ENGINEERING IIA, 3C8 MACHINE DESIGN. 2017. CRIB (M SUTCLIFFE)

1(a) - Instrument bike to measure crank speed & torque
- wear heart rate monitor as measure of work done
- use interpolation to bracket target heart rate (HR)
- vary brake torque, find speed for target HR
- plot torque-speed for range of HRs
- power for T-ω plot

(b) Max speed ⇒ max power

$P = \omega_0 T_0 (1 - \omega/\omega_0)$; $\frac{dP}{d\omega} = 0 \Rightarrow T_0 = \frac{2\omega T_0}{\omega_0}$, $\omega = \frac{\omega_0}{2}$, $P = \frac{T_0 \omega_0}{4}$

Max power = 80 Nm × 20 rad/s / 4 = 400 W

Locate this contour on graph. (much easier to do this question graphically)

- At α = 0, intersection of load line and P = 400 W line ⇒ $V \approx 12$ m/s

Now $V = \Omega R$, $G = \omega / \Omega$ ⇒ $G = \frac{\omega R}{V}$

At max power ω = 10 rad/s ⇒ $G_1 = \frac{10 \times 0.33}{12} \approx 0.275$

(many people got lost going down analytical route)

- For α = 0.1, lift load line by mgα = 100N

New intersection with 400 W line ⇒ $V \approx 4$ m/s, $G_2 = 0.825$

- α = -0.1, reduce load line by 100N ⇒ $V \approx 22$ m/s $G_3 \approx 0.15$

Total time = $10{,}000\left(\frac{1}{12} + \frac{1}{4} + \frac{1}{22}\right) = 3788$ s ≈ 63 minutes (65 minutes by analytical soln)

(c) Add rider characteristic to graph for G = 0.66

using $F_{max} = T_{max} G/R$, $V_{max} = \omega_{max} R/G$ → line on graph

- for α = 0, intersection to rider characteristic and load line at $V \approx 8.7$ m/s
- α = 0.1 " V = 3.6 m/s
- α = -0.1 " V = 14 m/s

(This step almost always missed especially when not using graph)

But for α = -0.1, F is negative. But freewheel limits F to +ve and instead freewheel gives F = 0, V = 20 m/s for this case.

New time = $10{,}000\left(\frac{1}{8.7} + \frac{1}{3.6} + \frac{1}{20}\right) \approx 74$ minutes

(d) Energy expended against air drag increases as V^2, so better to increase work input on uphill stages and reduce input going downhill to expend energy to better effect.

Version MPFS/4

Candidate Number:

EGT2
ENGINEERING TRIPOS PART IIA
Thursday 27 April 2017, Module 3C8, Question 1.

