac{2}{d} = \frac{4}{d} \quad \therefore R = \frac{d}{4}$

and $P' = p.d$

and $\frac{1}{E^*} = \frac{1-\nu_1^2}{E_1} + \frac{1-\nu_2^2}{E_2} = \frac{2(1-\nu^2)}{E} = \frac{2}{E'}$

$\therefore E^* = \frac{E'}{2}$

hence $p_0 = \left\{ p.d. \frac{E'}{2} . \frac{1}{\pi} . \frac{4}{d} \right\}^{\frac{1}{2}} = \underline{\underline{\left\{ \frac{2pE'}{\pi} \right\}^{\frac{1}{2}}}}$

b) total contact area is $n.2b.L$

where $L = n.d$.

and $b = 2 \left\{ \frac{P'R}{\pi E^*} \right\}^{\frac{1}{2}}$ (data sheet).

$$= 2 \left\{ p.d. \frac{d}{4} . \frac{1}{\pi} . \frac{2}{E'} \right\}^{\frac{1}{2}}$$

$$= \sqrt{2}\sqrt{p}\, d \frac{1}{\sqrt{\pi E'}} = d \sqrt{\frac{2p}{\pi E'}}$$

$\therefore$ area $= n. 2d \sqrt{\frac{2p}{\pi E'}}\, n\, d$

$$= \underline{\underline{2n^2 d^2 \sqrt{\frac{2p}{\pi E'}}}}$$

c) i) total force is $p.L^2 = p.(nd)^2$

force per contact is (n² contacts) $P = \frac{p\, n^2 d^2}{n^2} = pd^2$

from data sheet for circular contact $p_0 = \frac{1}{\pi}\left\{\frac{6PE^{*2}}{R^2}\right\}^{\frac{1}{3}}$

$R = \frac{d}{2}$ (crossed cylinders)

$$\therefore \quad p_0 = \frac{1}{\pi}\left\{\frac{6\, pd^2 E'^2\, 4}{d^2\, 4}\right\}^{\frac{1}{3}} = \underline{\underline{\frac{1}{\pi}\left\{6pE'^2\right\}^{\frac{1}{3}}}}$$

ii) contact area.

data sheet : $a = \left\{\frac{3PR}{4E^*}\right\}^{\frac{1}{3}}$

$$= \left\{3\, pd^2\, \frac{d}{2}\, \frac{1}{4}\, \frac{2}{E'}\right\}^{\frac{1}{3}}$$

$$= \left\{\frac{3}{4}\frac{pd^3}{E'}\right\}^{\frac{1}{3}}$$

area of each contact $= \pi a^2 = \pi\left\{\frac{3}{4}\, p\frac{d^3}{E'}\right\}^{\frac{2}{3}}$

total area $= n^2 \pi a^2 = n^2 \pi\left\{\frac{3}{4}\frac{pd^3}{E'}\right\}^{\frac{2}{3}}$

$$= \underline{\underline{n^2 d^2\, \pi\left\{\frac{3}{4}\frac{p}{E'}\right\}^{\frac{2}{3}}}}$$

comment

$$\text{ratio} \quad \frac{p_{line}}{p_{point}} = \left\{\frac{2pE'}{\pi}\right\}^{\frac{1}{2}} \frac{\pi}{\left\{6pE'^2\right\}^{\frac{1}{3}}}$$

$$= \sqrt{2\pi}\; 6^{-\frac{1}{3}} \left\{\frac{p}{E'}\right\}^{\frac{1}{6}} = 3.46 \left(\frac{p}{E'}\right)^{\frac{1}{6}}$$

$$\text{ratio} \quad \frac{\text{area}_{line}}{\text{area}_{point}} = \frac{2n^2 d^2}{n^2 d^2 \pi} \sqrt{\frac{2p}{\pi E'}} \cdot \left\{\frac{3}{4}\frac{p}{E'}\right\}^{-\frac{2}{3}}$$

$$= \frac{2\sqrt{2}}{\pi\sqrt{\pi}} \left\{\frac{4}{3}\right\}^{\frac{2}{3}} \left\{\frac{p}{E'}\right\}^{-\frac{1}{6}} = 0.62 \left\{\frac{p}{E'}\right\}^{-\frac{1}{6}}$$

Although it might be expected that the point contact case has a much higher pressure and lower area, in fact this depends on the value of P/E', with the break-even point being:

$$P/E' = 5.8 \cdot 10^{-4} \text{ for pressure}$$

$$P/E' = 0.057 \text{ for area.}$$

with values of P/E' smaller than this giving higher pressure and area for the line contact.