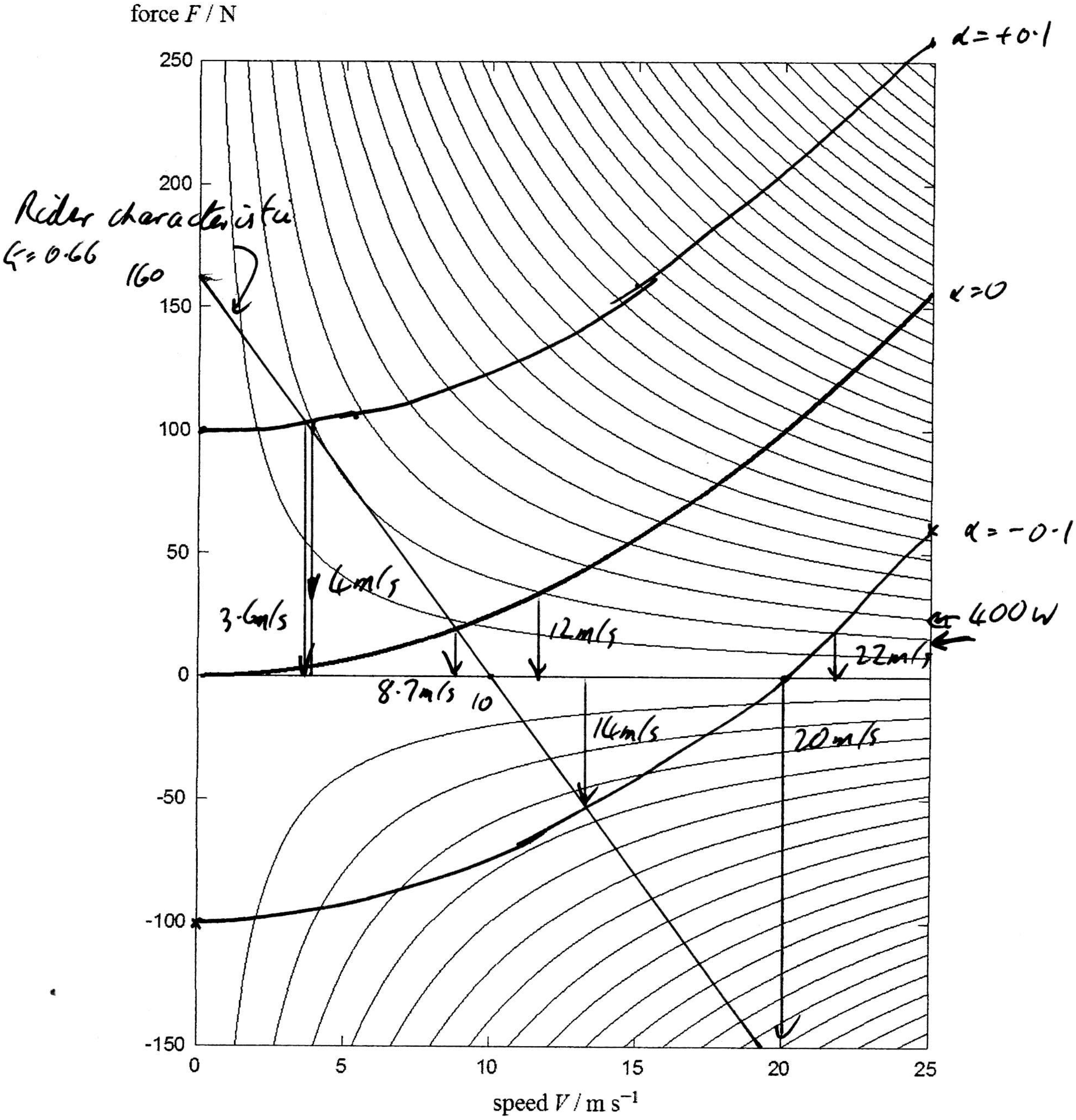
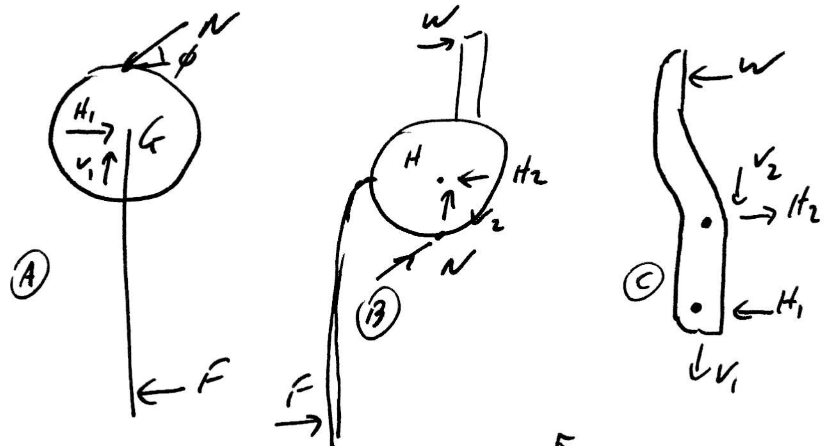


Fig. 2

2. (a)



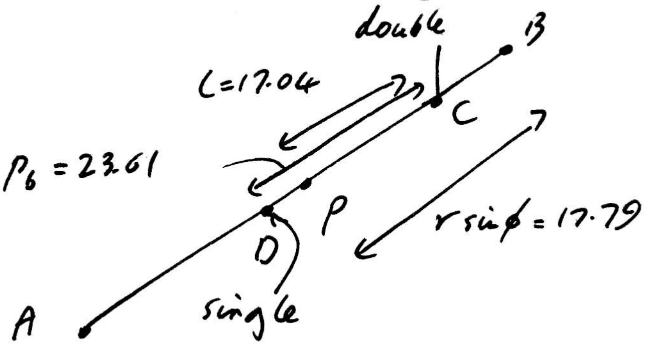
Needs careful FBD

(A): $M_G \circlearrowright \quad rN\cos\phi = 10rF$ (B) $M_H \circlearrowleft \quad 12rF + Nr\cos\phi - 2rW = 0$

Eliminate $N \Rightarrow W = 11F = \underline{1100\,N}$

(b) $N = 10F/\cos\phi = \underline{1064\,N}$

(c)



$\frac{l}{m} = (0.02926\,N^2 + N + 1)^{\frac{1}{2}} - 0.1710\,N$

$\Rightarrow l = 17.04\,mm$

where $N = 2r/m = 104/8 = 13$

$p_b = \frac{2\pi r\cos\phi}{N} = 23.61\,mm$

$E^* = \frac{1}{2}\left(\frac{E}{1-\nu^2}\right) = 39.3\,GPa$

N.B dimensions in mm, symmetrical

Double contact: $\frac{1}{R} = \frac{1}{17.79-17.04} + \frac{1}{17.79+17.04} \Rightarrow R = 0.73\,mm$

Single contact $\frac{1}{R} = \frac{1}{17.79+17.04-23.61} + \frac{1}{17.79-17.04+23.61} \Rightarrow R = 7.68\,mm$

(d) ~~R~~ $R_{DD}\times 2 < R_S$ so double contact is critical

$$p_0 = \left(\frac{P'E^*}{\pi R}\right)^{\frac{1}{2}} = \left(\frac{\frac{1064}{2\times10^{-2}} \times 39.3\times10^{9}}{\pi \times 0.73\times10^{-3}}\right)^{\frac{1}{2}} = 0.95\,GPa$$

2 contacts

facewidth

~~Reduce R~~ Increase R by having more teeth.

Increase face width.

In principle could reduce E but likely to reduce strength too.

Well answered question

3.

	S_1	C	A	S_2	P_1	P_2	
for(a) →	X	0	$\frac{XS_1}{P_1}\cdot\frac{P_1}{A}$	$-\frac{XS_1}{P_1}\cdot\frac{P_1}{S_2}$	$-X\frac{S_1}{P_1}$	$+X\frac{S_1}{P_1}$	carrier stationary
for (b) and (c) ↘	$X+Y$	Y	$Y+\frac{XS_1}{A}$	$Y-\frac{XS_1}{S_2}$			add arbitrary carrier turning

← speeds

tooth numbers

{few people used table ⇒ not well answered}

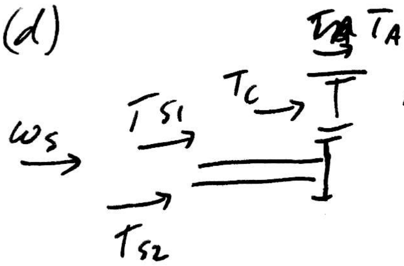
(a) $\left(\frac{A}{S_1}\right)_{\text{speeds}} = \frac{XS_1}{A}\Big/X = \frac{S_1}{A} = \frac{15}{36} = \frac{5}{12}$

(b) $(S_2)_{\text{speed}} = 0 \Rightarrow Y = X\frac{S_1}{S_2}$ $\quad \left(\frac{A}{S_1}\right)_{\text{speeds}} = \dfrac{\frac{S_1}{S_2} + \frac{S_1}{A}}{1+\frac{S_1}{S_2}} = \dfrac{\frac{30}{36}+\frac{30}{72}}{1+\frac{30}{36}} = \frac{15}{22}$

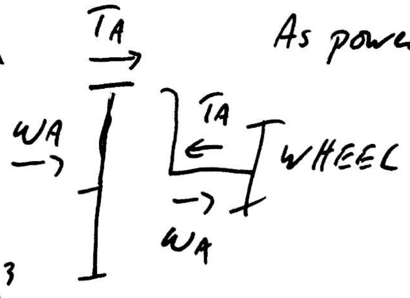
(c) $(S_1 = S_2)_{\text{speed}} \Rightarrow X+Y = Y - X\frac{S_1}{S_2} \Rightarrow X = 0 \Rightarrow \left(\frac{A}{S_1}\right)_{\text{speeds}} = 1$

{(a) is possible to do by chasing speeds, (b) and (c) need to use concept of superimposing carrier rigid body motion best done using table. Epicyclic speed equation cannot be used.}

(d)



Forces on shafts



As power out to wheel
$T_A\omega_A < 0$

$T_{S1} = T_i = P/\omega = \dfrac{50\times10^3}{2000\times2\pi/60} = 239\ \text{Nm}$