Typically we expect P/E' to be very small to avoid plasticity.

d) compliance

data sheet gives $\delta = \frac{a^2}{R} = \frac{1}{2}\left\{\frac{9}{2}\frac{P^2}{E^{*2}R}\right\}^{\frac{1}{3}}$

$$\delta = \frac{1}{2}\left\{\frac{9}{2}\frac{2^2 p^2 d^4\, 2}{E'^2\, d}\right\}^{\frac{1}{3}}$$

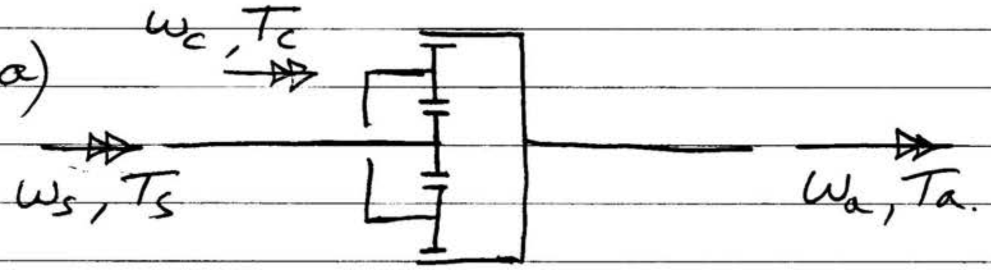
for one layer, but there are n rods,
so total deflection Δ is $\delta(n-1)$

$$\Delta = \frac{(n-1)}{2}\left\{\frac{36\, p^2 d^3}{E'^2}\right\}^{\frac{1}{3}}$$

$$\Delta = \frac{(n-1)}{2}\,\frac{36^{\frac{1}{3}}\, p^{\frac{2}{3}}\, d}{E'^{\frac{2}{3}}}$$

compliance $\frac{\Delta}{p} = \frac{(n-1)}{2}\, 36^{\frac{1}{3}}\,\frac{d}{p^{\frac{1}{3}} E'^{\frac{2}{3}}}$

3. a)



power conservation $\omega_s T_s + \omega_c T_c + \omega_a T_a = 0$

Find $\frac{T_s}{T_a}$, set $\omega_c = 0$ $\therefore$ $\omega_s T_s = -\omega_a T_a$

$$\frac{T_s}{T_a} = -\frac{\omega_a}{\omega_s}$$

epicyclic speed rule

$$\omega_s = (1+R)\omega_c - R\omega_a \qquad R = \frac{A}{S} = \frac{96}{24} = 4$$

$\omega_c = 0$ $\therefore$ $\omega_s = -R\omega_a$

$$\frac{-\omega_a}{\omega_s} = \frac{1}{R} = \frac{1}{4} = \underline{\underline{\frac{T_s}{T_a}}}$$

torque equilibrium $T_s + T_c + T_a = 0$

$\div T_a$ $\qquad \frac{T_s}{T_a} + \frac{T_c}{T_a} + 1 = 0.$

$$\frac{T_c}{T_a} = -\frac{T_s}{T_a} - 1 = -\frac{1}{4} - 1 = \underline{\underline{-\frac{5}{4}}}$$

b) $\quad P_s = \omega_s T_s$

$$T_s = \frac{P_s}{\omega_s} = \frac{50.10^3\ \text{W} \cdot 60\ \text{s/min}}{5000\ \text{revs/min}\ 2\pi\ \text{rad/rev}} = \frac{300}{\pi}\ \text{Nm}$$

from (a) $\quad T_a = 4T_s = \underline{\underline{\frac{1200}{\pi}}}\ \text{Nm}$

$$T_c = -\frac{5}{4}T_a = -5T_s = \underline{\underline{-\frac{1500}{\pi}}}\ \text{Nm.}$$

$\therefore \quad P_c = \omega_c T_c = \left(+2000 \cdot \frac{2\pi}{60}\right) \frac{1500}{\pi} \quad W$

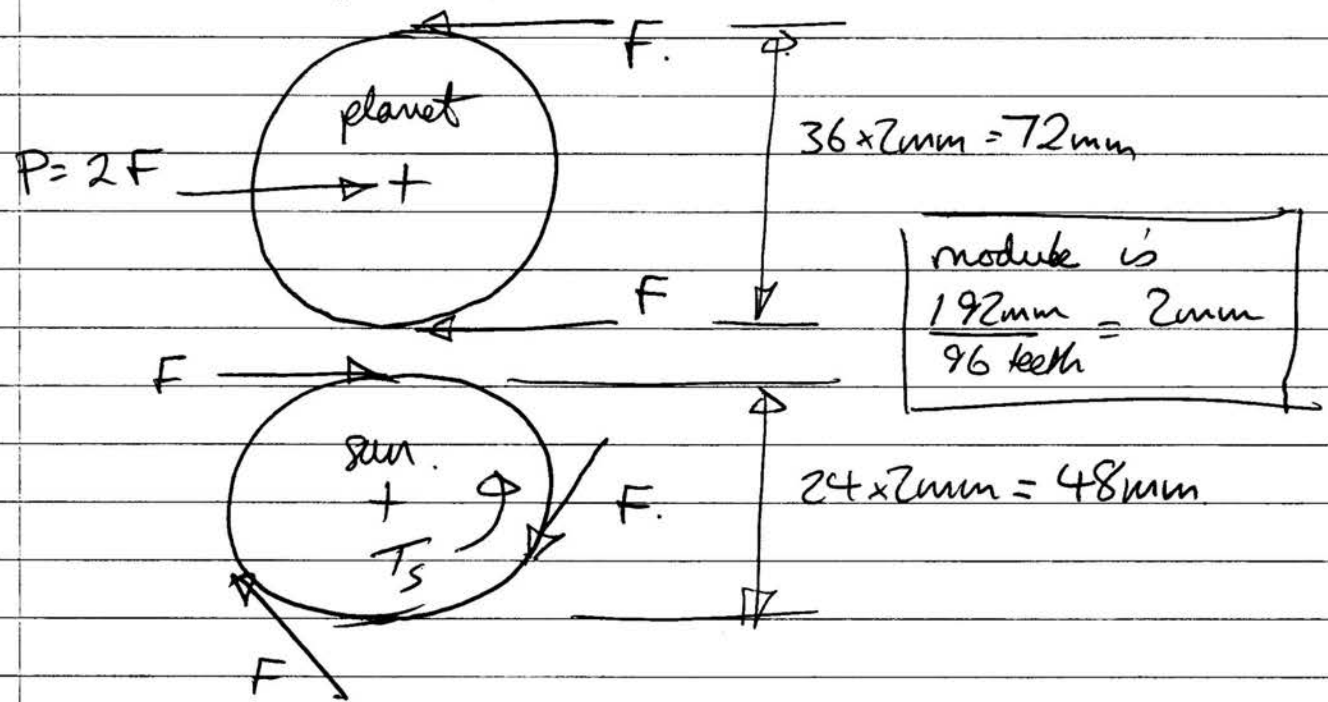
$\underline{\underline{P_c = 100\,kW}}$ into carrier

$P_s + P_c + P_a = 0$

$\therefore \; P_a = -P_c - P_s = -100\,kW - 50\,kW$

$\underline{\underline{P_a = -150\,kW}}$ i.e. 150 kW out of annulus

c) Find radial force on planet wheel bearing
Free body diagrams of planet and sun:



equilibrium of sun $\quad T_s = 3F \cdot 0.024\,m$

$3F = \frac{300\,Nm}{\pi \cdot 0.024\,m} = \frac{12500}{\pi}\,N$

radial force on planet bearing $P = 2F = \frac{12500}{\pi} \cdot \frac{2}{3}\,N$

$= \underline{\underline{2653\,N}}$

find rotation speed of bearing
need the speed of the planet relative to carrier
first find speed of annulus using epicyclic speed rule

$$\omega_s = (1+R)\omega_c - R\omega_a$$

$$\therefore \omega_a = \frac{(1+R)\omega_c - \omega_s}{R} = \frac{5\omega_c - \omega_s}{4}$$

$$= \frac{-5.2000 - 5000}{4} = -3750 \text{ rpm}$$

tabular method to deduce speeds when carrier is brought to a halt

	sun	carrier	annulus
speed / rpm	5000	-2000	-3750
add 2000 rpm	+2000	+2000	+2000
total	7000	0	-1750

thus planet speed $|\omega_{planet}|$ is $\omega_{sun} \cdot \frac{N_s}{N_p}$

$$= 7000 \cdot \frac{24}{36}$$

$$= 4667 \text{ rpm}$$

The number of revolutions in 1000 hours is
$4667 \text{ rpm} \times 60 \text{ mins/hr} \times 1000 \text{ hrs} = 280 \cdot 10^6 \text{ revs}$
thus $L = 280$ Mrevs.