$|T_A| = \dfrac{48\times10^3}{2000\times\frac{2\pi}{60}\times\frac{15}{22}} = 337\ \text{Nm}$, but this is −ve with chosen sign convention

↖ ω_A

$T_{S1} + T_{S2} + \cancelto{0}{T_C} + T_A = 0 \Rightarrow T_{S2} = -T_{S1} - T_A = 337 - 239 = 98\ \text{Nm}$

{Care needed to get signs right}

(e) Hold carrier, input to S_2, S_1 free $\Rightarrow \left(\frac{A}{S_2}\right)_{\text{speed}} = -\dfrac{S_1/A}{S_1/S_2}$ (see table)

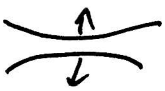
4 (a) Geometry:

$\equiv$

$z_1 = \frac{x}{2R_1}$ $\quad z_2 = \frac{x}{2R_2}$ $\qquad z_1 + z_2 = \frac{x}{2}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{x}{2}\left(\frac{1}{R}\right)$

equivalent R

Elasticity:



Equal and opposite forces: can sum deflections to give equivalent modulus E^*

(b) (i) Here δ is approach of centres.

$E^* = E'$
$R = R$
$P = \frac{4}{3}\pi R^3 \rho g$

From datasheet $\delta = \frac{1}{2}\left(\frac{9}{2}\frac{P^2}{E^*R}\right)^{\frac{1}{3}}$

$$\Rightarrow \frac{d\delta}{dP} = \frac{1}{2}\left(\frac{9}{2}\frac{1}{E^*R}\right)^{\frac{1}{3}} \times \frac{2}{3} P^{-\frac{1}{3}}$$

$$= \frac{1}{2}\left(\frac{9}{2}\frac{1}{E^*R}\right)^{\frac{1}{3}} \cdot \frac{2}{3}\left(\frac{3}{4\pi R^3 \rho g}\right)^{\frac{1}{3}} \Rightarrow \frac{dP}{d\delta} = 2(\pi R^4 E'^2 \rho g)^{1/3}$$

(ii) $$\tau_{max} = 0.310\, p_0 = \frac{0.310}{\pi}\left(\frac{6PE'^2}{R^2}\right)^{\frac{1}{3}}$$

$R = 30\,mm$
$E' = 230\,GPa$
$P = \frac{4}{3}\pi(20\times10^{-3})^3 \times 7000 \times 9.81$
$= 8.654\,N$

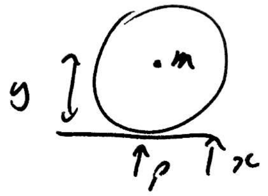
$$= \frac{0.310}{\pi}\left(\frac{6\times 8.654 \times (230\times10^9)^2}{(30\times10^{-3})^2}\right)^{\frac{1}{3}}$$

$$= 0.143\,GPa$$

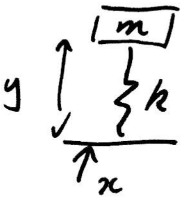
Put $\tau_{max} = \tau_y$ and use the approximation $\tau_y \simeq$ Hardness/6

$\Rightarrow$ need hardness $> 858\,MPa$ This step missed

(iii)



$\rightarrow$



$\leftarrow$ contact stiffness $k = \frac{dP}{d\delta}$ at equilibrium position, assumed constant for small oscillations

Case (b) with no damping

We want $|Y|/|X| < \frac{0.1}{2} = 0.05$

$m = \frac{4}{3}\pi R^3 \rho = 0.882\,kg$

$k = 2\left(\pi \times (30\times10^{-3})^4 \times (230\times10^9)^2 \times 7800 \times 9.81\right)^{\frac{1}{3}}$

$= 6.35\times10^7\,N/m$

$\omega_n = \sqrt{\frac{k}{m}} = 7022\,rad/s$

$$0.05 = \frac{(\omega/\omega_n)^2}{1-\left(\frac{\omega}{\omega_n}\right)^2} \Rightarrow \left(\frac{\omega}{\omega_n}\right)^2 < \frac{0.05}{1.05} = 0.0476$$

$$\omega < 0.218\,\omega_n < 1531\,rad/s$$
$$= 244\,Hz$$

b(i) and (ii) well answered but not (a) and b(iii)