Bearing life equation $L = a_1 a_{23} \left(\frac{C}{P}\right)^p$

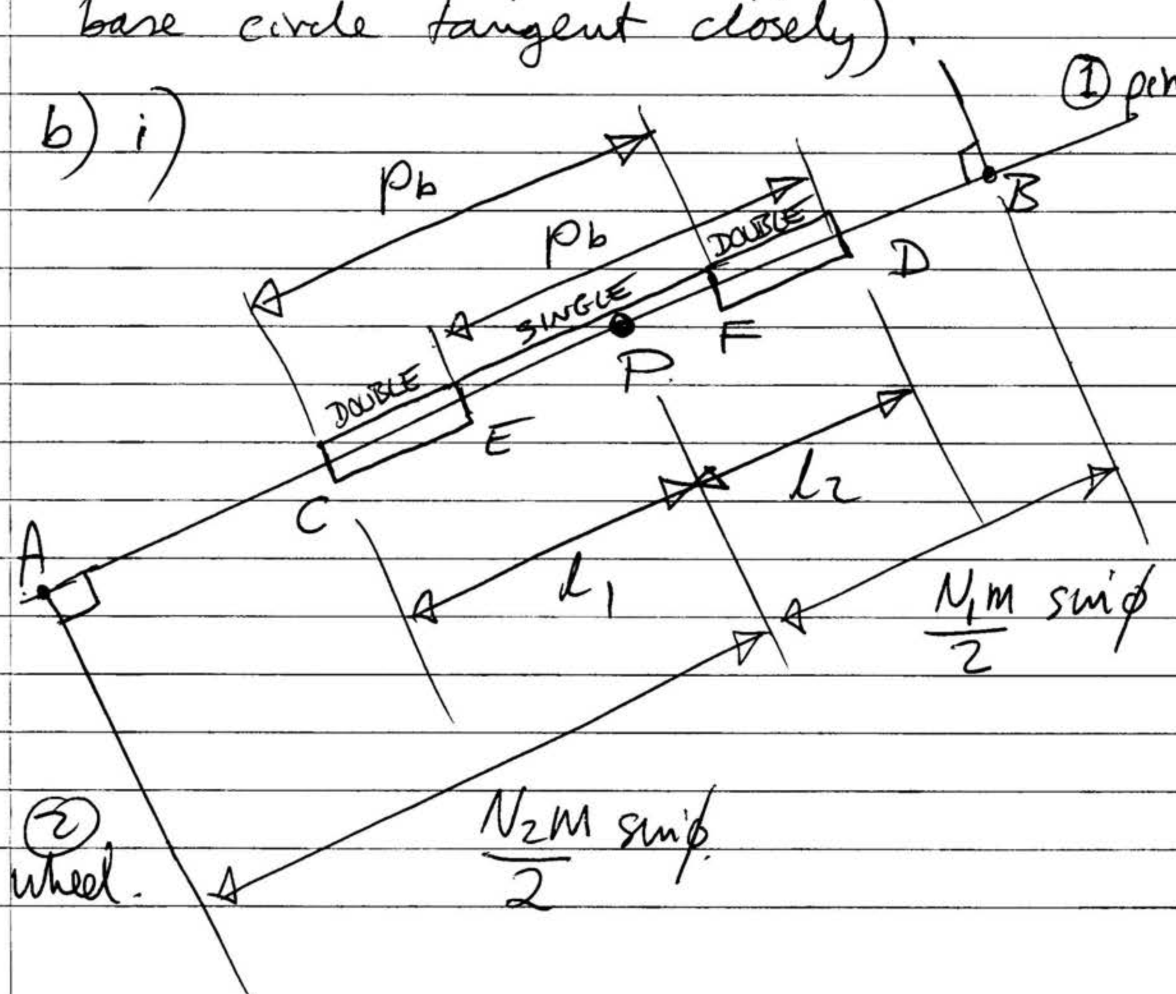
$p = \frac{10}{3}$ for roller brg, $a_{23} = 1$ for ideal lubrication,
$a_1 = 0.21$ for 1% prob failure, $P = 2653N$, $L = 280$
thus $C = 2653 \left(\frac{280}{0.21}\right)^{\frac{3}{10}} = 23 \text{ kN}$

4. a) A balanced design is one for which the allowable contact stress and allowable bending stress are achieved with the same torque.

In practice it is preferable for the gears to fail through contact stress rather than bending stress because a bending stress failure might result in tooth breakage and catastrophic failure of the whole gearbox. Contact stress usually results in progressive deterioration.

Increasing the module reduces the bending stress but does not affect the contact stress (except at very small tooth numbers, when the critical contact point approaches the base circle tangent closely).

b) i)



point D is critical double contact
point F is critical single contact

$$AD = AP + l_2 = \frac{N_2 m}{2}\sin\phi + l_2 = \frac{60.3}{2}.\sin 20 + 7.90 = \underline{\underline{38.69\,mm}}$$

$$DB = PB - l_2 = \frac{N_1 m}{2}\sin\phi - l_2 = \frac{31.3}{2}\sin 20 - 7.90 = \underline{\underline{8.00\,mm}}$$

$$\therefore \frac{1}{R_D} = \frac{1}{AD} + \frac{1}{DB} = \frac{1}{38.69} + \frac{1}{8.00} = \underline{\underline{6.63\,mm}}$$ critical double contact

$$AF = AP - l_1 + p_b = \frac{N_2}{2} m \sin\phi - l_1 + \pi m \cos\phi$$

$$= \frac{60.3}{2}\sin 20 - 7.35 + \pi.3\cos 20 = \underline{\underline{32.28\,mm}}$$

$$FB = AB - AF$$

$$= \frac{(N_1+N_2)}{2} m \sin\phi - AF = \frac{(31+60)}{2}3.\sin 20 - 32.28$$

$$= \underline{\underline{14.4\,mm}}$$

$$\therefore \frac{1}{R_F} = \frac{1}{AF} + \frac{1}{FB} = \frac{1}{32.28} + \frac{1}{14.4} = \underline{\underline{9.96\,mm}}$$ critical single contact.

ii) contact stress - assume line contact.

data sheet: $p_0 = \sqrt{\frac{P'E^*}{\pi R}}$

single contact $p_0 \propto \sqrt{\frac{P'}{R_F}}$, double contact (load sharing) $p_0 \propto \sqrt{\frac{P'}{2R_D}}$

hence worst case is single contact because

$9.96\,mm < 2 * 6.63\,mm$

allowable $P' = \frac{p_0^2 \pi R}{E^*} = \frac{(1200.10^6)^2\,\pi\, 9.96.10^{-3}}{115.10^9}$

$= 391.8\ kN/m.$

hence line force is $P'w = 391.8.10^3 \times 0.03$

$= 11754.3\ N$

pinion torque T_1 is line force * base circle radius

$$T_1 = 11754.3 * \frac{N_1 m \cos\phi}{2}$$

$$= 11754.3 \, . \, \frac{31 \, . \, 0.003 \, . \, \cos 20}{2}$$

$$\underline{\underline{T_1 = 513.6 \text{ Nm}}}$$ for contact stress

iii) bending stress, pinion is critical

data sheet $\sigma_b = \frac{P'_T}{Jm}$ where $P'_T = P' \cos\phi$

$$\therefore P'_T = \sigma_b Jm = 300.10^6 . 0.39 . 0.003$$

$$= 351 \text{ kN/m}$$ ← from table.

$$P_T = P'_T W = 351.10^3 \; 0.03$$

$$= 10.53 \text{ kN}$$

pinion torque $T_1 = P_T . \frac{N_1 m}{2}$

$$= 10.53.10^3 . \frac{31 . 0.003}{2}$$

$$\underline{\underline{T_1 = 490 \text{ Nm}}}$$ for bending stress

iv) Comparing results in (ii) and (iii) failure occurs at a slightly lower torque for bending failure. Thus the design is almost balanced, but an increase in module, maintaining gear diameters, would be desirable to ensure failure by contact stress.

ENGINEERING TRIPOS PART IIA 2016

ASSESSOR'S COMMENTS, MODULE 3C8: MACHINE DESIGN

Q1 Power matching

Calculation of maximum speed was done well, either by solving cubic or plotting load characteristic. The graph of maximum acceleration was often poorly annotated, lacked sufficient data points, or omitted negative values of acceleration. Many incorrectly assumed that a load characteristic passing through point A led to greater acceleration performance than point B. In the final part, many incorrectly assumed that the motor could operate beyond 1500 rad/s.

Q2 Contact mechanics

The least popular question, perhaps due to unfamiliar setting. Many candidates didn't distinguish between E' and E*. The effective radius of curvature was often calculated incorrectly (confusion over circular and line contacts). Answers were often dimensionally incorrect.

Q3 Epicyclic gear and bearing

The calculation of torque ratios was generally done well. Errors were sometimes made in the calculation of power (inconsistent sign conventions). The last part of the question on bearing calculation was challenging: there were frequent errors in determining the bearing speed and the radial force. Most candidates used the bearing life equation correctly.

Q4 Spur gears

Most candidates understood what was meant by 'balanced design'. There were some misconceptions about the effect of module on tooth bending and contact stress. Calculation of radius of curvature sometimes went wrong due to incomplete understanding of gear geometry (l_1, l_2, r_1, r_2 in wrong places, and single/double contact regions wrong).

D J Cole (Principal Assessor)